Gray code compression

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Abstract

An n-bit (cyclic) Gray code is a (cyclic) sequence of all n-bit strings such that consecutive strings differ in a single bit. We describe a construction of a cyclic n-bit Gray code whose graph of transitions is the d-dimensional hypercube Q_d if $n = 2^d$, or a subgraph of Q_d if $2^{d-1} < n < 2^d$. This provides a description of an algorithm which, given a positive integer n, constructs a transitional sequence of an n-bit code with the desired property. Following the inductive construction, the running time $T(n)$ of the algorithm may be expressed as

$$T(n) = \begin{cases} T(n/2) + O(2^n) & \text{if } n = 2^d \text{ and } d > 2, \\ T(n-1) + O(2^n) & \text{if } 2^{d-1} < n < 2^d \text{ and } d > 2, \\ O(1) & \text{if } n \leq 4. \end{cases}$$

Consequently, the time complexity of our construction is bounded by $O(N)$, where $N = 2^n$ is the output size, i. e., only constant amortized time is required per one element of the output sequence. Thus, such a construction allows to compress sequences that follow this code so that only $\Theta(\log \log n)$ bits per n-bit string are needed.

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