

# Gray code compression

Darko Dimitrov

Institut für Informatik, Freie Universität Berlin,  
Takustraße 9, D-14195 Berlin, Germany  
darko@inf.fu-berlin.de

## Abstract

An  $n$ -bit (cyclic) Gray code is a (cyclic) sequence of all  $n$ -bit strings such that consecutive strings differ in a single bit. We describe a construction of a cyclic  $n$ -bit Gray code whose graph of transitions is the  $d$ -dimensional hypercube  $Q_d$  if  $n = 2^d$ , or a subgraph of  $Q_d$  if  $2^{d-1} < n < 2^d$ . This provides a description of an algorithm which, given a positive integer  $n$ , constructs a transitional sequence of an  $n$ -bit code with the desired property. Following the inductive construction, the running time  $T(n)$  of the algorithm may be expressed as

$$T(n) = \begin{cases} T(n/2) + O(2^n) & \text{if } n = 2^d \text{ and } d > 2, \\ T(n-1) + O(2^n) & \text{if } 2^{d-1} < n < 2^d \text{ and } d > 2, \\ O(1) & \text{if } n \leq 4. \end{cases}$$

Consequently, the time complexity of our construction is bounded by  $O(N)$ , where  $N = 2^n$  is the output size, i. e., only constant amortized time is required per one element of the output sequence. Thus, such a construction allows to compress sequences that follow this code so that only  $\Theta(\log \log n)$  bits per  $n$ -bit string are needed.

It is joint work with Tomáš Dvořák, Petr Gregor, and Riste Škrekovski.