Gray Codes Faulting Matchings

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Abstract

A (cyclic) n-bit Gray code is a (cyclic) ordering of all 2^n binary strings of length n such that consecutive strings differ in a single bit. Equivalently, an n-bit Gray code can be viewed as a Hamiltonian path of the n-dimensional hypercube Q_n, and a cyclic Gray code as a Hamiltonian cycle of Q_n. In this presentation we study Hamiltonian paths and cycles of Q_n avoiding a given set of faulty edges that form a matching, briefly called (cyclic) Gray codes faulting a given matching. Given a matching M and two vertices u, v of Q_n, $n \geq 4$, our main result provides a necessary and sufficient condition, expressed in terms of forbidden configurations for M, for the existence of a Gray code between u and v faulting M. As a corollary, we obtain a similar characterization for a cyclic Gray code faulting M. In particular, in case that M is a perfect matching, Q_n has a (cyclic) Gray code faulting M if and only if $Q_n - M$ is a connected graph. This complements a recent result of Fink, who proved that every perfect matching of Q_n can be extended to a Hamiltonian cycle. Furthermore, our results imply that the problem of Hamiltonicity of Q_n with faulty edges, which is NP-complete in general, becomes polynomial for up to 2^{n-1} edges provided they form a matching.

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