Proof of Edwards-Walsh resolution theorem without Edwards-Walsh complexes

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Edwards-Walsh resolution Theorem

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For every compact metrizable space X with $\dim_{\mathbb{Z}} X \leq n$, there exists a compact metrizable space Z and a surjective map $\pi : Z \to X$ such that π is cell-like, and $\dim Z \leq n$.

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- a resolution
- $\dim$ and $\dim_G$, for an abelian group G ($\dim_{\mathbb{Z}}$)
- a cell-like map

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- a resolution
- dim and $\dim_G$, for an abelian group G ($\dim_{\mathbb{Z}}$)
- a cell-like map (and G-acyclic map)

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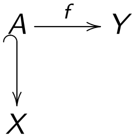

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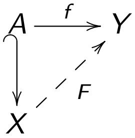

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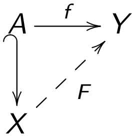

Standard notation: $Y \in \mathrm{AE}(X)$.
Also used: $\text{e-dim}\, X \leq Y$.
We will use: $X \, \tau \, Y$.

Characterization of dim and $\dim_G$ by extension of maps

Theorem

For any nonempty compact metrizable space X and $n \in \mathbb{Z}_{\geq 0}$,
$\dim X \leq n \;\Leftrightarrow\; X \tau S^n$.

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$K(G,n)$ = an Eilenberg-MacLane complex of type (G,n)
= a connected CW-complex having the property

$$\pi_i(K(G,n)) \cong \begin{cases} G & \text{if } i = n \\ 0 & \text{if } i \neq n. \end{cases}$$

How dim and $\dim_G$ relate to each other?

For a compact metrizable space X,

$$\dim_G X \leq \dim_{\mathbb{Z}} X \leq \dim X.$$

Theorem (P. S. Aleksandrov, 1936)

If X is a compact metrizable space with $\dim X < \infty$, then $\dim_{\mathbb{Z}} X = \dim X$.

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There are compact metrizable spaces with infinite dim and finite $\dim_{\mathbb{Z}}$:

• A. Dranishnikov, in 1988, constructed a compact metrizable space X with $\dim X = \infty$ and $\dim_{\mathbb{Z}} X \leq 3$

• J. Dydak and J. Walsh, in 1993, constructed a compact metrizable space X with $\dim X = \infty$ and $\dim_{\mathbb{Z}} X = 2$

Definition

A map $\pi : Z \to X$ between compact spaces is called:

- cell-like if for any CW-complex K and any $x \in X$, every map $f : \pi^{-1}(x) \to K$ is nullhomotopic,
- G-acyclic if for any $n \in \mathbb{N}$ and any $x \in X$, every map $f : \pi^{-1}(x) \to K(G, n)$ is nullhomotopic.

Clearly, $\pi : Z \to X$ is cell-like $\Rightarrow$ π is G-acyclic.

To see that a map $\pi : Z \to X$ between compact metrizable spaces is cell-like, it is sufficient to prove that, $\forall x \in X$, there is an inverse sequence (Z_i, p_i^{i+1}), of compact metrizable spaces Z_i, whose limit is (homeomorphic to) $\pi^{-1}(x)$ and such that for infinitely many $i \in \mathbb{N}$, $p_i^{i+1} : Z_{i+1} \to Z_i$ is null-homotopic.

$$Z_1 \xleftarrow{p_1^2} Z_2 \xleftarrow{p_2^3} \cdots \xleftarrow{p_{i-1}^i} Z_i \xleftarrow{p_i^{i+1}} Z_{i+1} \xleftarrow{p_{i+1}^{i+2}} \cdots \qquad \pi^{-1}(x)$$

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Z & \dim Z \leq n & \\
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X & \dim_{\mathbb{Z}} X \leq n & \dim X \text{ could be } \infty
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Note: (Surjective) cell-like map can raise dim.

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Theorem (A. Dranishnikov, 1988)

For every compact metrizable space X with $\dim_{\mathbb{Z}/p} X \leq n$, there exists a compact metrizable space Z and a surjective map $\pi : Z \to X$ such that π is $\mathbb{Z}/p$-acyclic, and $\dim Z \leq n$.

Other resolution Theorems

Dranishnikov resolution Theorem for $\mathbb{Z}/p$

Theorem (R. Edwards - J. Walsh, 1981)

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Theorem (M. Levin, 2005)

Let $n \in \mathbb{N}_{\geq 2}$. Then for every compact metrizable space X with $\dim_{\mathbb{Q}} X \leq n$, there exists a compact metrizable space Z and a surjective map $\pi : Z \to X$ such that π is $\mathbb{Q}$-acyclic, and $\dim Z \leq n$.

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$$Z \xrightarrow{\pi} X \qquad \dim Z \leq n \xrightarrow{\text{cell-like}} \dim_{\mathbb{Z}} X \leq n \qquad \dim Z \leq n \xrightarrow{\mathbb{Z}/p-\text{acyclic}} \dim_{\mathbb{Z}/p} X \leq n \qquad \dim Z \leq n \xrightarrow{\mathbb{Q}-\text{acyclic}} \dim_{\mathbb{Q}} X \leq n$$

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This does not work for any abelian group G:
if $G = \mathbb{Z}/p^{\infty} = \{\frac{m}{n} \in \mathbb{Q}/\mathbb{Z} \ : \ n = p^k \text{ for some } k \geq 0\}$
(quasi-cyclic p-group), then $\dim Z \not\leq n$, but $\dim Z \leq n + 1$.

Theorem (M. Levin, 2003)

Let G be an abelian group, $n \in \mathbb{N}_{\geq 2}$. Then for every compact metrizable space X with $\dim_G X \leq n$, there exists a compact metrizable space Z and a surjective map $\pi : Z \to X$ such that:

(a) π *is G-acyclic,*

(b) $\dim Z \leq n + 1$, *and*

(c) $\dim_G Z \leq n$.

Other resolution Theorems

Levin resolution Theorem for any G

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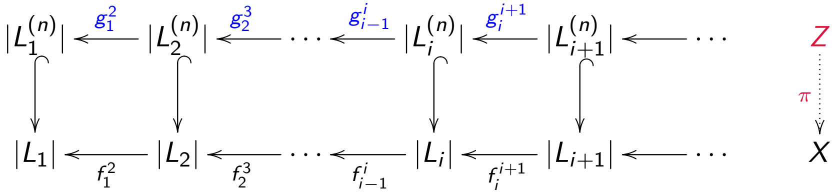
$$\begin{array}{c} Z \\ \downarrow \pi \\ X \end{array} \qquad \begin{array}{c} \dim Z \leq n \\ \downarrow \mathbb{Q}-\text{acyclic} \\ \dim_{\mathbb{Q}} X \leq n \end{array} \qquad \begin{array}{c} \dim Z \leq n+1,\ \dim_G Z \leq n \\ \downarrow G-\text{acyclic} \\ \dim_G X \leq n \end{array}$$

Original proof of Edwards-Walsh Theorem



- Choose an inverse sequence $(P_i = |L_i|, f_i^{i+1})$ of compact polyhedra, with simplicial, surjective bonding maps, whose limit is X (the space we started with). (Theorem by Freudenthal)
- Use this sequence as a foundation to build another inverse sequence $(|L_i^{(n)}|, g_i^{i+1})$ and an almost commutative ladder of maps, so that $\lim\,(|L_i^{(n)}|, g_i^{i+1}) = Z$ and the map $\pi : Z \to X$ with desired properties can be produced.

Original proof of Edwards-Walsh Theorem

$$
\begin{array}{ccccccccccccc}
|L_1^{(n)}| & \xleftarrow{g_1^2} & |L_2^{(n)}| & \xleftarrow{g_2^3} & \cdots & \xleftarrow{g_{i-1}^{i}} & |L_i^{(n)}| & \xleftarrow{g_i^{i+1}} & |L_{i+1}^{(n)}| & \longleftarrow & \cdots & & Z \\
\big\downarrow & & \big\downarrow & & & & \big\downarrow & & \big\downarrow & & & & \vdots\,\pi \\
|L_1| & \xleftarrow[f_1^2]{} & |L_2| & \xleftarrow[f_2^3]{} & \cdots & \xleftarrow[f_{i-1}^{i}]{} & |L_i| & \xleftarrow[f_i^{i+1}]{} & |L_{i+1}| & \longleftarrow & \cdots & & X
\end{array}
$$

- The hardest part of the construction is producing suitable $g_i^{i+1} : |L_{i+1}^{(n)}| \to |L_i^{(n)}|$.
- Edwards and Walsh used factoring maps through certain CW-complexes (to get good fiber properties for π): Edwards-Walsh complexes $\mathrm{EW}(L_i, \mathbb{Z}, n)$.

$$\begin{array}{c} \mathrm{EW}(L, G, n) \\ \downarrow \omega \\ |L| \end{array}$$

Let G be an abelian group, $n \in \mathbb{N}$ and L a finite, m-dimensional simplicial complex, $m \geq n$. An Edwards-Walsh resolution of L in dimension n for the group G is a pair $(\mathrm{EW}(L, G, n), \omega)$ consisting of a CW-complex $\mathrm{EW}(L, G, n)$ and a combinatorial map $\omega : \mathrm{EW}(L, G, n) \to |L|$ (that is, for each subcomplex L' of L, $\omega^{-1}(|L'|)$ is a subcomplex of $\mathrm{EW}(L, G, n)$) such that:

$$\begin{array}{ccc} \mathrm{EW}(L, G, n) & \omega^{-1}(|L'|) & \\ \Big\downarrow{\scriptstyle \omega} & & \\ |L| & |L'| & K(G, n) \end{array}$$

(i) $\omega^{-1}(|L^{(n)}|) = |L^{(n)}|$ and $\omega|_{|L^{(n)}|}$ is the identity map of $|L^{(n)}|$ onto itself,

(ii) for every simplex σ of L with $\dim \sigma > n$, the preimage $\omega^{-1}(\sigma)$ is an Eilenberg-MacLane complex of type $(\bigoplus G, n)$, where the sum here is finite, and

Original proof of Edwards-Walsh Theorem

Edwards-Walsh complexes

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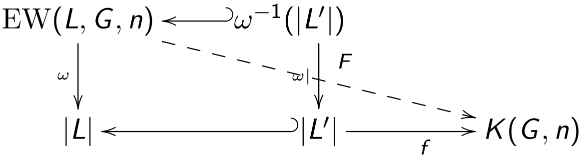
(iii) for every subcomplex L' of L and every map $f : |L'| \to K(G, n)$, the composition $f \circ \omega|_{\omega^{-1}(|L'|)} : \omega^{-1}(|L'|) \to K(G, n)$ extends to a map $F : \mathrm{EW}(L, G, n) \to K(G, n)$.

Original proof of Edwards-Walsh Theorem
Edwards-Walsh complexes



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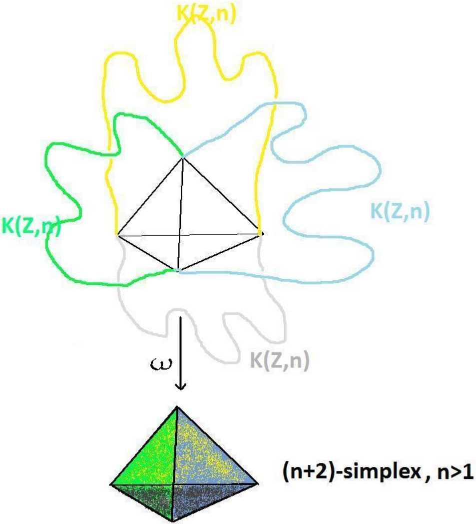
Lemma

For the groups $\mathbb{Z}$ and $\mathbb{Z}/p$, for any $n \in \mathbb{N}$ and for any simplicial complex L, there is an Edwards–Walsh resolution $\omega : \mathrm{EW}(L, G, n) \to |L|$ with the additional property for $n > 1$:

1. *the $(n+1)$-skeleton of $\mathrm{EW}(L, \mathbb{Z}, n)$ is equal to $L^{(n)}$;*
2. *the $(n+1)$-skeleton of $\mathrm{EW}(L, \mathbb{Z}/p, n)$ is obtained from $L^{(n)}$ by attaching $(n+1)$-cells by a map of degree p to the boundary $\partial\sigma$, for every $(n+1)$-dimensional simplex σ.*

Example of an Edwards-Walsh Complex $\mathrm{EW}(L, \mathbb{Z}, n)$



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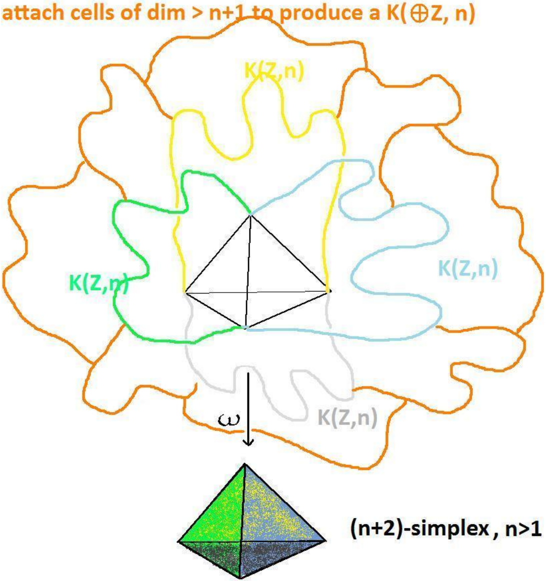
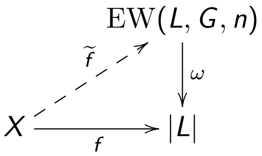


Original proof of Edwards-Walsh Theorem
Edwards-Walsh complexes

Edwards-Walsh complexes (resolutions) are useful because

Lemma

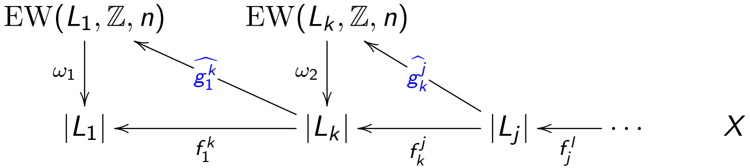
Let X be a compact metrizable space with $\dim_G X \leq n$, and let L be a finite simplicial complex. Then for every Edwards-Walsh resolution $\omega : \mathrm{EW}(L, G, n) \to |L|$, and for every map $f : X \to |L|$, there exists an approximate lift $\widetilde{f} : X \to \mathrm{EW}(L, G, n)$ of f.



$\widetilde{f}$ is an approximate (or combinatorial) lift of f with respect to ω if
$\forall x \in X,\ f(x) \in \Delta \Rightarrow \omega \circ \widetilde{f}(x) \in \Delta.$

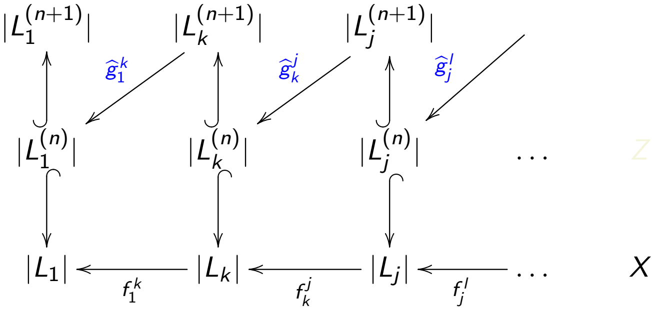
$\dim_G X \leq n \Leftrightarrow X \tau K(G,n)$

Original proof of Edwards-Walsh Theorem



- $\mathrm{EW}(L_i, \mathbb{Z}, n)^{(n+1)} = \mathrm{EW}(L_i, \mathbb{Z}, n)^{(n)} = L_i^{(n)}$.
- $\widehat{g_k}^{j}| : |L_j^{(n+1)}| \to \mathrm{EW}(L_k, \mathbb{Z}, n)^{(n+1)} = L_k^{(n)}$.

Original proof of Edwards-Walsh Theorem



$\hat{g}_k^j(|L_j^{(n+1)}|) \subset |L_k^{(n)}|$ is one of the facts making $\pi : Z \to X$ cell-like.

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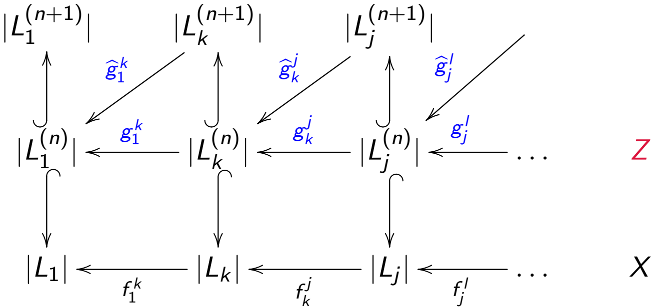

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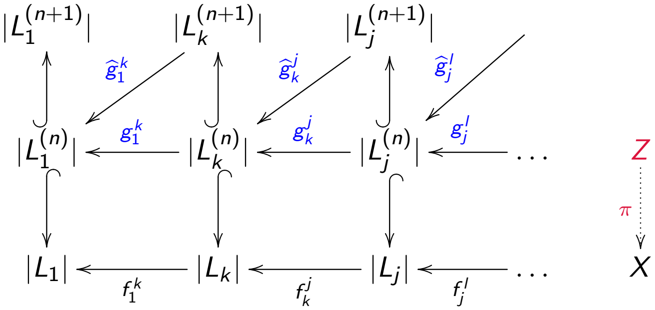

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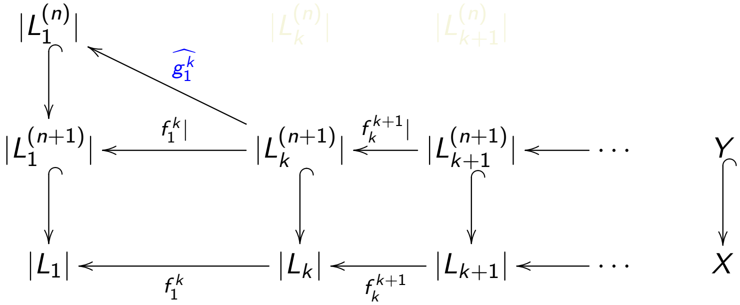
Alternative proof of Edwards-Walsh Theorem

$$|L_1^{(n)}| \qquad |L_2^{(n)}| \qquad |L_3^{(n)}| \qquad |L_4^{(n)}|$$

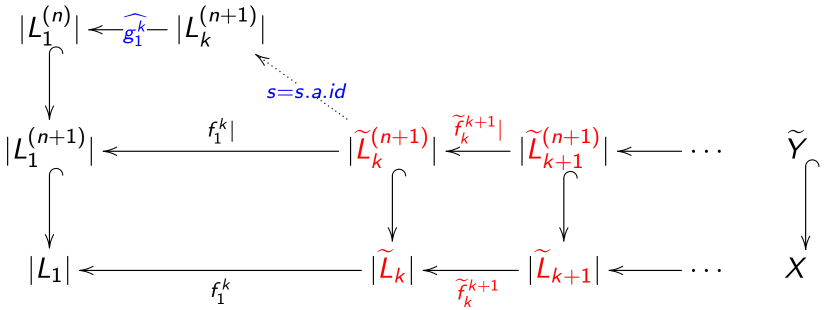
$$\begin{array}{ccccccccccc}
|L_1^{(n+1)}| & \xleftarrow{f_1^2|} & |L_2^{(n+1)}| & \xleftarrow{f_2^3|} & |L_3^{(n+1)}| & \xleftarrow{f_3^4|} & |L_4^{(n+1)}| & \longleftarrow & \cdots & & Y \\
\downarrow & & \downarrow & & \downarrow & & \downarrow & & & & \downarrow \\
|L_1| & \xleftarrow[f_1^2]{} & |L_2| & \xleftarrow[f_2^3]{} & |L_3| & \xleftarrow[f_3^4]{} & |L_4| & \longleftarrow & \cdots & & X
\end{array}$$

- Focus on the inverse sequence of $(n+1)$-skeleta, and let $Y = \lim(|L_i^{(n+1)}|, f_i^{i+1}|) \Rightarrow \dim Y \leq n+1$.
- Note that $\dim_{\mathbb{Z}} X \leq n \Rightarrow \dim_{\mathbb{Z}} Y \leq n$.



- Now by Aleksandrov's Theorem: $\dim Y < \infty \Rightarrow \dim_{\mathbb{Z}} Y = \dim Y$, so $\dim Y \leq n$.
- $\dim Y \leq n$ means we can produce $\widehat{g_1^k} : |L_k^{(n+1)}| \to |L_1^{(n)}|$ with needed properties without using $\mathrm{EW}(L_1, \mathbb{Z}, n)$.

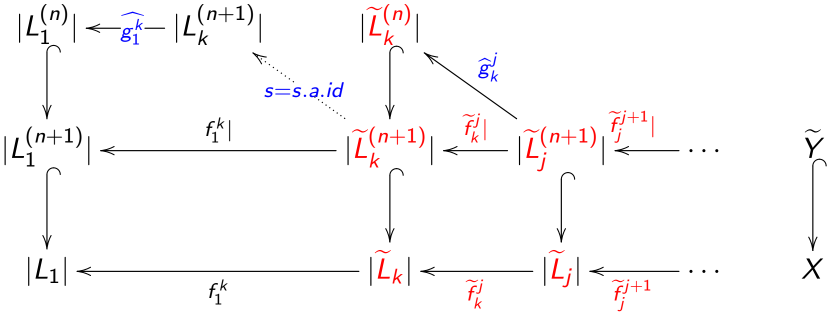
Alternative proof of Edwards-Walsh Theorem



Now we need to change $(|L_i|, f_i^{i+1})$: for $i \geq k$

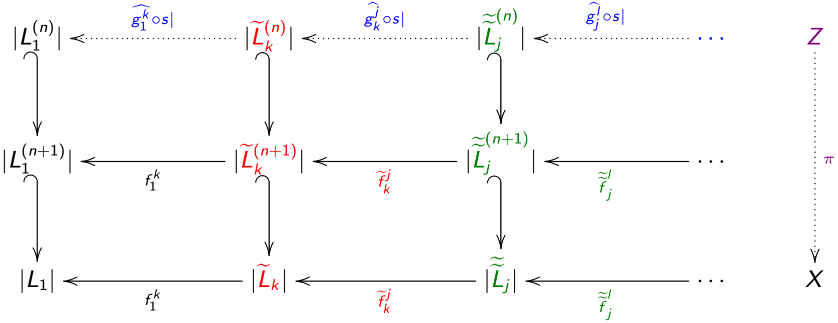
- subdivide L_i,
- replace f_i^{i+1} by their simplicial approximation w.r. to the new subdivisions (can be done so that limit $\approx X$).

Alternative proof of Edwards-Walsh Theorem



- Now we produce $\widehat{g}_k^j : |\widetilde{L}_j^{(n+1)}| \to |\widetilde{L}_k^{(n)}|$ with needed properties without using $\mathrm{EW}(\widetilde{L}_k, \mathbb{Z}, n)$.
- Connect $|\widetilde{L}_k^{(n+1)}|$ with $|L_k^{(n+1)}|$ using simplicial approximation to identity (s).

Alternative proof of Edwards-Walsh Theorem



- Z has $\dim Z \leq n$.
- Continuity of π follows from the ladder of maps being almost commutative.
- $\widehat{g_k}^j(|\widetilde{L}_j^{(n+1)}|) \subset |\widetilde{L}_k^{(n)}|$ contributes to cell-likeness of π.

Potential benefit – hybrid of the two approaches

- In resolution theorems involving abelian groups other than $\mathbb{Z}$, the Edwards-Walsh complexes or CW-complexes that play their role become more complicated, and algebra involved becomes trickier.
- It could be beneficial to use inverse sequences of $(n+1)$-skeleta (of the polyhedra forming the inverse sequence representing X), because Edwards-Walsh complexes built above $L^{(n+1)}$ are considerably simpler than ones built above L (these would not need complicated attachments that kill off higher and higher homotopy groups).
- We would need to use changing of the original inverse sequence that was representing X, but the technical ε-δ issues are usually easier to handle.
- Attempting to do this in joint work with L. Rubin.

The end

Thank you!