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CYCLIC GROUP ACTIONS ON Q AND HUREWICZ FIBRATION

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Section 1

For a compact metric space X, let G(X) be the space of self-homeomorphisms of X with the supremum metric. Let Q denote the Hilbert cube $\Pi_{i=1}^{\infty} J_i$, $J_i = [-1,1]$ and let $G_k(Q) \subset G(Q)$ denote the subspace consisting of all period k homeomorphisms, $k > 1$, each having a unique fixed point $0 = (0,0,\cdots)$. In this paper we show that every $\beta \in G_k(Q)$ is joined to the standard action α by a path λ in $G_k(Q)$, and that it induces a Hurewicz fibration $p\colon E \to [0,1]$ with E a Q-manifold and the fibers $p^{-1}(t)$ being the orbit spaces of the non-degenerate orbits of $\lambda(t)$. The formulation is motivated by the following theorem of Chapman-Ferry:

Theorem ([C-F]). *Let* $p\colon E \to [0,1]$ *be a Hurewicz fiber map with* E *a* Q-*manifold. If the fibers* $p^{-1}(t)$ *are compact* Q-*manifolds, then* p *is a trivial bundle map.*

To state our result more precisely, consider a map $\lambda\colon [0,1] \to G_k(Q)$. λ induces a level-preserving (l.p.) homeomorphism $H\colon Q \times [0,1] \to Q \times [0,1]$ by $H|_{Q\times\{t\}} = \lambda(t)$. Let E denote the orbit space of non-degenerate orbits of H and let $E_t \subset E$ be the orbits at level t. Define $p\colon E \to [0,1]$ by $p(E_t) = t$.

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Theorem. Given any $\beta_0, \beta_1 \in G_k(Q)$, *there is a path* $\lambda: [0,1] \to G_k(Q)$ *joining* β_0, β_1 *such that* $p: E \to [0,1]$ *is a Hurewicz fibration.*

The general question concerning cyclic group actions on Q is whether β_0 is necessarily conjugate to β_1; that is, whether there is an $h \in G(Q)$ such that $\beta_1 = h\beta_0 h^{-1}$. If we are able to assert that p is in fact a trivial bundle map, then it implies $E_0 \cong E_1$ ("$\cong$" means homeomorphic to) which then shows that β_0, β_1 are conjugate. The last assertion, unfortunately, is not yet known. A partial solution was given in [W_2] where it was shown that the answer is yes when restricted to the cyclic actions which have a basis of contractible, invariant neighborhoods about the fixed point. Other results which generalized those of [W_2] are given in [B-We] and [E-H]. In [We] a non-trivial action in Hilbert cube hyperspace was shown to satisfy the criterion in [W_2] and in [L], it is demonstrated that free finite group actions on compact Q-manifolds can be factored into actions on finite dimensional manifolds.

Theorem 1(3) of [C-F] and our theorem implies

Corollary. The composition $p \cdot \mathrm{proj}: E \times [0,1) \to E \to [0,1]$ *is a trivial bundle map.*

Notation. Composition of two maps f and g is denoted either by gf or $g \cdot f$.

2. A Canonical Isotopy in $G_k(Q)$

In this section we show that any two members α, β in $G_k(Q)$ can be canonically joined by a path in $G_k(Q)$. Let

$C(I,G_k(Q))$ denote the space of maps (sup metric) of $I = [0,1]$ into $G_k(Q)$.

Lemma 2.1. *Fix any* $\alpha \in G_k(Q)$. *There is a map* $\mu: G_k(Q) \to C(I,G_k(Q))$ *such that for any* β, $\mu(\beta)(0) = \alpha$, $\mu(\beta)(1) = \beta$.

Proof. Given any $\beta \in G_k(Q)$, we shall construct a path from β to α in $G_k(Q)$ by exhibiting a level-preserving (l.p.) homeomorphism $H: Q \times [0,1] \to Q \times [0,1]$ such that each $h_t = H|_{Q\times\{t\}} \in G_k(Q)$, $h_0 = \alpha$, $h_1 = \beta$ and H depends continuously on β. To simplify notation we shall construct a l.p. homeomorphism of $Q \times R^*$ onto itself, where $R^* = R \cup \{-\infty,\infty\}$ and then reparametrize R^* to $[0,1]$.

We write Q as $J^2 \times J^2 \times \cdots$, where $J^2 = [-1,1]^2$. For points $(x_1,x_2,\cdots)$, $(y_1,y_2,\cdots)$ in Q, $x_i,y_i \in J^2$, denote

$$(x_1',x_2',\cdots) = \beta(x_1,x_2,\cdots) \text{ and}$$
$$(y_1',y_2',\cdots) = \alpha(y_1,y_2,\cdots).$$

First we define h_n at level n by

$$h_n(x_1,\cdots,x_{n+1},y_1,x_{n+2},y_2,\cdots) = (x_1',\cdots,x_{n+1}',y_1',x_{n+2}',y_2',\cdots)$$

for $n \geq 0$, and

$$h_{-n}(y_1,\cdots,y_n,x_1,y_{n+1},x_2,\cdots) = (y_1',\cdots,y_n',x_1',y_{n+1}',x_2',\cdots)$$

for $-n < 0$. Let

$$h_\infty(x_1,x_2,\cdots) = (x_1',x_2',\cdots)$$

and

$$h_{-\infty}(y_1,y_2,\cdots) = (y_1',y_2',\cdots).$$

Next we define h_{n-t} for $0 \leq t \leq 1$. For any integer n define $\theta_{|n|} \in G(Q)$ by interchanging the $(|n|+1)^{\text{th}}$ and $(|n|+2)^{\text{th}}$ coordinates, and in general, $(|n|+2j-1)^{\text{th}}$ and $(|n|+2j)^{\text{th}}$ coordinates, $j = 1,2,\cdots$.

By $[W_1]$ $\theta_{|n|}$ is isotopic to the identity in $G(Q)$ and the isotopy fixes the point $(0,0,\cdots)$ and leaves the first $|n|$ coordinates of each point unchanged. Denote such an isotopy by $\{\theta_{|n|,t}\}_{0\le t\le 1}$ with $\theta_{|n|,0}$ = id, and $\theta_{|n|,1} = \theta_{|n|}$. Define

$$(*) \qquad h_{n-t} = \theta_{|n|,t} h_n \theta_{|n|,t}^{-1}.$$

Then $\{h_{n-t}\}_{0\le t\le 1}$ induces a path in $G_k(Q)$ between h_n and h_{n-1}. Putting all the $\{h_{n-t}\}$ together we get a l.p. homeomorphism $H: Q \times R^* \to Q \times R^*$ such that $h_{-\infty} = \alpha$ and $h_\infty = \beta$. Finally, the dependency of H on β follows trivially from the construction.

Section 3

In this section we establish lemma 3.3 which will be used in section 4. For any $t \in \mathbf{R}$, let λ_t denote the "square" rotation of the complex space $\mathbb{C}$ in the sense that, by writing $\mathbb{C}$ as the union of concentric squares (with center 0 and sides parallel to the coordinate axes), each point z travels to the point z' along the unique square to which it belongs, and that Arg z' = Arg z + t. Denote $\lambda_{\pi/4}$ by λ. Define $\tilde{\lambda}, \tilde{\lambda}_t: \mathbb{C} \times \mathbb{C} \to \mathbb{C} \times \mathbb{C}$ by

$$\tilde{\lambda}((x_1,y_1),(x_2,y_2)) = ((x_1',y_1'),(x_2',y_2'))$$
$$\tilde{\lambda}_t = \lambda_t \times \text{id},$$

where

$$(x_1',x_2') = \lambda(x_1,x_2) \text{ and } (y_1',y_2') = \lambda(y_1,y_2).$$

Lemma 3.1. *(i)* $\tilde{\lambda}^{-1}(\lambda_t \times \lambda_t)\tilde{\lambda} = \lambda_t \times \lambda_t$ *for all* $t \in \mathbf{R}$ *and (ii)* $\tilde{\lambda}^{-1}\tilde{\lambda}_\pi\tilde{\lambda}(z_1,z_2) = (z_2,z_1)$.

Proof. The proof is a result of routine computation and will be omitted.

Remark. Intuitively the justification for concluding (ii) is as follows: The map $(x_1,x_2) \to (x_2,x_1)$ is a result of (a) applying λ to the point (x_1,x_2), (b) following by the reflexion γ across the y-axis ($(x,y) \to (-x,y)$) and (c) applying λ^{-1} to the image $\gamma\cdot\lambda(x_1,x_2)$.

Using the data above we obtain

Lemma 3.2. $(\tilde{\lambda}\tilde{\lambda}_{s\pi}\tilde{\lambda}^{-1})(\lambda_t \times \lambda_t)(\tilde{\lambda}\tilde{\lambda}_{s\pi}\tilde{\lambda}^{-1})^{-1} = \lambda_t \times \lambda_t$ *for any* $t \in \mathbf{R}$ *and for all* $s \in [0,1]$.

Proof. This is a trivial application of Lemma 3.1 (i).

Next let $J_n^2 \subset \mathbb{C}$ be the square $[-1,1]^2$. For fixed $k > 1$, define

$$\alpha,\theta\colon J_1^2 \times J_2^2 \times \cdots \to J_1^2 \times J_2^2 \times \cdots \text{ by}$$
$$\alpha = \lambda_{2\pi/k} \times \lambda_{2\pi/k} \times \cdots \text{ and}$$
$$\theta(z_1,z_2,\cdots) = (z_2,z_1,z_4,z_3,\cdots).$$

Lemma 3.3. *There is a path* $\{\theta_t\}$ *in* $G(Q)$ *such that* $\theta_0 = \text{id}$, $\theta_1 = \theta$, *each* θ_t *fixes* $0 = (0,0,\cdots)$, *and* $\theta_t\alpha\theta_t^{-1} = \alpha$ *for all* $0 \le t \le 1$.

Proof. By Lemma 3.2 there is a path $\{\phi_s = \tilde{\lambda}\tilde{\lambda}_{s\pi}\tilde{\lambda}^{-1}\}$ in $G(\mathbb{C} \times \mathbb{C})$ such that $\phi_0 = \text{id}$, $\phi_1(z_1,z_2) = (z_2,z_1)$ and $\phi_s(\lambda_{2\pi/k} \times \lambda_{2\pi/k})\phi_s^{-1} = \lambda_{2\pi/k} \times \lambda_{2\pi/k}$ for all $s \in [0,1]$. Now apply $\{\phi_s\}$ to each pair (J_{2n-1},J_{2n}), $n \ge 1$.

Section 4

In this section we establish the main technical lemmas. As in section 2, we write $Q = J_1^2 \times J_2^2 \times \cdots$, where $J_n^2 \subset \mathbb{C}$. Let $\alpha,\theta\colon Q \to Q$ be defined as in section 3. If $f\colon X \to X$ and $g\colon Y \to Y$ are maps, we define a map of pairs $\phi\colon (X,f) \to (Y,g)$ to be a map $\phi\colon X \to Y$ satisfying $\phi\cdot f = g\cdot\phi$. For any $\beta \in G_k(Q)$, let $H: Q \times [0,1] \to Q \times [0,1]$ be the isotopy joining α to β as described in the proof of Lemma 2.1. Denote $M = Q\setminus\{0\}$, $H_0 = H|_{M\times[0,1]}$, $M_t = M \times \{t\}$, $\alpha_0 = \alpha|_{M_0}$ and $\beta_1 = \beta|_{M_1}$.

Lemma 4.1. *For any map* $g: (M_1,\beta_1) \to (M_0,\alpha_0)$, *there is a retraction* $G: (M \times [0,1],H_0) \to (M_0,\alpha_0)$ *such that* $G|_{M_1} = g$.

Remark. As a consequence of the contruction, G_0 actually depends continuously on g.

Proof. We adopt the notation as in the proof of Lemma 2.1 where we identify $M \times [0,1]$ as $M \times (R \cup \{-\infty,\infty\})$. We also let $h_t = H_0|_{M_t}$. To begin, define $G|_{M_{-\infty}} = \mathrm{id}$ and $G|_{M_\infty} = g$ and for integer $n \geq 0$, define $g_n = G|_{M_n}: M_n \to M_{-\infty}$ by

$$g_n(x_1,\cdots,x_{n+1},y_1,x_{n+2},y_2,\cdots,n)$$
$$= (x_1',\cdots,x_{n+1}',y_1,x_{n+2}',y_2,\cdots,-\infty),$$

where $(x_1,\cdots,x_{n+1},y_1,x_{n+2},y_2,\cdots) \in J_1^2 \times J_2^2 \times \cdots$ and

$$(x_1',x_2',\cdots) = g(x_1,x_2,\cdots).$$

Similarly, for $-n < 0$, define

$$g_{-n}(y_1,\cdots,y_n, x_1,y_{n+1},x_2,\cdots,-n)$$
$$= (y_1,\cdots,y_n,x_1',y_{n+1},x_2',\cdots,-\infty).$$

The map of g_n clearly satisfies (A) $g_nh_n = \alpha_0g_n$, and thus, is a map of (M_n,h_n) into $(M_{-\infty},\alpha_0)$. Next we shall define $g_{n-t} = G|_{M_{n-t}}: M_{n-t} \to M_{-\infty}$ for $0 \leq t \leq 1$. But first, for any integer n, let $\{\theta_{|n|,t}\}_{0\leq t\leq 1}$ be given as in the proof of Lemma 2.1. By Lemma 3.3 we may choose $\{\theta_{|n|,t}\}$ to satisfy (B) $\theta_{|n|,t}^{-1}\cdot\alpha_0\cdot\theta_{|n|,t} = \alpha_0$ for all t. We then observe that $\theta_{|n|,t}$ can be regarded as a homeomorphism of M_n onto M_{n-t}. Now we define (C) $g_{n-t} = \theta_{|n|,t}g_n\theta_{|n|,t}^{-1}$. We assert that $g_{n-t}\cdot h_{n-t} = \alpha_0\cdot g_{n-t}$. To prove this, we have

$$\begin{aligned}\alpha_0\cdot g_{n-t} &= \alpha_0\cdot\theta_{|n|,t}\cdot g_n\cdot\theta_{|n|,t}^{-1}\\ &\overset{(B)}{=} \theta_{|n|,t}\cdot\alpha_0\cdot g_n\cdot\theta_{|n|,t}^{-1}\\ &\overset{(A)}{=} \theta_{|n|,t}\cdot g_n\cdot h_n\cdot\theta_{|n|,t}^{-1}\\ &\overset{(C)}{=} g_{n-t}\theta_{|n|,t}\cdot h_n\cdot\theta_{|n|,t}^{-1}\\ &\overset{(*)}{=} g_{n-t}\cdot h_{n-t}.\end{aligned}$$

Finally it is routine to verify that, when putting all the levels $\{h_t\}$ together, we get a retraction G as required.

For the next lemma, let G be as above and let $G'\colon (M\times[0,1],H_0)\to(M\times[0,1],\alpha_0\times \mathrm{id})$ be defined by $G'(x,t) = (G(x,t),t)$. Denote $G'|_{M_t} = g_t'$.

Two maps $\phi_0,\phi_1\colon (X,f)\to(Y,g)$ are *homotopic* if there is a homotopy of maps $\phi_t\colon (X,f)\to(Y,g)$ joining ϕ_0 to ϕ_1.

Lemma 4.2. *Given the data above, suppose* $g\colon (M_1,\beta_1)\to(M_0,\alpha_0)$ *and* $f\colon (M_0,\alpha_0)\to(M_1,\beta_1)$ *are maps such that* fg *is homotopic to the identity in* (M_1,β_1) *by* $\{\phi_t\}$. *There is, then, a level-preserving map* $F\colon (M\times[0,1],\alpha_0\times \mathrm{id})\to(M\times[0,1],H_0)$, *such that* $F|_{M_0} = \mathrm{id}$, $F|_{M_1} = f$ *and* $F\cdot G'$ *is level preservingly homotopic to the identity in* $(M\times[0,1],H_0)$.

Proof. Again as in section 2 we regard [0,1] as $R\times\{-\infty,\infty\}$ and adopt the notations established in Lemma 4.1. Denote

$$f(x_1,x_2,\cdots,-\infty) = (x_1',x_2',\cdots,+\infty).$$

Define, for $n\geq 0$, $f_n = F|_{M_n}\colon (M_n,\alpha_0)\to(M_n,h_n)$ by

$$\begin{aligned}&f_n(x_1,\cdots,x_{n+1},y_1,x_{n+2},y_2,\cdots,n)\\ &\quad= (x_1',\cdots,x_{n+1}',y_1,x_{n+2}',y_2,\cdots,n)\end{aligned}$$

and for $-n < 0$

$$f_{-n}(y_1,\cdots,y_n,x_1,y_{n+1},x_2,\cdots,-n) = (y_1,\cdots,y_n,x_1',y_{n+1},x_2',\cdots,-n).$$

Recall that $g_n' = G'|_{M_n}: (M_n,h_n) \to (M_{+n},\alpha_0)$. We assert that $f_n \cdot g_n'$ is homotopic to id in (M_n,h_n). To see this suppose $n \geq 0$. By definition

$$f_n \cdot g_n'(x_1,\cdots,x_{n+1},y_1,x_{n+2},y_2,\cdots,n) = (\tilde{x}_1,\cdots,\tilde{x}_{n+1},y_1,\tilde{x}_{n+2},y_2,\cdots,n),$$

where $(\tilde{x}_1,\tilde{x}_2,\cdots) = f \cdot g(x_1,x_2,\cdots)$. Since $f \cdot g$ is homotopic to id in (M_1,β_1), by definition of h_n, $f_n \cdot g_n'$ is homotopic to id in (M_n,h_n). The case for $-n < 0$ is similar.

Now let $\{\phi_t\}$ denote a homotopy in (M_n,h_n) so that $\phi_0 = $ id and $\phi_1 = f_n \cdot g_n'$. At any level $n-t$, $0 \leq t \leq 1$, define $f_{n-t} = F|_{M_{n-t}}: M_{n-t} \to M_{n-t}$ by

$$f_{n-t} = \theta_{|n|,t} \cdot f_n \cdot \theta_{|n|,t}^{-1}.$$

We assert

(1) $h_{n-t} \cdot f_{n-t} = f_{n-t} \cdot \alpha_0$ and

(2) $f_{n-t} \cdot g_{n-t}'$ is homotopic to id in (M_{n-t},h_{n-t}).

To see (1), $h_{n-t} \cdot f_{n-t} = h_{n-t} \cdot \theta_{|n|,t} \cdot f_n \cdot \theta_{|n|,t}^{-1}$ $= \theta_{|n|,t} \cdot h_n \cdot f_n \cdot \theta_{|n|,t}^{-1} = \theta_{|n|,t} \cdot f_n \cdot \alpha_0 \cdot \theta_{|n|,t}^{-1}$ $= (\theta_{|n|,t} \cdot f_n \cdot \theta_{|n|,t}^{-1})(\theta_{|n|,t} \cdot \alpha_0 \cdot \theta_{|n|,t}^{-1}) = f_{n-t} \cdot \alpha_0$. To see (2), define a homotopy $\{\gamma_s\}$ of M_{n-t} into itself by $\gamma_s = \theta_{|n|,t} \cdot \phi_s \cdot \theta_{|n|,t}^{-1}$. Then $\gamma_0 = $ id and

$$\begin{aligned}\gamma_1 &= \theta_{|n|,t} \cdot \phi_1 \cdot \theta_{|n|,t}^{-1} = \theta_{|n|,t} \cdot f_n \cdot g_n' \cdot \theta_{|n|,t}^{-1} \\ &= \theta_{|n|,t}(\theta_{|n|,t}^{-1} \cdot f_{n-t} \cdot \theta_{|n|,t})(\theta_{|n|,t}^{-1} \cdot g_{n-t}' \\ &\quad \cdot \theta_{|n|,t})\theta_{|n|,t}^{-1} \\ &= f_{n-t} \cdot g_{n-t}'\end{aligned}$$

Furthermore, for any $s \in [0,1]$,

$$\begin{aligned}
h_{n-t}\cdot\gamma_s &= (\theta_{|n|,t}\cdot h_n\cdot\theta_{|n|,t}^{-1})(\theta_{|n|,t}\cdot\phi_s\cdot\theta_{|n|,t}^{-1})\\
&= \theta_{|n|,t}\cdot h_n\cdot\phi_s\cdot\theta_{|n|,t}^{-1}\\
&= \theta_{|n|,t}\cdot\phi_s\cdot h_n\cdot\theta_{|n|,t}^{-1}\\
&= (\theta_{|n|,t}\cdot\phi_s\cdot\theta_{|n|,t}^{-1})(\theta_{|n|,t}\cdot h_n\cdot\theta_{|n|,t}^{-1})\\
&= \gamma_s h_{n-t}.
\end{aligned}$$

So $\{\gamma_s\}$ is a homotopy in (M_{n-t},h_{n-t}) between $f_{n-1}g'_{n-1}$ and id. This proves assertion (2).

5. Proof of the Theorem

Without loss of generality, we may assume β_0 is the map $\alpha = \lambda_{2\pi/k} \times \lambda_{2\pi/k} \times \cdots$ as defined in section 3. Let $H: Q \times [0,1] \to Q \times [0,1]$ be the l.p. homeomorphism given in the proof of Lemma 2.1. Denote $H_0 = H|_{M\times[0,1]}$ where $M = Q\backslash\{0\}$. We shall continue to employ the notations established previously.

Imbed E into $E \times [0,1]$ by $i: e \to (e,p(e))$ and let $p': E \times [0,1] \to [0,1]$ be the projection map. We assert that there is a f.p. retraction of $E \times [0,1]$ onto $i(E)$. Since p' is a Hurewicz fibration, so is $p'|_{i(E)}$ and hence p.

Let us now prove the assertion. The Q-manifolds $p^{-1}(0) = M_0/\alpha_0$ and $p^{-1}(1) = M_1/\beta_1$ are Eilenberg-Maclane spaces of type $(Z_k,1)$. Hence there is a homotopy equivalence $f_*: p^{-1}(0) \to p^{-1}(1)$. Let $g_*: p^{-1}(1) \to p^{-1}(0)$ be a homotopy inverse of f_*. f_* and g_* induce maps $f: (M_0,\alpha_0) \to (M_1,\beta_1)$ and $g: (M_1,\beta_1) \to (M_0,\alpha_0)$ such that $f\cdot g: (M_1,\beta_1) \to (M_1,\beta_1)$ is homotopic to id in (M_1,β_1). Denote such a homotopy by $\{\phi_t\}$. Now let $G: (M \times [0,1],H_0) \to (M_0,\alpha_0)$ and $G': (M \times [0,1],H_0) \to (M \times [0,1],\alpha_0 \times \mathrm{id})$ be the maps described in section 4 and let $F: (M \times [0,1],\alpha_0 \times \mathrm{id}) \to (M \times [0,1],H_0)$ be the level-preserving map given by Lemma 4.2. We have the following properties:

(1) $G|_{M_1} = g$

(2) $F|_{M_0} = \mathrm{id}$, $F|_{M_1} = f$ and

(3) $F\cdot G'$ is l.p. homotopic to id in $(M \times [0,1], H_0)$ by $\{\gamma_t\}$.

Passing to the orbit spaces, G, G', F and $\{\gamma_t\}$ induce maps $G_*: E \to p^{-1}(0)$, $G_*^!: E \to p^{-1}(0) \times [0,1]$, $F_*: p^{-1}(0) \times [0,1] \to E$ and a f.p. homotopy $\gamma_s^*: E \to E$ between $F_*\cdot G_*^!$ and id where $\gamma_0^* = F_*G_*^!$ and $\gamma_1^* = \mathrm{id}$. Define a map

$$q: (E \times [0,1] \times \{0\}) \cup (i(E) \times [0,1]) \to i(E)$$

by

$$q(x,t,0) = (i(F_*(G_*(x),t)),0) \text{ for }$$
$$(x,t,0) \in E \times [0,1] \times \{0\}$$

and

$$q(i(x_t),s) = (i(\gamma_s^*(x_t)),s) \text{ where } x_t \in p^{-1}(t).$$

We verify easily that q is well-defined and $q|_{i(E)\times\{1\}} = \mathrm{id}$. We wish to extend q fiber-preservingly (preserving the middle-coordinates) to all of $E \times [0,1] \times [0,1]$. If q' is such an extension, then the restriction $q'|_{E\times[0,1]\times\{1\}}$ is a fiber-preserving retraction of $E \times [0,1]$ onto $i(E)$. The usual techniques of homotopy extension imply that we need only to extend q fiber-preservingly to a neighborhood of $A = (E \times [0,1] \times \{0\}) \cup (i(E) \times [0,1])$ in $E \times [0,1] \times [0,1]$. To achieve this it is sufficient to construct a neighborhood N of A which fiber-preservingly retracts onto A. Since the obit maps $M \times [0,1] \to E$ and $M \times \{t\} \to p^{-1}(t)$ are covering maps, each $a \in A$ has either a local fiber-collared or fiber-bi-collared neighborhood in $E \times [0,1] \times [0,1]$. The usual proofs of M. Brown that locally collared (or bi-collared) implies collared (or bi-collared) apply equally well in the fibered case (see, for example, [R]). So N exists and the proof of the theorem is complete.

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