TOPOLOGY PROCEEDINGS

Volume 2, 1977

Pages 341–343

http://topology.auburn.edu/tp/

Research Announcement:

CONFLUENT IMAGES OF RATIONAL CONTINUA

by

J. Grispolakis and E. D. Tymchatyn

Topology Proceedings

Web:	http://topology.auburn.edu/tp/
Mail:	Topology Proceedings
	Department of Mathematics & Statistics
	Auburn University, Alabama 36849, USA
E-mail:	topolog@auburn.edu
ISSN:	0146-4124

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CONFLUENT IMAGES OF RATIONAL CONTINUA

J. Grispolakis and E. D. Tymchatyn

In 1974, A. Lelek posed the following question: Do confluent mappings preserve rational continua? (see [4], Problem I). E. D. Tymchatyn in [6] constructed a rational continuum and a confluent mapping of it onto a non-rational continuum. However, this rational continuum is non-acyclic and the mapping is not h-confluent. Then, the following questions were posed by A. Lelek in [5], and by J. Grispolakis in [2]: "Do confluent mappings preserve acyclic rational continua?" and "Do h-confluent mappings preserve rational continua?" We answer both questions in the negative by constructing an acyclic rational continuum and a strongly confluent mapping of it onto a non-rational continuum.

By a *continuum* we mean a connected, compact, metric space. A continuum is said to be *rational* provided it admits a basis of open sets with countable boundaries. A mapping $f:X \to Y$ of a continuum X onto a continuum Y is said to be *strongly confluent* (resp., h-*confluent*) provided for each connected subset K of Y each component (resp., quasi-component) of $f^{-1}(K)$ is mapped onto K. The mapping f is said to be *confluent* provided for each subcontinuum K of Y each component of $f^{-1}(K)$ is mapped onto K. Strongly confluent mappings are h-confluent and h-confluent mappings are confluent. Finally, the mapping $f:X \to Y$ is said to be H-*pseudo confluent* provided for each subset Z of Y and each point $z \in Z$ we have

$$Q(Z,z) = \cup \{f[Q(f^{-1}(Z),x)] \mid x \in f^{-1}(z)\} ,$$

where $Q(A,a)$ is the quasi-component of A at the point $a \in A$. It has been proved in [2] that the class of open mappings and the class of monotone mappings are proper subclasses of the class of H-pseudo confluent mappings. The following theorem has been proved in [2]:

Theorem 1 ([2]). *H-pseudo confluent mappings preserve rational continua.*

Example 1. There exist an arclike rational continuum X and a strongly confluent mapping f of X onto a non-rational continuum Y.

We obtain the continuum X as an inverse limit of arclike continua and the mapping f as an at most two-to-one strongly confluent mapping.

The continuum X is a rational continuum of rim-type 3, which contains no subcontinuum of rim-type 2. This provides another counterexample to Lelek's problem 13 in [3]. The first example resolving this problem on the negative was given by B. B. Epps, Jr. in [1].

A complete version of the present paper will be published elsewhere.

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University of Saskatchewan

Saskatoon, Saskatchewan

Canada S7N OWO