TOPOLOGY PROCEEDINGS
Volume 3, 1978
Pages 347–362
http://topology.auburn.edu/tp/

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Topology Proceedings

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ISSN:	0146-4124

CONTINUA WHICH ADMIT ONLY CERTAIN CLASSES OF ONTO MAPPINGS

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The purpose of this article is to present a rather complete study of those classes of continua which admit only confluent (resp. semi-confluent, weakly confluent, pseudo-confluent) onto mappings. The first results were obtained by H. Cook [3] who proved that if X is a hereditarily indecomposable continuum, then every mapping from any continuum onto X is confluent, and by D. R. Read [20] who proved that the converse is true, that is, if X is a continuum such that every mapping from any continuum onto X is confluent, then X is hereditarily indecomposable.

In what follows we study the class of continua X with the property that every mapping from any continuum onto X is weakly confluent. Finally, at the end of the paper we study the classes of continua X with the property that every mapping from any continuum onto X is semi-confluent (resp., pseudo-confluent).

1. Definitions and Preliminaries

By a *continuum* is meant a connected, compact, metric space. By a *mapping* is always meant a continuous function. A mapping f: X → Y of a continuum X onto a continuum Y is said to be *confluent* [2], *semi-confluent* [18], or *weakly*

[1]The first author was supported by a University of Saskatchewan postdoctoral fellowship. The second author was supported in part by NRC (Canada) Grant no. A5616.

confluent [15] provided that for each subcontinuum K of Y, the following conditions are satisfied, respectively:

(c) for each component C of $f^{-1}(K)$ we have $f(C) = K$;

(s) for any pair C_1, C_2 of components of $f^{-1}(K)$ we have that either $f(C_1) \subset f(C_2)$ or $f(C_2) \subset f(C_1)$;

(w) there exists a component C of $f^{-1}(K)$ such that $f(C) = K$.

The mapping $f: X \to Y$ is said to be *pseudo-confluent* [17] provided for each irreducible subcontinuum K of Y there exists a component C of $f^{-1}(K)$ such that $f(C) = K$. Clearly, (c) implies (s), (s) implies (w) and (w) implies pseudo-confluent. A continuum X is said to be in *Class (W)* provided that every mapping from any continuum onto X is weakly confluent. A. Lelek asked in [16] for a characterization of Class (W) and remarked that continua in Class (W) are unicoherent and not triods. A continuum is said to be *unicoherent* provided it cannot be written as the union of two proper subcontinua whose intersection is not connected. A continuum X is said to be *hereditarily unicoherent* provided that every subcontinuum of X is unicoherent. A continuum K is said to be a *triod* provided $K = K_1 \cup K_2 \cup K_3$ for some subcontinua K_1, K_2 and K_3 of K whose intersection is connected and such that no one is contained in the union of the other two. A continuum is said to be *atriodic* provided that it does not contain any triod. A continuum is said to be *arc-like* (resp., *tree-like*) provided it admits finite open covers of arbitrarily small mesh whose nerves are arcs (resp., one-dimensional acyclic connected polyhedra).

The following results give some classes of continua which are in Class (W):

1.1. *Theorem* (Cook [3]). *Every hereditarily indecomposable continuum is in Class* (W).

1.2. *Theorem* (Read [20]). *Every arc-like continuum is in Class* (W).

1.3. *Theorem* (Feuerbacher [5]). *Every non-planar circle-like continuum is in Class* (W).

2. Some Sufficient Conditions for Class (W)

If X is a continuum, then by C(X) we denote the space of all subcontinua of X topologized by the Hausdorff metric H (or equivalently by the Vietoris topology). In [21] Whitney proved that for any continuum X there exists a mapping μ: C(X) $\to$ $[0,\infty)$ such that $\mu(\{x\}) = 0$ for each $x \in X$, and if A, B $\in$ C(X) with $A \subset B \neq A$, then $\mu(A) < \mu(B)$. Any such mapping will be called a *Whitney map*. It was remarked in [4, p. 1032] that μ is a monotone mapping. We say that the continuum X has the *covering property* provided that for any Whitney map μ for C(X) and any $t \in (0, \mu(X))$ no proper subcontinuum of $\mu^{-1}(t)$ covers X (see [13]). B. Hughes in an unpublished work proved the following:

2.1. *Theorem* (Hughes). *Every continuum with the covering property is in Class* (W).

He also asked whether the converse of Theorem 2.1 is true. This was the first attempt towards a characterization of Class (W) (for a discussion see [19, Chapter 14]). In [6]

Grispolakis, Nadler and Tymchatyn proved that the converse of Theorem 2.1 is true for the class of circle-like continua.

Hughes' result has proved to be useful in proving that certain classes of continua are in Class (W) by proving that they have the covering property. The following theorem was proved by using techniques involving coverings.

2.2. *Theorem* (Grispolakis, Nadler and Tymchatyn [6, (4.1)]). *Let* X *be a circle-like continuum with no local separating continua. Then* X *has the covering property, and hence,* X *is in Class* (W).

Recently, the following theorem was proved by using techniques involving coverings and the structure of atriodic continua.

2.3. *Theorem* (Grispolakis and Tymchatyn [7, (5.3)]). *Let* X *be an atriodic continuum which admits finite open covers of arbitrarily small mesh whose nerves are graphs with Betti numbers not exceeding some positive integer* N. *Suppose that* X *satisfies the following condition:*

(i) if the continuum K *locally separates* X *between some points* p *and* q, *then* K *separates* X *between* p *and* q. *Then* X *has the covering property, and hence,* X *is in Class* (W).

Theorem 2.3 gives a useful criterion for a continuum to be in Class (W). It is clear that we obtain as corollary that arc-like continua, non-planar circle-like continua, and circle-like continua with no local separating continua are in Class (W). We can also see that the Case-Chamberlin continuum [1] and any continuum constructed in a similar manner is in Class (W). We isolate the following result which is obtained as a corollary to Theorem 2.3.

2.4. *Theorem* ([7]). *Let* X *be a tree-like atriodic continuum. Then* X *has the covering property, and hence,* X *is in Class* (W).

Ingram has proved that there exists an uncountable collection of non-homeomorphic atriodic tree-like continua on the plane such that none of them is arc-like [10], and he has proved that all of them are in Class (W).

So far we have seen different classes of continua which are in Class (W), but the only higher dimensional continua in Class (W) we have seen are the hereditarily indecomposable ones. In Section 3 we shall see that any continuum can be embedded in a compactification of the half line which is in Class (W).

3. Two Characterizations of Class (W)

A continuum X is said to be *absolutely* C*-*smooth* provided that whenever X is embedded in a continuum Y and $X = \lim_{i\to\infty} X_i$ for a sequence $\{X_i\}_{i=1}^{\infty}$ of subcontinua of Y, then $C(X) = \lim_{i\to\infty} C(X_i)$ (see [6] and [19]). The following result was obtained in [6] and it was asked whether the converse is true.

3.1. *Theorem* (Grispolakis, Nadler and Tymchatyn [6]). *Every continuum which has the covering property is absolutely* C*-*smooth.*

In [8], the converses of both Theorem 2.1 and Theorem 3.1 were proved and another proof of Theorem 2.1 was given. The proofs made use of the mapping cylinder and of the fact that continuum-valued upper semi-continuous functions preserve connectedness.

3.2. *Theorem* (Grispolakis and Tymchatyn [8]). *For any continuum* X *the following are equivalent:*

(a) X *is in Class* (W);

(b) X *has the covering property;*

(c) X *is absolutely* C*-*smooth.*

The following result now follows by using Theorem 3.2 and results in [7].

3.3. *Theorem* ([8]). *Let* X *be a compactification of the half line* $[0, \infty)$. *Then* X *is in Class* (W) *if and only if*

$$C(X) = \mathrm{Cl}[C([0, \infty))],$$

where $C([0, \infty))$ *denotes the subspace of* $C(X)$ *which consists of all non-empty subcontinua of* $[0, \infty)$.

3.4. *Theorem. Any compactification of the half line* $[0, \infty)$ *with remainder a continuum in Class* (W) *is also in Class* (W).

Proof. Let Y be a continuum in Class (W), and let X be a compactification of $[0, \infty)$ with remainder Y. We shall prove that $C(X) \subset \mathrm{Cl}[C([0, \infty))]$. For this let $A \in C(X)$. If $A \cap [0, \infty) \neq \emptyset$, then it is obvious that $A \in \mathrm{Cl}[C([0, \infty))]$. Assume, therefore, that $A \subset Y$. Clearly, $Y \in \mathrm{Cl}[C([0, \infty))]$. By Theorem 3.2, Y is absolutely C*-smooth, and hence, there exists a sequence $\{A_i\}_{i=1}^{\infty}$ of arcs in $[0, \infty)$ such that

$A = \operatorname{Lim}_{i\to\infty} A_i$. Thus, $A \in \mathrm{Cl}[C([0, \infty))]$. Since $\mathrm{Cl}[C([0, \infty))] \subset C(X)$, we have that $C(X) = \mathrm{Cl}[C([0, \infty))]$. The theorem now follows from 3.3.

The following result shows that the hereditarily indecomposable continua are <u>not</u> the only higher dimensional continua which are in Class (W).

3.5. *Theorem. Every continuum* X *can be embedded in a compactification* Y *of the half line such that* Y *is in Class* (W).

Proof. Let Q denote the Hilbert cube. For each positive integer i let $\pi_i: Q \to [0,1]$ be the i^{th} coordinate projection. We suppose without loss of generality that $X \subset \pi_1^{-1}(1)$.

Let $\{A_i\}_{i=1}^{\infty}$ be a countable dense set in C(X). We may suppose, without loss of generality, that each A_j appears infinitely often in the sequence $\{A_i\}_{i=1}^{\infty}$. For each $i \in \{1,2,...\}$ let $K_i = \{a_{i,1},...,a_{i,n_i}\}$ be a finite subset of A_i such that $A_i \subset \bigcup_{j=1}^{n_i} S(a_{i,j}, 1/2i)$ and $K_i \cap K_j = \emptyset$ for $i \neq j$. (Note $S(p,\varepsilon)$ denotes the open ε-ball centered at p for $p \in Q$ and $\varepsilon > 0$). For each $i \in \{2,3,...\}$ let $L_i = \{a_{i-1,n_{i-1}} = b_{i,1}, b_{i,2},...,b_{i,m_i} = a_{i,1}\}$ be a finite subset of X such that $d(b_{i,j}, b_{i,j+1}) < 1/i$ for each $j \in \{1,...,m_i-1\}$. For each $i \in \{1,2,...\}$ and for each $j \in \{1,...,n_i\}$ let

$$c_{i,j} = (1 - 1/i, \pi_2(a_{i,j}), \pi_3(a_{i,j}), ...).$$

For each $i \in \{2,3,...\}$ and for each $j \in \{1,...,m_i\}$ let

$$d_{i,j} = (1 - \frac{1}{i-1} + \frac{j-1}{(m_i-1)i(i-1)}, \pi_2(b_{i,j}), \pi_3(b_{i,j}), \ldots).$$

For each $i \in \{2,3,\ldots\}$ let

$$D_i = d_{i,1}d_{i,2} \cup d_{i,2}d_{i,3} \cup \ldots \cup d_{i,m_i-1}d_{i,m_i}$$

where pq denotes the convex arc in Q with ends p and q. Then D_i is a polygonal arc. For each $i \in \{1,2,\ldots\}$ let C_i be a polygonal arc in the Hilbert cube $\pi_1^{-1}(1 - 1/i)$ with endpoints $c_{i,1}$ and c_{i,n_i} such that C_i is contained in the 1/i-neighborhood (in $\pi_1^{-1}(1 - 1/i)$) of $\{c_{i,1},\ldots,c_{i,n_i}\} \subset C_i$. Let $E = C_1 \cup \bigcup_{i=2}^{\infty} (C_i \cup D_i)$ and let $Y = E \cup X$. Then E is a half-line and Y is a compactification of E such that $A_i \in \text{Cl}[C(E)]$ for each $i \in \{1,2,\ldots\}$. Hence, $C(Y) = \text{Cl}[C(E)]$. By 3.3, Y is in Class (W).

4. Some Necessary Conditions for Class (W)

Among the most useful necessary conditions for Class (W) is the following result which was proved in [6, (3.6)] and was used repeatedly in [7] and [9] (for a more general result see [9, 3.1]).

4.1. *Proposition* (Grispolakis, Nadler, Tymchatyn [6]). *If* X *is a continuum in Class* (W) *and* A *is a subcontinuum of* X *that locally separates* p *and* q *in* X, *then* A *separates* p *and* q *in* X.

As we mentioned at the beginning of this paper, continua in Class (W) are not triods. Theorem 3.5, though, tells us that continua in Class (W) may contain triods. The first person to construct such an example was B. Hughes. The theorem that follows tells us that it is impossible to construct such an example in the plane.

4.2. *Theorem* (Grispolakis and Tymchatyn [9]). *For a planar continuum to be in Class* (W) *it is necessary that it be atriodic.*

In [11] Ingram asked for a characterization of tree-like continua which are in Class (W). Combining Theorem 2.4 and Theorem 4.2 we obtain the following partial answer to Ingram's question.

4.3. *Theorem* ([9]). *A planar tree-like continuum is in Class* (W) *if and only if it is atriodic.*

It is known that two-dimensional planar continua contain 2-cells, and hence, they contain triods. Thus, planar continua in Class (W) are one-dimensional.

4.4. *Theorem. Let* X *be a planar continuum which admits finite open covers of arbitrarily small mesh whose nerves are graphs with Betti numbers not exceeding some positive integer* N. *Then* X *is in Class* (W) *if and only if the following conditions are satisfied:*

(i) X *is atriodic,*

(ii) if the continuum K *locally separates* X *between some points* p *and* q, *then* K *separates* X *between* p *and* q.

Proof. The necessity of the conditions (i) and (ii) is given by Theorem 4.2 and Proposition 4.1, respectively. The sufficiency of the conditions (i) and (ii) is given by Theorem 2.3.

Example 5.4 in [7] shows that the condition imposed on the covers of the continuum X cannot be omitted.

Problem 1. What intrinsic conditions characterize planar continua which are in Class (W)?

Let P be a property of topological spaces. We say that P is a *Whitney property* provided that for each continuum X with property P, each Whitney map $\mu: C(X) \to [0,\infty)$ and each $t \in [0, \mu(X))$, $\mu^{-1}(t)$ has property P. Whitney properties were first studied by Krasinkiewicz [12] and by Krasinkiewicz and Nadler [13]. It was proved in [19, (14.76.8)] that being in Class (W) hereditarily, and hence having the covering property hereditarily, are Whitney properties. It was asked in [19, (14.76.9-10)] whether the property of being in Class (W) or the covering property are Whitney properties.

4.5. *Theorem* (Grispolakis and Tymchatyn [9, 4.5]). *Being in Class* (W) *or having the covering property are not Whitney properties.*

Next we prove a result on upper semi-continuous decompositions of continua into continua which are in Class (W). An irreducible continuum X is said to be of *type* λ (see [14, p. 216]) provided that there exists a monotone mapping $\phi: X \to I$ of X onto the unit interval $I = [0, 1]$ with preimages of points being nowhere dense s[illegible]ntinua. The subcontinua $\phi^{-1}(t)$ are called the *tranches* of X. A tranche $\phi^{-1}(t)$ is said to be a *tranche of cohesion* provided that $t = 0$ or $t = 1$ or

$$\phi^{-1}(t) = \mathrm{Cl}[\phi^{-1}([0,t))] \cap \mathrm{Cl}[\phi^{-1}((t,1]))].$$

4.6. *Theorem. Let* X *be an irreducible continuum of type* λ *such that each tranche is a tranche of cohesion and each non-degenerate tranche is in Class* (W). *Then* X *is in Class* (W).

Proof. Let $f: Y \to X$ be a mapping of a continuum Y onto X, and let $\phi: X \to [0,1]$ be a finest monotone mapping of X onto [0,1]. Since the arc [0,1] is in Class (W), the mapping $\phi o f: Y \to [0,1]$ is weakly confluent. Notice also that every subcontinuum of X is either a subset of some tranche or of the form $\phi^{-1}[a,b])$ for some a, b with $0 \leq a \leq b \leq 1$. We first prove that if $K = \phi^{-1}([a,b])$ with $a < b$, then there exists a component C of $f^{-1}(K)$ mapped onto K. Since $\phi o f$ is weakly confluent, there exists a component C of $(\phi o f)^{-1}([a,b])$ such that $\phi o f(C) = [a,b]$. We have that $f(C) \subset \phi^{-1}([a,b])$ and f(C) is a continuum that meets both $\phi^{-1}(a)$ and $\phi^{-1}(b)$. Hence, $f(C) = \phi^{-1}([a,b])$.

Consider, now, a subcontinuum K of X of the form $K = \phi^{-1}(a)$ for some $a \in [0,1]$, and assume, without loss of generality, that $a > 0$. Let $a_1, a_2, \ldots$ be an increasing sequence in [0,a) converging to a. Since $\phi^{-1}(a)$ is a tranche of cohesion, we have that

$$\phi^{-1}(a) = \operatorname*{Lim}_{i\to\infty} \phi^{-1}([a_i,a]).$$

For each i there exists a continuum C_i in Y such that $f(C_i) = \phi^{-1}([a_i,a])$. Assume, without loss of generality, that $C = \operatorname*{Lim}_{i\to\infty} C_i$. Then C is a subcontinuum of Y such that $f(C) = \phi^{-1}(a)$.

Finally, let K be a subcontinuum of $\phi^{-1}(a)$ for some $a \in [0,1]$. As we proved above there exists a subcontinuum

C of Y such that $f(C) = \phi^{-1}(a)$. Since $\phi^{-1}(a)$ is a continuum in Class (W), the mapping $f|C: C \to \phi^{-1}(a)$ is weakly confluent. Thus, there exists a subcontinuum L of C such that $f(L) = K$. This completes the proof that f is weakly confluent. Hence, X is in Class (W).

4.7. *Corollary. Let* G *be a continuous collection of continua in Class* (W) *(some may be degenerate) such that* $\bigcup G$ *is an irreducible continuum. Then* $\bigcup G$ *is in Class* (W).

The following example shows that the hypothesis in Theorem 4.6 that each tranche be a tranche of cohesion is necessary.

4.8. *Example.* Let

$X = \{(-x,y) | y = \sin \frac{\pi}{x}, \ 0 < x \leq 1\} \cup \{(0,y) | -1 \leq y \leq 1\} \cup$

$\{(x,y) | y = \frac{1}{2}\sin \frac{\pi}{x}, \ 0 < x \leq 1\}.$

Then X is an irreducible continuum of type λ such that every tranche is degenerate except one which is an arc, and hence, a continuum in Class (W). Notice that the only non-degenerate tranche $K = \{(0,y) | -1 \leq y \leq 1$ is not a tranche of cohesion, and that the continuum X is not in Class (W).

Problem 2. Characterize those irreducible continua of type λ which are in Class (W) provided that each non-degenerate tranche is in Class (W).

5. Semi-Confluent and Pseudo-Confluent Mappings

The first result of this section is similar to the Cook-Read result mentioned at the beginning of this paper.

5.1. *Theorem. A continuum* X *is hereditarily indecomposable if and only if every mapping from any continuum onto* X *is semi-confluent.*

Proof. By Cook [3], if X is a hereditarily indecomposable continuum, then every mapping from any continuum onto X is confluent, and hence, semi-confluent. Conversely, let X be a continuum which is not hereditarily indecomposable, and let $K = A \cup B$ be a decomposable continuum where A and B are proper subcontinua of K. Let $a \in A\setminus B$, $b \in B\setminus A$ and $c \in A \cap B$, and let U and V be two open subsets of A and B, respectively, with $a \in U$, $b \in V$, $Cl(U) \subset A\setminus B$ and $Cl(V) \subset B\setminus A$. Then the component C_1 of c in $A\setminus U$ meets Bd(U), and the component C_2 of c in $B\setminus V$ meets Bd(V), since A and B are continua. Therefore, $C_1 \cup C_2$ is a continuum such that $C_1 \cup C_2 \subset A \cup B\setminus\{a,b\}$, $[C_1 \cup C_2] \cap (A\setminus B) \neq \emptyset \neq [C_1 \cup C_2] \cap (B\setminus A)$. Let A_1 and B_1 be homeomorphic copies of A and B, respectively, such that A_1, B_1 and X are pairwise disjoint, and let $Z = X \cup A_1 \cup B_1$. Then there exists a natural mapping $f: Z \to X$ of Z onto X which maps X onto X by the identity, and A_1 and B_1 homeomorphically onto A and B, respectively. Define an equivalence relation $\sim$ on Z by setting $x \sim y$ if and only if $x = y$ or $f(x) = f(y) = a$ or $f(x) = f(y) = b$. Let Y be the quotient space $Z/\sim$ and let $g: Y \to X$ be the unique mapping such that $g\circ\phi = f$, where $\phi: Z \to Y$ is the quotient map. Then it is obvious that g is an onto mapping such that $g^{-1}(C_1 \cup C_2)$ has two components L_1 and L_2 with $L_1 \subset A_1$, $L_2 \subset B_2$ with the property that

$$f(L_1) \not\subset f(L_2) \quad \text{and} \quad f(L_2) \not\subset f(L_1).$$

Thus, the mapping g is not semi-confluent. This proves that if X is a continuum such that every mapping from any continuum onto X is semi-confluent, then X is hereditarily indecomposable.

We say that a continuum X is in *Class* (P) provided that every mapping from any continuum onto X is pseudo-confluent. It is obvious that every continuum which is in Class (W) is also in Class (P). It was proved in [9, 5.1] that Class (P) is strictly larger than Class (W).

By using methods as in the proof of Theorem 3.2, the following characterizations of Class (P) were given in [9]:

5.2. *Theorem* (Grispolakis and Tymchatyn [9]). *Let* X *be a continuum. Then the following are equivalent:*

(a) X *is in Class* (P);

(b) for any Whitney map $\mu\colon C(X) \to [0, \infty)$ *and for each* $t \in [0, \mu(X)]$, *if* Λ *is a subcontinuum of* $\mu^{-1}(t)$ *such that* $\bigcup \Lambda = X$, *then every irreducible subcontinuum* $A \in \mu^{-1}(t)$ *belongs to* Λ;

(c) if $X \subset Y$ *for some continuum* Y, *and* $\{X_i\}_{i=1}^{\infty}$ *is a sequence of subcontinua of* Y *converging to* X, *then for each irreducible subcontinuum* K *of* X *there exists a sequence of continua* $\{K_i\}_{i=1}^{\infty}$ *converging to* K *such that* $K_i \subset X_i$ *for each* i.

5.3. *Theorem* ([9]). *Let* X *be an atriodic continuum. Then the following are equivalent:*

(a) X *is in Class* (W);

(b) X *is in Class* (P).

We remark that the Theorems 3.4, 4.1, 4.7, 4.8 and

Corollary 4.4 still hold true if we replace "Class (W)" by "Class (P)". We also mention that Example 4.5 in [9] proves also the following result:

5.4. *Theorem. Being in Class* (P) *is not a Whitney property.*

Finally, in connection with Theorem 4.6 where "Class (W)" is replaced by "Class (P)", we pose the following:

Problem 3. Characterize those irreducible continua of type λ which are in Class (P) provided that each non-degenerate tranche is in Class (P).

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