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A COVERING PROPERTY WHICH IMPLIES ISOCOMPACTNESS II*

H. H. Wicke and J. M. Worrell, Jr.

1. Introduction

We discussed in [WoW] (referred to subsequently as I) a method of defining weak covering properties, a method having forerunners in [W_2] and closely related to some concepts of [HV]. In particular, we defined weak $[\alpha,\infty)^r$-refinability, a weakening of the $[\alpha,\infty)^r$-refinability of [HV]. The weak property for $\alpha = \omega_1$ generalizes weak $\delta\theta$-refinability and implies isocompactness. The main theorem of I states that for $\kappa \geq \omega_0$, every weakly $[\kappa,\infty)^r$-refinable space such that every subset of cardinality $>\kappa$ has a 2-limit point has the property that closed ultrafilters with the κ-intersection property are fixed.

The main purpose of this paper is to characterize the covering properties of I and certain generalizations of them with the use of covering properties defined by a simultaneous generalization of star refinement and order at a point as well as a generalization of the distributivity property of weak θ-refinability. We use the term *barycentric* in defining such properties in a natural extension of the terminology of [E]. The main theorem, which involves eight types of covering properties, is 3.1. A simple corollary is the following.

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1.1. *Theorem. Suppose that* X *is a space and* H *is an open cover* X *of regular cardinality such that there is a collection* Q *of open collections with* $|Q| \leq \omega_0$ *and for each* $p \in X$ *there exist* $\mathcal{V} \in Q$ *and* $\mathcal{H}_p \subseteq \mathcal{H}$ *such that* $|\mathcal{H}_p| \leq \omega_0$ *and* $\{V \in \mathcal{V}: p \in V\}$ *is not empty and refines* $\mathcal{H}_p$. *Then there is a countable collection* Q' *of open collections refining* $\mathcal{H}$ *such that for all* $p \in X$, *there exists* $\mathcal{V} \in Q'$ *such that* $0 < |\{V \in \mathcal{V}: p \in V\}| < |\mathcal{H}|$.

The use of star refinements in describing a covering property involving finitude at a point is illustrated by the well known theorem of Stone [S] which we formulate so as to make the analogy to 3.1 clear.

1.2. *Theorem* [S]. *A* T_1*-space* X *is paracompact if and only if for every open cover* $\mathcal{H}$ *of* X *there is an open refinement* $\mathcal{U}$ *of* $\mathcal{H}$ *such that for each* $p \in X$ *there exists* $\mathcal{H}_p \subsetneq \mathcal{H}$ *such that* $|\mathcal{H}_p| = 1$ *and* $\{V \in \mathcal{U}: p \in V\}$ *refines* $\mathcal{H}_p$.

Worrell in [W_1] showed that an analogous characterization of metacompactness could be given if the condition of star refinement is relaxed.

1.3. *Theorem* [W_1]. *A space* X *is metacompact if and only if for every open cover* $\mathcal{H}$ *of* X *there exists an open refinement* $\mathcal{U}$ *of* $\mathcal{H}$ *such that for each* $p \in X$ *there exists* $\mathcal{H}_p \subsetneq \mathcal{H}$ *such that* $|\mathcal{H}_p| < \omega_0$ *and* $\{V \in \mathcal{U}: p \in V\}$ *refines* $\mathcal{H}_p$.

Subsequently θ-refinability was characterized in an analogous way [W_3]. See [W_2], for an earlier formulation.

In addition to the main characterization theorem we give some applications, proving some analogues to theorems of I. We also give examples illustrating dependence of some of the covering properties on cardinalities. For additional background see [WW_3, WoW].

2. Definitions and Notation

We use ω_0 and ω_1 to denote the first and second infinite cardinals, respectively equal to the corresponding limit ordinals. Greek letters will denote cardinal or ordinal numbers according to context. If κ is a cardinal, κ^+ denotes its successor cardinal.

2.1. *Notation.* If X is a set and $\mathcal{V}$ is a collection of subsets of X and $A \subseteq X$, then $(\mathcal{V})_A = \{V \in \mathcal{V}: A \cap V \neq \emptyset\}$. In the particular case where $A = \{p\}$, we write $(\mathcal{V})_p$. (Such notation has been used by H. Junilla.)

We introduce here a slightly more elaborate terminology than that of I, partly because of the scope of the method of proof of Theorem 3.1. The neighborhoodwise refinements are used here only in Theorem 3.1.

2.2. *Definitions.* Let κ and μ be cardinal numbers. An open cover $\mathcal{H}$ of a space X is said to have a *weak* $\langle\kappa,\mu\rangle$-*refinement* ($\langle\kappa,\mu\rangle$-*refinement*) if and only if there is a collection Q such that

(1) Each member of Q is an open collection (open cover) which refines $\mathcal{H}$.

(2) $|Q| < \kappa$.

(3) For all $p \in X$, there exists $\mathcal{V} \in Q$ such that $0 < |(\mathcal{V})_p| < \mu$.

An open cover $\mathcal{H}$ is said to have a *weak* $\langle\kappa,\mu\rangle$*-barycentric refinement* ($\langle\kappa,\mu\rangle$*-barycentric refinement*) if and only if there is a collection Q such that (2) and the appropriate part of (1) above are satisfied and,

(4) For all $p \in X$, there exist $\mathcal{V} \in Q$ and a collection $\mathcal{H}_p \subseteq \mathcal{H}$ such that $|\mathcal{H}_p| < \mu$ and $(\mathcal{V})_p$ is not empty and refines $\mathcal{H}_p$.

An open cover $\mathcal{H}$ is said to have a *(weak)* $\langle\kappa,\mu\rangle$*-neighborhoodwise refinement,* correspondingly a *(weak)* $\langle\kappa,\mu\rangle$*-neighborhoodwise barycentric refinement* provided there exists a collection Q satisfying the conditions (1)-(2); and (3), correspondingly, (4) are replaced by:

(3') For all $p \in X$, there exist $\mathcal{V} \in Q$ and a neighborhood W_p of p such that $p \in \cup\mathcal{V}$ and $|(\mathcal{V})_{W_p}| < \mu$, correspondingly,

(4') For all $p \in X$, there exist $\mathcal{V} \in Q$, a neighborhood W_p of p, and $\mathcal{H}_p \subseteq \mathcal{H}$ such that $|\mathcal{H}_p| < \mu$, $p \in \cup\mathcal{V}$ and $(\mathcal{V})_{W_p}$ refines $\mathcal{H}_p$.

Terminology. In the various cases of Definition 2.2, we say that the collection Q *determines* the corresponding type of refinement.

2.3. *Remark.* The types of refinement defined in Definition 2.1 of I in the terminology of 2.3 above are, respectively, a weak $\langle\mu,\mu\rangle$-refinement, a $\langle\mu,\mu\rangle$-refinement. In view of this, a space X is *(weakly)* $[\kappa,\infty)^r$*-refinable* if every open cover $\mathcal{H}$ of X such that $|\mathcal{H}|$ is regular and $|\mathcal{H}| \geq \kappa$ has a (weak) $\langle|\mathcal{H}|,|\mathcal{H}|\rangle$-refinement.

We restate here Worrell's characterization of θ-refinability using the terminology of 2.2.

2.4. *Theorem* [W_3]. *Suppose every open cover* $\mathcal{H}$ *of a space* X *has a* $\langle \omega_1, \omega_0 \rangle$*-barycentric refinement. Then* X *is* θ*-refinable.*

3. A Characterization Theorem

In this section we show that certain of the weak covering properties of Definition 2.2 can be characterized in terms of barycentric refinements. The regularity of the cardinality of uncountable covers plays a role as is evident from the proof of 3.1. In the case of weak refinements, regularity does not matter as Theorem 3.4 shows. If countable open covers $\mathcal{H}$ have $\langle |\mathcal{H}|, |\mathcal{H}| \rangle$-barycentric refinements, so that the spaces are countably metacompact, then the restriction to regularity in the non-weak case is no longer needed, as Theorem 3.3 shows. Theorem 3.2 shows how countable metacompactness can be used in the singular case of countable cofinality.

3.1. *Theorem. Let* X *be a topological space and let* $\mathcal{H}$ *be an open cover of* X *of regular cardinality* μ. *Let* κ *be a cardinal number. Then:*

(1) $\mathcal{H}$ *has a (weak)* $\langle \kappa, \mu \rangle$*-barycentric refinement if and only if* $\mathcal{H}$ *has a (weak)* $\langle \kappa, \mu \rangle$*-refinement.*

(2) $\mathcal{H}$ *has a (weak)* $\langle \kappa, \mu \rangle$*-neighborhoodwise barycentric refinement if and only if* $\mathcal{H}$ *has a (weak)* $\langle \kappa, \mu \rangle$*-neighborhoodwise refinement.*

Proof. The sufficiency is clear in all cases. We prove necessity. For each $p \in X$, let W_p denote $\{p\}$ for the proofs of the theorems of part (1) and let W_p denote a neighborhood of p for the proofs of the theorems of part (2). It will be clear that essentially the same argument will suffice for all 4 theorems. Let Q be a collection determining a fixed one of the types of $\langle\kappa,\mu\rangle$-barycentric refinements of the cover $\mathcal{H}$ involved in the statement of the theorem. We show that $\mathcal{H}$ has the corresponding type of $\langle\kappa,\mu\rangle$-refinement. Let $\mathcal{H} = \{H_\delta: \delta < \mu\}$. For each $V \in \mathcal{V} \in Q$, let $\alpha(V,\mathcal{V}) = \min\{\delta < \mu: H_\delta \supseteq V\}$, and for each $\alpha < \mu$, let $\mathcal{G}(\alpha,\mathcal{V}) = \{V \in \mathcal{V}: \alpha = \alpha(V,\mathcal{V})\}$. Let $\mathcal{J}(\mathcal{V}) = \{\cup\mathcal{G}(\alpha,\mathcal{V}): \alpha < \mu\}\setminus\{\emptyset\}$. Note that if $\cup\mathcal{G}(\alpha,\mathcal{V}) = \cup\mathcal{G}(\beta,\mathcal{V}) \neq \emptyset$, then $\alpha = \beta$. For if $V \in \mathcal{G}(\alpha,\mathcal{V})$, $V \subseteq H_\alpha$. Since $V \subseteq \cup\mathcal{G}(\beta,\mathcal{V}) \subseteq H_\beta$, $\alpha \leq \beta$. Similarly, $\beta \leq \alpha$. For each $\mathcal{V} \in Q$, let $C(\mathcal{V}) = \{p \in \cup\mathcal{V}$: there exists $\mathcal{H}_p$ such that $(\mathcal{V})_{W_p}$ refines $\mathcal{H}_p$ and $|\mathcal{H}_p| < \mu\}$. If $p \in C(\mathcal{V})$, let $\sigma_0(p) = \sup\{\delta: H_\delta \in \mathcal{H}_p\}$. Since $|\mathcal{H}_p| < \mu$ and μ is regular, $\sigma_0(p) < \mu$. Suppose $B \in \mathcal{J}(\mathcal{V})$ and $W_p \cap B \neq \emptyset$. Then for some α, $B = \cup\mathcal{G}(\alpha,\mathcal{V})$. There exists $V \in \mathcal{G}(\alpha,\mathcal{V})$ such that $W_p \cap V \neq \emptyset$, so there is $H_\delta \in \mathcal{H}_p$ such that $V \subseteq H_\delta$. Hence $\alpha \leq \delta \leq \sigma_0(p)$. Hence $|(\mathcal{J}(\mathcal{V}))_{W_p}| < \mu$. This argument proves the weak cases of (1) and (2). To prove the other two cases one only needs to note that if $\mathcal{V}$ covers X, so does $\mathcal{J}(\mathcal{V})$.

3.2. *Theorem. Let* X *be a countably metacompact* $[\omega_1,\infty)^r$*-refinable space. Then every infinite open cover* $\mathcal{H}$ *of* X *has a* $\langle|\mathcal{H}|,|\mathcal{H}|\rangle$*-refinement.*

Proof. Let $\mathcal{H}$ be an infinite open cover of X. If $|\mathcal{H}|$ is regular, it has a $\langle|\mathcal{H}|,|\mathcal{H}|\rangle$-refinement. Suppose $\alpha = |\mathcal{H}|$

is singular and let $\kappa = cf(\alpha)$. Then $\mathcal{H}$ can be expressed as $\cup\{\mathcal{M}_\gamma : \gamma < \kappa\}$ where $0 < |\mathcal{M}_\gamma| < \alpha$ for all $\gamma < \kappa$. For each $\gamma < \kappa$, let $V_\gamma = \cup\mathcal{M}_\gamma$. Then $\mathcal{U} = \{V_\gamma : \gamma < \kappa\}$ is an open cover of X of regular cardinality. Hence there exists a collection Q of open covers of X such that $\cup Q$ refines $\mathcal{U}$, $|Q| < \kappa$, and for all $p \in X$ there exists $\mathcal{V} \in Q$ such that $|(\mathcal{V})_p| < \kappa$. For each $\gamma < \kappa$ and $\mathcal{V} \in Q$ let $\mathcal{W}(\mathcal{V},\gamma) = \{H \cap W: W \in \mathcal{V}$ and $V_\gamma = F(W,\mathcal{U})$ and $H \in \mathcal{M}_\gamma\}$ where $F(W,\mathcal{U})$ = the first element of $\mathcal{U}$ which includes W. Let $\mathcal{W}(\mathcal{V}) = \cup\{\mathcal{W}(\mathcal{V},\gamma): \gamma < \kappa\}$. For each $p \in X$, choose $\mathcal{V} \in Q$ such that $\beta = |(\mathcal{V})_p| < \kappa$. For $W \in (\mathcal{V})_p$, there exists $\gamma < \kappa$ such that $V_\gamma = F(W,\mathcal{U})$. Hence there is $H \in \mathcal{M}_\gamma$ such that $p \in H \cap W$. Since $|(\mathcal{V})_p| < \kappa$, $|\{\gamma$: for some $W \in (\mathcal{V})_p$, $V_\gamma = F(W,H)\}| < \kappa$. Also each $|(\mathcal{W}(\mathcal{V},\gamma))_p| \leq |\mathcal{M}_\gamma| \cdot \beta < \alpha$. Hence $|(\mathcal{W}(\mathcal{V}))_p| < \alpha$. Thus $\{\mathcal{W}(\mathcal{V}): \mathcal{V} \in Q\}$ determines a $\langle|\mathcal{H}|,|\mathcal{H}|\rangle$-refinement of $\mathcal{H}$.

3.3. *Theorem. Suppose that every infinite open cover* $\mathcal{H}$ *of regular cardinality of a space* X *has a* $\langle|\mathcal{H}|,|\mathcal{H}|\rangle$*-barycentric refinement. Then every infinite open cover* $\mathcal{H}$ *of* X *has a* $\langle|\mathcal{H}|,|\mathcal{H}|\rangle$*-refinement; in particular,* X *is countably metacompact.*

Proof. By Theorem 3.1, every countable open cover $\mathcal{H}$ of X has a $\langle|\mathcal{H}|,|\mathcal{H}|\rangle$-refinement. Thus the space is countably metacompact. Since Theorem 3.1 implies that X is $[\omega_1,\infty)^r$-refinable, the conclusion follows from Theorem 3.2.

Note that for the weak case, the theorem corresponding to Theorem 3.3 is trivial.

3.4. *Theorem. Suppose that every open cover* $\mathcal{H}$ *of* *regular cardinality* $\geq \kappa$ *of a space* X *has a weak* $\langle |\mathcal{H}|, |\mathcal{H}|\rangle$-*barycentric refinement. Then every open cover* $\mathcal{H}$ *of cardinality* $\geq \kappa$ *has a weak* $\langle |\mathcal{H}|, |\mathcal{H}|\rangle$-*refinement.*

Proof. That open covers $\mathcal{H}$ of regular cardinality $\geq \kappa$ have weak $\langle |\mathcal{H}|, |\mathcal{H}|\rangle$-refinements follows immediately from Theorem 3.1. If $\alpha = |\mathcal{H}|$ is singular, we may write $\mathcal{H} = \cup\{\mathcal{M}_\gamma : \gamma < cf(\alpha)\}$ where $0 < |\mathcal{M}_\gamma| < \alpha$ for all $\gamma < cf(\alpha)$. Thus $Q = \{\mathcal{M}_\gamma : \gamma < cf(\alpha)\}$ determines a weak $\langle |\mathcal{H}|, |\mathcal{H}|\rangle$-refinement of $\mathcal{H}$.

4. Applications

In this section we make a few applications. The main Theorem 4.1 is a consequence of the characterization Theorem 3.1 and Theorem 3.1 of I. It can be given a direct proof, of course. The corollaries stated are of interest also, since even the weakest one, 4.2, is new and is not merely a restatement of the weakly $\delta\theta$-refinable case.

4.1. *Theorem. Suppose* X *is a space and* κ *is an infinite cardinal such that every well ordered increasing open cover* $\mathcal{H}$ *of* X *of regular cardinality* $>\kappa$ *has a weak* $\langle |\mathcal{H}|, |\mathcal{H}|\rangle$-*barycentric refinement. If every subset of* X *of cardinality* $>\kappa$ *has a* 2-*limit point, then no free closed ultrafilter on* X *has the* κ-*intersection property.*

Proof. Theorem 3.1 shows that every well ordered increasing open cover $\mathcal{H}$ of regular cardinality $>\kappa$ has a weak $\langle |\mathcal{H}|, |\mathcal{H}|\rangle$-refinement. Thus the result follows from Theorem 3.1 of I.

4.2. *Corollary. Suppose* X *is a space and every well* *ordered increasing open cover $\mathcal{H}$ of* X *of regular uncountable cardinality has a weak* $\langle |\mathcal{H}|, |\mathcal{H}| \rangle$*-refinement. Then:*

(1) If X *is countably compact, then* X *is compact.*

(2) If X *is* T_1 *and* ω_1*-compact, then* X *is closed-complete.*

Proof. If X is countably compact every set of cardinality $\geq\omega_0$ has an ω-accumulation point. Since every closed filter in a countably compact space has the ω_0-intersection property, it follows from Theorem 4.1 that there are no free closed ultrafilters on X. This proves (1). If X is T_1 and ω_1-compact then by Theorem 4.1 there are no free closed ultrafilters with the countable intersection property.

An easy corollary of the preceding result is worth stating since the covering property involved in its hypothesis is implied by weak $\delta\theta$-refinability, and thus, by numerous other covering properties (see [WW_3] for discussion and references).

4.3. *Corollary. Suppose that* X *is a space and every open cover of* X *of regular uncountable cardinality has a weak* $\langle \omega_1, \omega_1 \rangle$*-refinement. Then*

(1) If X *is countably compact,* X *is compact.*

(2) If X *is* T_1 *and* ω_1*-compact,* X *is closed-complete.*

(3) If X *is* T_1 *and no closed discrete subspace of* X *has cardinality* $>\kappa\geq\omega_0$*, then no free closed ultrafilter on* X *has the* κ*-intersection property.*

4.4. *Remark.* If a space X is weakly $\delta\theta$-refinable [WW_3], then it satisfies the hypothesis of 4.2.

5. Some Examples

The theorem of this section presents a class of examples which show the dependence of certain of the concepts of Definition 2.2 on the cardinalities involved. The examples are Hausdorff first countable scattered spaces and, although not regular, they are pararegular [WW_2], a concept which can effectively substitute for regularity for spaces having bases of countable order.

5.1. *Theorem. For every regular infinite cardinal* κ, *there is a scattered first countable Hausdorff space* X *of cardinality* κ^+ *having a base* $\mathcal{B}$ *such that* $|(\mathcal{B})_x| \leq \kappa$ *for all* $x \in X$ *(hence every open cover has a* $\langle 2,\kappa^+\rangle$*-refinement) but such that no open cover by sets of* $\mathcal{B}$ *has a* $\langle \kappa^+,\kappa\rangle$*-refinement.*

Proof. Let κ be a regular infinite cardinal and let $F(\kappa^+)$ be the set of all limit ordinals $<\kappa^+$ which are limits of increasing countable sequences of ordinals. For each $\lambda \in F(\kappa^+)$ choose an increasing sequence $\{\mu(n,\lambda): n \in \mathbf{N}\}$ of non limit ordinals such that $\lambda = \lim_{n\to\infty} \mu(n,\lambda)$. Let

$S_I(\kappa^+) = \{(\alpha,\beta) \in \kappa^+ \times \kappa^+: \alpha$ is not a limit ordinal or $\alpha \in F(\kappa^+)$ and $\beta > \alpha\}$,

and

$$S_{II}(\kappa^+) = \{(\alpha,\alpha): \alpha \in F(\kappa^+)\}, \text{ and}$$
$$S(\kappa^+) = S_I(\kappa^+) \cup S_{II}(\kappa^+).$$

For $(\alpha,\beta) \in S_I(\kappa^+)$, let $\{(\alpha,\beta)\}$ be a neighborhood base at (α,β). For $(\alpha,\alpha) \in S_{II}(\kappa^+)$, define $D(n,\alpha) = ([\mu(n,\alpha),\alpha[\times \{\beta: \beta \geq \alpha\}) \cup \{(\alpha,\alpha)\}$. A neighborhood base at $(\alpha,\alpha) \in S_{II}(\kappa^+)$ is defined by $\{D(n,\alpha): n \in \mathbf{N}\}$. With these neighborhood assignments $S(\kappa^+)$ is a scattered first countable Hausdorff non-regular space and is thus basically complete [WW_1, Theorem 3.8]. The space is weakly θ-refinable, since the subspaces $S_I(\kappa^+)$ and $S_{II}(\kappa^+)$ are discrete. Also $S(\kappa^+)$ is quasi-developable. Let $\mathcal{B}$ be the base consisting of the union of the neighborhood bases defined above. Suppose $(\alpha,\beta) \in S(\kappa^+)$. Then $|\{\lambda \in F(\kappa^+): \lambda \leq \beta\}| \leq |\beta| \leq \kappa$. Since (α,β) is not in any set $D(n,\lambda)$ with $\beta < \lambda$, it follows that $|(\mathcal{B})_{(\alpha,\beta)}| \leq \kappa$.

Suppose $\mathcal{H}$ is any cover of $S(\kappa^+)$ by sets of $\mathcal{B}$ and Q is a collection of open covers of $S(\kappa^+)$ such that $\cup Q$ refines $\mathcal{H}$ and $|Q| \leq \kappa$. Let $\mathcal{V} \in Q$. For each $(\alpha,\alpha) \in S_{II}(\kappa^+)$ choose a $D(n(\alpha),\alpha)$ which is a subset of some element of $\mathcal{V}$ containing (α,α). Then $\alpha \mapsto \mu(n(\alpha),\alpha)$ determines a regressive function on the stationary set $F(\kappa^+)$, so by the Pressing Down Lemma, there is a stationary set $E(\mathcal{V}) \subseteq F(\kappa^+)$ and $\gamma(\mathcal{V}) < \kappa^+$ such that $\mu(n(\alpha),\alpha) = \gamma(\mathcal{V})$ for all $\alpha \in E(\mathcal{V})$. For each $\mathcal{V} \in Q$ there is such a $\gamma(\mathcal{V})$ and $E(\mathcal{V})$. Let $\xi = \sup\{\gamma(\mathcal{V}): \mathcal{V} \in Q\}$. Then for $\mathcal{V} \in Q$, $\alpha > \xi$ and $\alpha \in E(\mathcal{V})$, $\{\xi\} \times \{\beta: \beta \geq \alpha\} \subseteq D(n(\alpha),\alpha)$. There exists $\nu(\mathcal{V})$ such that $\xi < \nu(\mathcal{V}) < \kappa^+$ and $|\{\alpha \in E(\mathcal{V}): \xi < \alpha < \nu(\mathcal{V})\}| = \kappa$. Thus all points (ξ,η) with $\eta \geq \nu(\mathcal{V})$ are in κ sets in $\mathcal{V}$. Let $\tau < \kappa^+$ exceed all $\nu(\mathcal{V})$ for $\mathcal{V} \in Q$. Then any point (ξ,η) with $\eta \geq \tau$ is in κ sets in every $\mathcal{V} \in Q$. Thus $\mathcal{H}$ has no $\langle \kappa^+,\kappa \rangle$-refinement.

5.2. *Remark.* It follows from the above proof and [WW_1, Theorem 3.8, 4.3] and [WW_2, Theorem 3.1] that each of the spaces $S(\kappa^+)$ is a pararegular space having λ-bases hereditarily, and is an open continuous image of a complete metric 0-dimensional space of the same weight and cardinality as $S(\kappa^+)$.

References

[E] R. Engelking, *General topology*, Polish Scientific Publishers, Warsaw, 1977.

[HV] R. E. Hodel and J. E. Vaughan, *A note on* [a,b]-*compactness*, Gen. Top. Appl. 4 (1974), 179-189.

[S] A. H. Stone, *Paracompactness and product spaces*, Bull. Amer. Math. Soc. 54 (1948), 977-982.

[WW_1] H. H. Wicke and J. M. Worrell, Jr., *Topological completeness of first countable Hausdorff spaces* I, Fund. Math. 75 (1972), 209-222.

[WW_2] ________, *Topological completeness of first countable Hausdorff spaces* II, Fund. Math. 91 (1976), 11-27.

[WW_3] ________, *Point-countability and compactness*, Proc. Amer. Math. Soc. 55 (1976), 427-431.

[W_1] J. M. Worrell, Jr., *A characterization of metacompact spaces*, Portugal Math. 25 (1966), 171-174.

[W_2] ________, *On collections of domains inscribed in a covering of a space in the sense of Alexandroff and Urysohn*, Portugal Math. 26 (1967), 405-420.

[W_3] ________, *Some properties of full normalcy and their relation to Čech completeness*, Notices Amer. Math. Soc. 14 (1967), 555.

[WoW] ________ and H. H. Wicke, *A covering property which implies isocompactness* I, Proc. Amer. Math. Soc. (to appear).

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