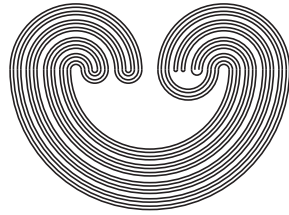

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Research Announcement:

THE SPAN OF MAPPINGS AND SPACES

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THE SPAN OF MAPPINGS AND SPACES

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Let X, Y be metric spaces, and let $f: X \rightarrow Y$ be a mapping. By p_1 and p_2 we denote the standard projections of the product $X \times X$ onto X , i.e., $p_1(x, x') = x$ and $p_2(x, x') = x'$ for $(x, x') \in X \times X$. The span $\sigma(f)$ of the mapping f is the least upper bound of the set of real numbers α with the following property: there exist connected sets $C_\alpha \subset X \times X$ such that $p_1(C_\alpha) = p_2(C_\alpha)$ and $\alpha \leq \text{dist}[f(x), f(x')]$ for $(x, x') \in C_\alpha$ (see [2], p. 99). The span $\sigma(X)$ of the space X is the span of the identity mapping on X (see [4], p. 209). The purpose of the present paper⁽¹⁾ is to announce some results which relate to spans of mappings and have a number of interesting consequences for spans of spaces. A complete version will be published elsewhere.

The proofs of the following four propositions are rather straightforward.

1. If $f: X \rightarrow Y$, then $0 \leq \sigma(f) \leq \sigma(Y) \leq \text{diam } Y$.
2. If $f: X \rightarrow Y$ and X is compact, then
 $\text{Inf}\{d[f^{-1}(y), f^{-1}(y')]: \sigma(f) \leq \text{dist}(y, y')\} \leq \sigma(X)$.
3. If $f: X \rightarrow Y$, X is compact and $0 < \epsilon \leq \text{diam } Y$, then
 $0 < \text{Inf}\{d[f^{-1}(y), f^{-1}(y')]: \epsilon \leq \text{dist}(y, y')\}$.
4. If $f: X \rightarrow Y$ and X is compact, then $\sigma(X) = 0$ implies $\sigma(f) = 0$.

¹This paper was presented during the Thirteenth Spring Topology Conference at Ohio University, on March 17, 1979.

Note that proposition 4 follows from propositions 2 and 3. By S we denote the unit circle on the plane, and by T we denote the union of two tangent circles each of radius $1/2\pi$. We consider T to be a metric space with the geodesic metric ρ . In other words, $\rho(y, y')$ is the length of the shortest arc joining the points y and y' in T for $y, y' \in T$, so that the diameter of T is one. We say that a mapping is *essential* if it is not homotopic to a constant mapping.

5. *Lemma.* If $f: S \rightarrow T$ is an essential mapping and $0 \leq \epsilon \leq \frac{1}{2}$, then there exist a continuum K and two surjective mappings $\phi, \psi: K \rightarrow S$ such that

$$\rho[f\phi(x), f\psi(x)] = \epsilon \quad (x \in K).$$

6. *Theorem.* If $f: X \rightarrow T$ is an essential mapping, X is compact, $\dim X \leq 1$ and $0 \leq \epsilon \leq \frac{1}{2}$, then there exists a continuum $K \subset X \times X$ such that $p_1(K) = p_2(K)$ and $\rho[f(x), f(x')] = \epsilon$ for $(x, x') \in K$.

The following four statements are corollaries to theorem 6.

7. If $f: X \rightarrow T$ is an essential mapping, X is compact and $\dim X \leq 1$, then $\sigma(f) \geq \frac{1}{2}$.

8. If $f: X \rightarrow T$ is an essential mapping, X is compact, $\dim X \leq 1$ and $0 < \epsilon \leq \frac{1}{2}$, then

$$0 < \text{Inf}\{d[f^{-1}(y), f^{-1}(y')]: \rho(y, y') = \epsilon\} \leq \sigma(X).$$

9. If X is compact and $\sigma(X) = 0$, then each mapping $f: X \rightarrow T$ is inessential.

10. If X is a continuum and $\sigma(X) = 0$, then X is tree-like.

It is known [4] that continua of span zero are one-dimensional if non-degenerate. By corollary 9, the mappings defined on them and having values in one-dimensional polyhedra [3] are all inessential, and then corollary 10⁽²⁾ can be obtained via a well-known characterization of tree-like continua [1]. Also, notice that $\sigma(T) = \frac{1}{2}$. Hence, by proposition 1 and corollary 7, we get $\sigma(f) = \sigma(T)$ for all essential mappings f of one-dimensional compact metric spaces into T . It remains as an open problem to determine a wider class of mappings $f: X \rightarrow Y$ such that $\sigma(f) = \sigma(Y)$.

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²A recent result of James F. Davis establishes the equality between the span and the semi-span [5] for a certain class of continua. Using the tree-likeness of continua of span zero (corollary 10), it implies, among other things, that they have the fixed point property.