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PROBLEM SECTION

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PROBLEM SECTION

CONTRIBUTED PROBLEMS

The contributor's name appears in parentheses after each problem. In most cases, there is an article by the contributor in this volume which contains material related to the problem, and sometimes a specific item in the article is referenced. After some problems there also appears a reference to an article in an earlier volume of these PROCEEDINGS.

The problems under each heading are numbered independently, and pick up where volume 3 left off.

A. Cardinal Invariants

8. (*Saks*) A set $C \subset \beta\omega - \omega$ is a *cluster set* if there exist $x \in \beta\omega - \omega$ and a sequence $(x_n: n \in \omega)$ in $\beta\omega$ such that $C = \{\bar{D} \in \beta\omega - \omega: x = \bar{D} - \lim x_n, \{n: x_n \neq x\} \in \bar{D}\}$. [Here a point of $\beta\omega$ is identified with the ultrafilter on ω that converges to it.] Is it a theorem of ZFC that $\beta\omega - \omega$ is not the union of fewer than 2^c cluster sets? [See especially Theorem 3.1.]

See also C24, C25, C26, C27, C28, P10, and P11.

B. Generalized Metric Spaces and Metrization

15. (*Junnila*) Is every strict p -space submetacompact? [See vol. 3, pp. 375-405. This problem is a generalization of Problem B4, vol. 2.]

16. (*Junnila*) Does there exist, in ZFC, a set X and two topologies τ and π on X such that $\tau \subset \pi$, every π -open

set is an F_σ -set with respect to τ , the space (X, π) is metrizable but the space (X, τ) is not a σ -space?

17. (*van Wouwe*) Is each GO-space X that is hereditarily a Σ -space, metrizable? What if X is compact?

18. (*Burke*) Does every regular space X with a σ -locally countable base have a σ -disjoint base?

C. Compactness and Generalizations

24. (*Saks, attributed to Comfort*) Does there exist a family of spaces $\{X_i: i \in I\}$ with $|I| = 2^c$, $\prod_{i \in I} X_i$ is not countably compact, and $\prod_{i \in J} X_i$ is countably compact, whenever $J \subset I$ and $|J| < 2^c$? [This is a special case of Problem C7, vol. 2. An affirmative answer to any of A8, P10, or P11 would be sufficient to construct such a family.]

25. (*Saks*) Do there exist spaces X and Y such that X^k and Y^k are countably compact for all cardinals k , but $X \times Y$ is not countably compact?

26. (*Comfort*) Let $\alpha \geq \beta \geq \omega$. An infinite space X is called *pseudo- (α, β) -compact* if for every family $\{U_\xi: \xi < \alpha\}$ of nonempty open subsets of X , there exists $x \in X$ such that

$$|\{\xi < \alpha: W \cap U_\xi \neq \emptyset\}| \geq \beta$$

for every neighborhood W of x . If β is singular and $1 < m < \omega$, does there exist a Tychonoff space X such that X^{m-1} is pseudo- (β, β) -compact and X^m is not pseudo- (α, ω) -compact?

27. (*Comfort*) Let $\alpha > \beta \geq \omega$ with $\text{cf}(\alpha) = \omega$. Is there a Tychonoff space X such that X^m is pseudo- (α, β) -compact for all $m < \omega$ and X^ω is not pseudo- (α, β) -compact?

28. (*Nyikos*) Does there exist a separable, first countable, countably compact, T_2 (hence regular) space which

is not compact? [Yes if $BF(c)$ or the ω_1 -tunnel axiom holds.]

29. (Nyikos) Does there exist a first countable, countably compact, noncompact regular space which does not contain a copy of ω_1 ? [Yes if \clubsuit ; also yes in any model which is obtained from a model of \clubsuit by iterated CCC forcing, so that "yes" is compatible with $MA + \neg CH$.]

See also A8, B17, L2, P10, and P11.

E. Separation and Disconnectedness

7. (Kunen) Is there a locally compact, extremally disconnected space which is normal but not paracompact? [Vol. 3, pp. 407-428] [Yes if there exists a weakly compact cardinal.]

L. Topological Algebra

2. (Grant) If every finite power of a group is minimal (or totally minimal, or a $B(A)$ group), must arbitrary powers of the group have the same property?

O. Theory of Retracts; Extension of Continuous Functions

8. (Sennott) If S is a closed subspace of a normal space X such that (X, S) has the γ -ZIP, must $S \times Y$ be C -embedded in $X \times Y$ for every metric space Y of weight $\leq |S|$? [See Theorem 3.1, vol. 3, p. 511.]

9. (Sennott) Characterize metric spaces Y such that if X is a topological space and S is C -embedded in X , then $S \times Y$ is C -embedded in $X \times Y$. Is the space of rational numbers in this class? [See vol. 3, pp. 507-520.]

See also T5 and T6.

P. Products, Hyperspaces, and Similar Constructions

10. (Saks) Does there exist a subset D of $\beta\omega - \omega$ such that $|D| = 2^{\mathfrak{c}}$ and whenever $(x_n: n \in \omega)$ and $(y_n: n \in \omega)$ are sequences in $\beta\omega$ and $x, y \in \beta\omega - \omega$, $d, d' \in D$, and $x = d - \lim x_n$ and $y = d' - \lim y_n$, then $x \neq y$? [See Example 2.3.]

11. (Saks) Does there exist a set D of weak P -points such that $|D| = 2^{\mathfrak{c}}$ and if $x \in \text{cl } A$ for some countable subset A of $\bigcup \{F(d): d \in D\}$, then there exists a countable subset C of D such that $x \notin \text{cl } B$ for all countable subsets B of $\bigcup \{F(d): d \in D \setminus C\}$? Here $F(d)$ is the set of all nonisolated images of d under self-maps of $\beta\omega$ induced by self-maps of ω . [See Section 4.]

See also A8, C24, C25, C26, C27, O8, and O9.

T. Algebraic and Geometric Topology

5. (Pak) Let $\mathcal{J} = \{E, P, B, Y\}$ be an orientable Hurewicz fibering. Is it true that if E satisfies the J -condition, then B and Y do also? Is the converse question true?

6. (Pak) Enlarge the class of Jiang spaces.

INFORMATION ON EARLIER PROBLEMS

Classic Problem III, vol. 1. In vol. 2, it was incorrectly stated that T. Przymusiński has announced the construction, assuming CH, of a normal space with a σ -disjoint base which is not paracompact, and (in a footnote added in proof) that he had withdrawn his claim. The truth is that Przymusiński never made such a claim; the editors made the mistake of reporting a rumor as a fact.

B8, vol. 3. H. Wicke has introduced costratifiable

bases, which generalize θ -bases and point-countable bases, and a simultaneous generalization of costratifiable bases and $\delta\theta$ -bases (AMS Notices, 1978).

B10 and B11, vol. 3. D. Burke [these PROCEEDINGS, vol. 4, p. 25] has shown that a submetacompact (" θ -refinable") regular space with a σ -locally countable base is developable. Thus Problems B10 and B11 have affirmative answers where submetacompact regular spaces are concerned. [B10: Is every space with a σ -locally countable base quasi-developable? B11: Is every collectionwise normal space with a σ -locally countable base metrizable--equivalently, paracompact?] To the article by C. Aull where these problems were first raised, one might add the information that D. Burke [AMS Notices, 1978] showed that T_2 quasi-developable spaces and spaces with primitive bases are preserved under perfect maps and that Kofner [Pac. J. Math., to appear] has shown that the class of quasi-metrizable spaces is also preserved under perfect maps. Also, H. R. Bennett's example of a paracompact, non-metrizable space in MOBI [AMS Proceedings, vol. 26] shows that the class of spaces with σ -locally countable bases is not preserved under compact open mappings.

O5, vol. 3 (Gruenhage, Kosłowski, and Nyikos) Is a non-metrizable AR homeomorphic to I^κ for some κ ? *Solution.* No, Shchepin. The cone over I^κ is also an AR and is not homeomorphic to I^κ for $\kappa > \omega$.

O6, vol. 3 (Nyikos) If X is a BAR, does there exist a BAR Y such that $X \times Y \approx 2^\kappa$ for some κ ? Is $X^{\kappa \approx 2^\kappa}$ for large enough κ ? *Solution.* Yes, Shchepin.

O6, vol. 3. Does there exist an intrinsic characterization of BAR's either among compact spaces or among dyadic spaces? *Partial solution (Shchepin)* Among dyadic spaces of weight $\leq \aleph_1$, the BAR's are characterized by the Bockstein separation property: disjoint cozero sets are contained in disjoint F_δ 's. However, this no longer holds for BAR's of higher weight.

A20, vol. 2 (bottom p. 670). Under GCH there are for every regular κ two normal initially κ -compact spaces whose product is not initially κ -compact. For $\kappa = \omega$, MA suffices. This also answers B15 of the Lecture Notes. *E. K. van Douwen*, The product of two normally initially κ -compact spaces.

B10 (from the Lecture Notes). There are two countably compact groups whose product is not countably compact under MA; under CH they can be hereditarily normal. *E. K. van Douwen*, The product of two countably compact groups.