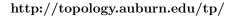
# TOPOLOGY PROCEEDINGS

Volume 5, 1980

Pages 243-245



# **Research Announcement:**

# A NOTE ON THE EQUI-WEAK TOPOLOGY

by

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**ISSN:** 0146-4124

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#### A NOTE ON THE EQUI-WEAK TOPOLOGY

### Donald F. Reynolds and John W. Schleusner

In [1] it is shown that for a given family F of real-valued functions on a set X, there is a weakest topology on X for which F is equicontinuous. This is accomplished by associating with  $F \subseteq R^X$  another family A(F) such that F is equicontinuous if and only if A(F) is continuous. Then the weak topology induced by A(F) is defined to be the equi-weak topology induced by F and is denoted by  $\mathcal{J}^F$ .

In the same paper, an analogy is noted between the role of equi-weak topologies in the theory of pseudometrizable spaces and the role of weak topologies in the theory of completely regular spaces. In particular, we have

Theorem 1. A topology for X is pseudometrizable if and only if it is an equi-weak topology induced by a family of real-valued functions.

The analogy is maintained if we require the functions to separate points and the topology to be Hausdorff.

Theorem 2. A topology for X is metrizable if and only if it is an equi-weak topology induced by a point-separating family of real-valued functions.

If we suppose that X is already endowed with a topology, we can carry the analogy one step further. For just as any subcollection of the continuous functions on X will induce (via the weak topology) a coarser completely regular topology,

so any equicontinuous subcollection of C(X) will induce (via the equi-weak topology) a coarser pseudometrizable topology.

Theorem 3. Let (X,J) be a topological space and let  $F \subset C(X)$ . Then  $J^F \subset J$  if and only if F is equicontinuous.

Proof. Since F is equicontinuous in  $\mathcal{I}^{\mathbf{F}}$ , it is also equicontinuous in any finer topology,  $\mathcal{I}$  in particular. Conversely, F is equicontinuous only if A(F) is continuous. But  $\mathcal{I}^{\mathbf{F}}$  is the coarsest topology on X for which A(F) is continuous. Hence  $\mathcal{I}^{\mathbf{F}} \subseteq \mathcal{I}$ .

In addition to its role in extending the analogy which was established previously, Theorem 3, in conjunction with Theorem 2, provides us with a strikingly simple characterization of submetrizability. A space is said to be submetrizable if its topology contains a metrizable subtopology.

Theorem 4. A space X is submetrizable if and only if C(X) contains a point-separating equicontinuous subcollection.

Proof. Let  $\mathcal{I}_{\mathbf{d}}$  be a metrizable subtopology of  $\mathcal{I}$ . By Theorem 2,  $\mathcal{I}_{\mathbf{d}} = \mathcal{I}^{\mathbf{F}}$  for some point-separating family F. Then by Theorem 3, F is equicontinuous. Conversely, let  $\mathbf{F} \subset \mathbf{C}(\mathbf{X})$  be equicontinuous and separate points. By Theorem 2,  $\mathcal{I}^{\mathbf{F}}$  is metrizable, and by Theorem 3,  $\mathcal{I}^{\mathbf{F}} \subset \mathcal{I}$ .

#### References

 D. F. Reynolds and J. W. Schleusner, Weak topologies and equicontinuity, Proc. Amer. Math. Soc. 80 (2) (1980), 349-352.

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