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W.R. Utz

1. Introduction

This paper gives an overview of results known for the problem of embedding a self-homeomorphism of a topological space into a continuous flow. A way to view the problem is that of determining exactly which homeomorphisms may occur in a flow. When viewed in this way the problem is revealed as both non-trivial and fundamental in topological dynamics.

In the study of embedding one assumes a given space and a given self-homeomorphism of the space and then seeks conditions on that pair to insure the existence of a continuous flow on the space for which the given homeomorphism occurs for a fixed value from the group of reals.

Let X be a topological space and G(+) a topological group. The ordered triple (X,G,π) is a dynamical system if (1) $\pi: X \times G \to X$ is continuous, (2) $\pi(\pi(x,g_1),g_2) = \pi(x,g_1+g_2)$ and (3) $\pi(x,e) = x$ for $e \in G$, the identity, and every $x \in X$. When G is the additive group of reals, the dynamical system is called a continuous flow. If G is the sub-group of integers, then the system is a discrete flow and is simply the integral iterates of the homeomorphism $\pi(x,1)$.

The problem of embedding a self-homeomorphism $f: X \to X$ in a continuous flow is that of finding a dynamical system (X,G,π), G = reals, such that $\pi(x,1) = f(x)$.

The problem just described is called the restricted embedding problem since the flow is restricted to the space X. If the space, X, for which f is a self-homeomorphism may be enlarged to secure a flow containing f, then the problem (which is easier to solve because of this freedom) is called unrestricted. This problem is discussed in Section 5.

Before specializing X we wish to mention that Gary D. Jones has considered the more abstract problem of embedding to groups from dense subgroups with the base space, X, fixed. Let (X,G^*,π^*) be a given dynamical system where G^* is an algebraic subgroup of G which is dense in G, then Jones [13] shows that if X is a complete metric space and π^* is uniformly continuous, then (X,G^*,π^*) can be embedded in a dynamical system (X,G,π).

Also, we call attention to the results of P. L. Sharma and T. L. Hicks [23] wherein they give sufficient conditions for the simultaneous embedding of two self-homeomorphisms of the reals in a continuous flow on the reals.

In the sections that follow we describe results for subsets of the line, simple closed curves and 2-cells (Section 2), the plane (Section 3) and for diffeomorphisms (Section 4).

2. Embeddings for the Line, Simple Closed Curve, Disc

If $f: X \to X$ can be embedded in a continuous flow then $g: X \to X$ can be, also, if g and f are topologically equivalent. Moreover, $f: X \to X$ can be embedded in a continuous flow if, and only if, $g: Y \to Y$ can be embedded in a

continuous flow where Y is homeomorphic to X and g is the induced homeomorphism. Thus, results for connected subsets of the line, for a circle and for the closed unit disc are valid for their respective topological classes.

Embedding results for a subset of a line are old since this is very close to the study of the iterations of certain functions. If f is a monotone increasing, continuous real function from a connected subset of the reals onto that set, then the embedding problem is equivalent to the problem of continuous powers of the function.

It is easily seen that for a connected subset of the line (more generally, any subset of the plane) a self-homeomorphism must be orientation preserving to be a candidate for embedding. This condition is also sufficient in the case of a connected subset of a line, R.

Theorem 1. *A self-homeomorphism of a connected subset of* R *can be embedded in a continuous flow if, and only if, the homeomorphism is orientation preserving (order preserving).*

Theorem 1 has been proved in this setting by N. J. Fine and G. E. Schweigert [5], M. K. Fort [9], Sharma and Hicks [23] and others. A constructive proof was given by the author [27] using results of Morgan Ward [28].

The following theorem characterizes self-homeomorphisms of a simple closed curve embeddable in a continuous flow.

Theorem 2 (Foland and Utz [8]). *Let* f *be an orientation preserving homeomorphism of a simple closed curve,* X,

onto itself. It is possible to embed f *in a continuous flow if, and only if, either*

1) X *contains a fixed point under* f, *or*

2) f *is periodic on* X, *or*

3) f *is transitive on* X. *That is, for some* $x \in X$, *the orbit of* x *under* f, $\sum_{-\infty}^{\infty} f^i(x)$, *is everywhere dense in* X.

Definitive results for a closed 2-cell are not available but the following theorems give sufficient conditions for embeddability.

A self-homeomorphism, f, of a metric space (X,ρ) is said to be almost periodic if $\varepsilon > 0$ implies the existence of a relatively dense sequence, $\{m_i\}$, of integers such that $\rho(x,f^{m_i}(x)) < \varepsilon$ for all $x \in X$ and $i = \pm 1, \pm 2, \cdots$.

Theorem 3 (Foland [6]). *Any almost periodic, orientation preserving self-homeomorphism of a closed* 2*-cell can be embedded in a continuous flow on the* 2*-cell.*

Foland [6] also shows that the embedding of Theorem 3 is unique (in the sense of orbit closure decomposition of the continuous flow) in case the given homeomorphism is strictly almost periodic (i.e., not periodic). He gives an example of a periodic self-homeomorphism of the disc and two embeddings into continuous flows for which the orbit decompositions of the continuous flows are different. Uniqueness, in the sense of orbit decompositions of the continuous flow, exists in the case of embeddings on a simple closed curve and on connected subsets of R.

Theorem 3 improves an earlier theorem of [9] where regular almost periodicity was required of f.

The following theorem is a consequence of Jones' study of embeddings in the plane (described in Section 3).

Theorem 4 (Jones [14]). *Let* f *be an orientation preserving self-homeomorphism of the closed* 2*-cell* D. *Assume that the set of fixed points,* N, *of* f *on* D *is finite and contained in* D - int D. *If for each pair of points* x, y $\in$ D-N *there exists an arc* A $\subset$ D-N *joining* x *and* y *such that* $f^n(A)$ *tends to a fixed point as* $n \to \pm\infty$, *then* f *can be embedded in a continuous flow on* D.

The theorem improves a theorem of [13] wherein all arcs tend to the same fixed point.

3. Embeddings for the Plane

Embedding theorems for the plane, R^2, are primarily due to Stephen A. Andrea [1], Robin Ault [2] and Gary D. Jones [15].

Any flowable homeomorphism (i.e., one that is a candidate for embedding in a flow) must be orientation preserving since if the given homeomorphism, f, is flowable, then $\pi(x,1) = f$, $\pi(x,0)$ is the identity and through $\pi(x,t)$, $0 \leq t \leq 1$, they are isotopic.

A contracting homeomorphism f, of the plane is one for which there is a real number r, $0 < r < 1$, such that $\rho(f(x),f(y)) \leq r\ \rho(x,y)$ for all $x, y \in R^2$. For such a homeomorphism there is a unique fixed point. Foland has given a constructive proof of the following theorem.

Theorem 5 (Foland [7]). *If* f *is an orientation preserving contracting self-homeomorphism of* R^2, *then* f *can be embedded in a continuous flow on* R^2.

Using a theorem of T. Homma and S. Kinoshita [11] one secures a slightly more general result.

Theorem 6 (Jones [15]). *Let* f *be an orientation preserving self-homeomorphism of* R^2 *with one fixed point* x_0. *Suppose that* $f^n(x) \to \infty$ *as* $n \to -\infty$ *and* $f^n(x) \to x_0$ *as* $n \to \infty$ *for all* $x \neq x_0$. *Then* f *can be embedded in a continuous flow on* R^2.

Gy Targonski [25] (with R. Graw) has announced a non-embeddability condition for R^2: If a homeomorphism from a simply connected subset of the plane into itself has a k-cycle ($k \geq 2$) but no fixed point, then it is not embeddable.

Hereafter, in this section, we will only consider fixed point free orientation preserving self-homeomorphisms of the plane.

Let f be an orientation preserving self-homeomorphism of R^2 without fixed points. A set $A \subset R^2$ is said to be f-divergent if corresponding to any compact set $K \subset R^2$, there exists a natural number $N(K)$ such that $f^n(A) \cap K = \phi$ for $|n| > N$.

Andrea [1] has defined the binary relation, $\sim$, on R^2 as $x \sim y$, $x, y \in R^2$, if and only if, there is an f-divergent arc having x, y as endpoints (the arc is to be degenerate if $x = y$). The relation $\sim$ is an equivalence relation for

orientation preserving, fixed point free homeomorphisms and Andrea calls the equivalence classes the fundamental regions of f.

Theorem 7 (Andrea [1]). f *is equivalent to a translation if, and only if, it has one fundamental region.*

That is, an orientation preserving, fixed point free self-homeomorphism of the plane may be embedded in a flow which is topologically equivalent to a translation if, and only if, the homeomorphism has exactly one fundamental region.

The number of fundamental regions for a flowable homeomorphism may be any finite number except 2 (Andrea [1]), a countable infinity or an uncountable infinity (Ault [2]).

The fundamental regions need not be open in R^2. They are arcwise connected, unbounded, invariant under f and if a simple closed curve is contained in a fundamental region, then its interior is also.

In the two figures we give examples of fundamental regions of homeomorphisms of R^2 where there are 5 fundamental regions.

In Figure 1 the homeomorphism moves points one unit of arc length along the curves. Of the fundamental regions, U_{-1} and U_1 are open, while the other three are closed sets. The homeomorphism is visibly embeddable in a flow.

The homeomorphism of Figure 2 (Andrea [1]) is not embeddable in a flow even though it is orientation preserving and

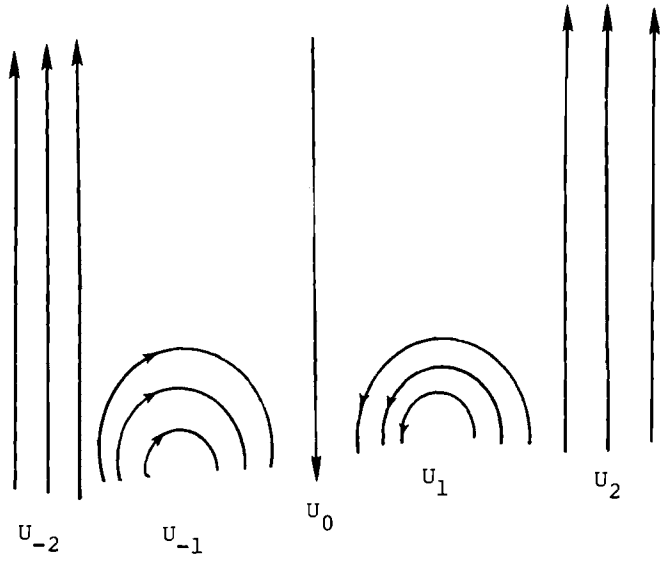

Figure 1

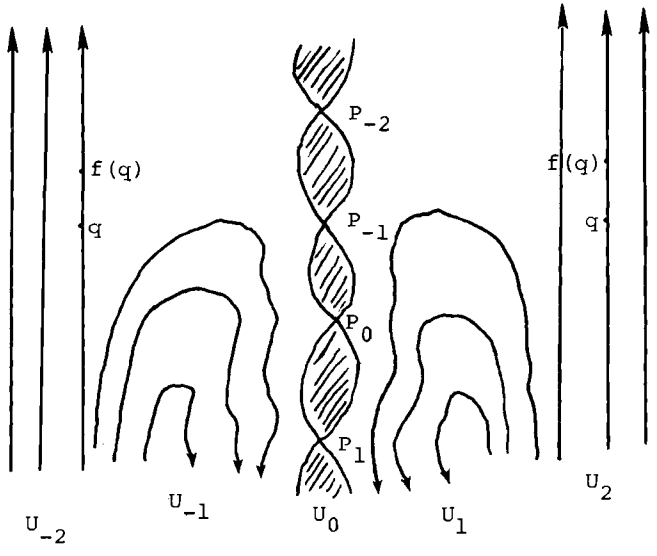

Figure 2

fixed point free. In U_0 the cells are moved downward to the next cell so that the pointset $\{p_i\}$ is invarient as a set. If the cells have diameter 1, the transformation on the complementary set is to be the indicated movement through arc-length 1. It's intuitively evident that the required flow lines could not get through the pointset $\{p_i\}$.

Jones [15] defines relative equivalence in R^2 with respect to f. If $A \subset R^2$, then $x \sim y \pmod{A}$ if x, y are endpoints of an arc $C \subset A$ for which $f^n(C) \to \infty$ as $n \to \pm\infty$.

A proper flowline for f is a subset B of R^2 for which (a) $f(B) = B$, (b) B is homeomorphic to the real line and (c) $B \cup \{\infty\}$ is a Jordan curve on the sphere $R^2 \cup \{\infty\}$.

Jones gives special attention to the following subclass of all self-homeomorphisms of R^2.

Class J. Assume of f that it is an orientation preserving, fixed-point free self-homeomorphism of R^2 for which (1) the fundamental regions, U_i, are finite in number, (2) if $x \in U_i - \text{int } U_i$, then $x \in C \subset U_i - \text{int } U_i$ where C is a proper flowline, (3) if $x_1, x_2 \in \text{int } U_i$, then $x_1 \sim x_2 \pmod{U_i}$.

Theorem 8 (Jones [15]). *Let* f *be a Class* J *homeomorphism. Suppose that* U_1 *and* U_i, $i = 2,3,\cdots,n$, *are not separated. Then*

(a) U_1 *is open*

(b) $f|U_1$ *can be embedded in a continuous flow, and*

(c) *if* $f|\overline{U}_1$ *can be embedded in a continuous flow* π_1, *then* f *can be embedded in a continuous flow* π, *where* $\pi(x,r) = \pi_1(x,r)$ *if* $(x,r) \in \overline{U}_1 \times \text{Reals}$

(d) *if* $f|U_1$ *can be embedded in a continuous flow* π_1, *such that* π_1 *restricted to* $U_1 \times [0,1]$ *is uniformly continuous, then* f *can be embedded in a continuous flow* π.

Jones [15] gives cannonical forms for the cases where f generates m = 3,4,5,6 fundamental regions and indicates how this sequence may be continued for all natural numbers, m.

The work of Ault does not require that the number of fundamental regions be finite. The class of homeomorphisms considered are fixed point free and satisfy the following conditions.

Class A. Assume that f is an orientation preserving self-homeomorphism of the plane. The fundamental regions of f are to satisfy these conditions: (1) Each fundamental region is invariant under f. (2) Each fundamental region is a set of one of the following forms: (a) a Jordan line; (b) an open connected set, bounded by a collection of disjoint Jordan lines; or (c) a set an in (b), plus some of the boundary lines. (3) The collection of Jordan lines which are either whole fundamental regions, or boundary lines of fundamental regions, forms a regular curve family.

Each member of the family described in (3) is called a fundamental line.

An f-cell, put intuitively, is a maximal subset of a given closed half-plane in which f can be considered equivalent to a translation (i.e., embedded in a translation). The base line of an f-cell is a boundary line of the f-cell.

Theorem 9 (Ault [2]). *If* f *is fixed point free and flowable, then* f *is in Class* A. *If* U *is any* f*-cell whose base line is a fundamental line of* f, *then* $f|\overline{U}$ *is flowable.*

Theorem 10 (Ault [2]). *Let* f *be a fixed point free, Class* A *self-homeomorphism of the plane. Then, there is a finite or infinite sequence* $U_0, U_1, \cdots$ *of* f*-cells such that*

(1) *the base line of each* U_n *is a fundamental line*

(2) *the* U_n *cover* R^2

(3) *the* U_n *are pairwise disjoint, except that* U_0 *and* U_1 *meet along their common base line*

(4) *the* U_n *form a locally-finite family.*

Given any such sequence, f *is flowable if, and only if, for each* $n \geq 0$, $f|\overline{U}_n$ *is flowable.*

4. Differentiable Embeddings

One may ask for conditions sufficient to embed a C^r-diffeomorphism in a C^s-flow ($s \leq r$) in contrast to the topological problem where $r = s = 0$. There are some results for $s > 0$ but the conditions to insure differentiability for the flow, even C^1, are destined to be restrictive as revealed by theorems that assure one that such cases are not numerous.

For example, J. Palis has shown

Theorem 11 (Palis [22]). *The subset of* C^r*-diffeomorphisms for a compact* C^∞ *manifold without boundary that embed in* C^1*-flows is of first category.*

It isn't even easy to embed a C^r-diffeomorphism in a

topological continuous flow if the base space is a C^∞ manifold without boundary.

Theorem 12 (Palis [21]). *If* M *is a* C^∞ *manifold without boundary and if* U *is any neighborhood of the identity in the group of* C^r *diffeomorphisms of* M, *then there exists an open set* $V \subset U$ *such that no element of* V *embeds in a topological flow.*

For linear manifolds it is possible to identify enough conditions to insure embeddability of a smooth homeomorphism in a differentiable flow.

For example, consider the circle, S, as the interval [0,1] with endpoints identical and consider the set $\Sigma \subset$ Diff [0,1] (the group of C^∞ homeomorphisms of [0,1] with C^∞ inverses) defined by $g \in \Sigma$ if $g(x) < x$ for all $0 < x < 1$ and $g'(0) \neq 1 \neq g'(1)$. Since S is compact there is an induced distance function, d, given by the uniform C^0 topology on Diff [0,1].

Theorem 13 (Kopell [16]). *Given* $f \in \Sigma$ *and* $\varepsilon > 0$, *it is possible to find a* $g \in$ Diff [0,1] *such that* $d(g,f) < \varepsilon$ *and* g *embeds in a* C^∞ *flow.*

Diffeomorphisms of connected subsets of the line have been studied extensively, especially by P. F. Lam [17], with emphasis on the embedding of a C^1-diffeomorphism in a C^1-flow.

Let X be a connected subset of R. Clearly, a differentiable self-homeomorphism, f, of X must satisfy $f'(x) > 0$

to be embeddable in a differentiable flow. If, in addition, f is C^1 and has no fixed point, then f can be embedded in a C^1 continuous flow on X (Bödewadt [4]). If f has one fixed point, then the problem is more difficult. If one adds the condition that f be C^2 to $f'(x) > 0$ then f can be embedded in a C^1 flow (Szekeres [24]). M. K. Fort [9] and Gordon G. Johnson [12] have given conditions to replace the C^2 condition of Szekeres.

By adding a variety of conditions to that of being C^1, Lam [18,19,20] has secured numerous sufficient conditions for the embeddability of f on any connected subset of R.

Ault [2] has given an example of a C^n diffeomorphism $f: R^2 \to R^2$ which is flowable but not C^n flowable ($n > 0$). Also, Ault shows that even to assume f to be a C^∞ diffeomorphism will not make any one of the conditions of Class A to be superfluous.

5. The Unrestricted Embedding Problem

In the previous sections the given self-homeomorphism of X was embedded in a continuous flow on X, itself. This is called a restricted embedding. If one is permitted to enlarge X then the embedding problem always has a solution. Such an embedding is called unrestricted.

That is, one is now given a subgroup, G^*, of a group G and a dynamical system (X,G^*,π^*). One seeks a dynamical system (Y,G,π) such that Y contains X homeomorphically; $\pi(y,g^*)$, $g^* \in G^*$, is invariant on a subset Z of Y homeomorphic to X and $\pi | Z \times G^*$ is topologically equivalent to $\pi^*: X \times G^* \to X$.

There are two well-known [8] embeddings for self-homeomorphisms of any separable metric space and one of them, the twisted cylinder (or suspension), is valid for any topological space.

We first describe the twisted cylinder solution of the problem as generalized by Jones [13].

Let G be a topological group and G^* be a discrete subgroup of G. Let (X,G^*,π^*) be given where X is any topological space. Let $(x_1,g_i) \sim (x_2,g_2)$ provided $g_1 - g_2 \in G^*$ and $\pi^*(x_1,g_1 - g_2) = x_2$ where $(x_i,g_i) \in X \times G$ for i = 1,2. The relation $\sim$ is an equivalence relation. Let $Y = (X \times G)/\sim$. If $(x_1,g_1) \in \hat{x} \in Y$ by $\pi(\hat{x},g) = \hat{y}$. With this definition (Y,G,π) is a dynamical system. We take $Z = \{\hat{x} \in Y \mid (x,o) \in \hat{x}\}$ and define the homeomorphism $h: X \to Z$ by $h(x) = \hat{x}$, for $(x,o) \in \hat{x}$. Z is homeomorphic to X and $\pi \mid Z \times G^*$ is topologically equivalent to $\pi^*: X \times G^* \to X$.

The properties of the new space, Y, are a function of both X and π^*. Evidently there has not been a study of this question in any depth. That is, the topological properties of Y for specific spaces X and mappings π^*. However, with very little effort [9] one may see that independent of π^*, Y is connected, regular, perfectly separable, separable or compact if X has the respective property.

A second solution to the unrestricted embedding problem is known when G^* is the integers, G is the reals and X is a separable metric space. This solution depends on a flow of Bebutoff [3] and is called the Bebutoff solution of the problem.

Let F be the set of all real functions f(y) defined and continuous for all $y \in R$. The Bebutoff metric for F is

$$\rho(f,g) = \sup_{A>0} \min \{ \sup_{|y| \leq A} |f(y) - g(y)|, 1/A \}.$$

For $f \in F$, $t \in R$, define $\phi(f,t) = f(y+t) \in F$ to secure a continuous flow on F.

Now, to secure the base space, Y, for the Bebutoff flow, let Y be all sequences $f = (f_1, f_2, \cdots)$, $f_1 \in F$ and $0 \leq f_1(y) \leq 1$ for $y \in R$. For $f, g \in Y$

$$d(f,g) = \sum_{i=1}^{\infty} 2^{-1} \rho(f_i, g_i)$$

provides a metric in Y. For each $f \in Y$, $t \in R$ define

$$\pi(f,t) = (\phi(f_1,t), \phi(f_2,t), \cdots)$$

to secure the Bebutoff flow on Y.

If X is a separable metric space, then any discrete flow given by a self-homeomorphism $T: X \to X$ can be embedded in the Bebutoff flow. To achieve this embedding, let Q_ω denote all sequences $x = (x_1, x_2, \cdots)$, $x_i \in R$, $0 \leq x_i \leq 1$ with metric

$$\sigma(x,y) = \sum_{i=1}^{\infty} 2^{-i} |x_i - y_i|$$

for $x, y \in Q_\omega$. It's well-known that Q_ω is a universal metric space and so there is a homeomorphism $H: X \to X^* \subset Q_\omega$. Corresponding to the given self-homeomorphism T, define $M = H T H^{-1}$ on X^*. For $x = (x_1, x_2, \cdots) \in X^*$ define

$$f_i(u) = x_i^{(n)} + (u-n)(x_i^{(n+1)} - x_i^{(n)});$$

$$n \leq u \leq n + 1; \quad n = 0, \pm 1, \pm 2, \cdots,$$

$i = 1,2,3,\cdots$, where

$$x^{(n)} = (x_1^{(n)}, x_2^{(n)}, \cdots) \equiv M^n(x).$$

Let $f = (f_1, f_2, \cdots)$ where the f_i are defined above. Then, $f \in Y$. Let H^* denote the homeomorphism just described of

X^* into Y and let $H^*(X^*) = Z \subset Y$. Then, $\pi(Z,t)$ is an invariant subset of Y and $\pi(f,t)$ in a continuous flow on Z. Z is homeomorphic to X, $\pi(f,1)$ leaves Z invariant and $\pi(f,1)$ is topologically equivalent to the given homeomorphism, T.

Jones [13] notices that of the two solutions of the problem given, they can't be the same for all initial homeomorphisms T. For if both apply (i.e., X is a separable metric space) then they are surely different if T has a fixed point because π does not leave the point fixed for the twisted cylinder solution but does leave it fixed for the Bebutoff solution. Lacking a fixed point, the solutions are topologically equivalent.

Theorem 14 (Jones [13]). *Let* f *be a self-homeomorphism of a metric space* X. *Let* (Y,R,π) *be any solution of the unrestricted problem for* f, *where for convenience of notation we assume that* $X \subset Y$, *such that*

(1) Y *is a metric space,*

(2) $\{\pi(x,t)\}$ *in an equicontinuous family of homeomorphisms, and*

(3) *if* $x \in X$ *and* $N \cap X \neq \phi$ *for every neighborhood,* N, *of* $\pi(x,t_1)$, *then* t_1 *is an integer.*

Then, $(\pi(X,R),R,\pi)$ *is topologically equivalent to the twisted cylinder solution of the embedding problem.*

Specifically, as a corollary, one has the following theorem.

Theorem 15 (Jones [13]). *Let* $f: X \to X$ *be a self-homeomorphism of the separable metric space,* X. *The*

twisted cylinder solution and the Bebutoff solution of the unrestricted embedding problem for f *are topologically equivalent if, and only if,* f *has no fixed points.*

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