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Research Announcement:

COMMON FIXED POINT THEOREMS

by

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Definition. (Takahashi) Let X be a metric space and I be closed unit interval. A mapping W: $X \times X \times I \to X$ is said to be a *convex structure* on X if for all x, y $\in$ X and $\lambda \in I$ the following condition is satisfied

$$d(u,W(x,y;\lambda)) \leq \lambda d(u,x) + (1-\lambda)d(u,y)$$

for all u in X. A metric space with a convex structure is called a *convex metric* space.

Theorem 1. *Let* K *be a nonempty compact convex subset of a convex metric space* X. *If* S *is a left reversible semigroup of nonexpansive mappings of* K *into itself, then* K *contains a common fixed point of* S.

Theorem 2. *Let* X *be a compact metric space and* G: $X \to X$ *be a linearly ordered semigroup of mappings. Suppose* G *has diminishing orbital diameter and there exists* $g \in G$ *with* $g \neq I$ *such that*

(i) G *is continuous mapping with diminishing orbital diameter,*

(ii) G *is Archimedean at* g.

Then G *has a common fixed point.*

Theorem 3. *Let* X *be a convex metric space having property* (C) *and* H *be a closed convex subset of* X. *Let* K *be a bounded, closed convex subset of* H *with normal structure.*

If T: K $\to$ H *is nonexpansive and if* T: $\partial_H K \to K$ ($\partial_H K$ *is the relative boundary of* H $\cap$ K *in* H), *then* T *has a fixed point in* K.

Also we prove a common fixed point theorem for commuting linearly ordered semigroup of nonexpansive mappings having convex diminishing orbital diameter.

Theorem 1 generalizes results of De Marr [1], Mitchell [4] and Takahashi [5,6]. Theorem 2 and 3 extend the results of Kirk [2], [3] respectively.

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