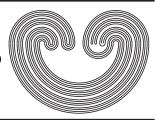
TOPOLOGY PROCEEDINGS

Volume 6, 1981 Pages 201–202



http://topology.auburn.edu/tp/

Research Announcement: COMMON FIXED POINT THEOREMS

by

S. A. NAIMPALLY, K. L. SINGH, AND J. D. M. WHITFIELD

Topology Proceedings

Web:	http://topology.auburn.edu/tp/
Mail:	Topology Proceedings
	Department of Mathematics & Statistics
	Auburn University, Alabama 36849, USA
E-mail:	topolog@auburn.edu
ISSN:	0146-4124

COPYRIGHT © by Topology Proceedings. All rights reserved.

COMMON FIXED POINT THEOREMS

S.A. Naimpally, K.L. Singh, and

J.H.M. Whitfield

Definition. (Takahashi) Let X be a metric space and I be closed unit interval. A mapping W: $X \times X \times I \rightarrow X$ is said to be a *convex structure* on X if for all x, y \in X and $\lambda \in I$ the following condition is satisfied

 $d(u,W(x,y;\lambda)) < \lambda d(u,x) + (1-\lambda)d(u,y)$

for all u in X. A metric space with a convex structure is called a *convex metric* space.

Theorem 1. Let K be a nonempty compact convex subset of a convex metric space X. If S is a left reversible semigroup of nonexpansive mappings of K into itself, then K contains a common fixed point of S.

Theorem 2. Let X be a compact metric space and G: $X \rightarrow X$ be a linearly ordered semigroup of mappings. Suppose G has diminishing orbital diameter and there exists $g \in G$ with $g \neq I$ such that

(i) G is continuous mapping with diminishing orbital diameter,

(ii) G is Archimedean at g.

Then G has a common fixed point.

Theorem 3. Let X be a convex metric space having property (C) and H be a closed convex subset of X. Let K be a bounded, closed convex subset of H with normal structure. If T: K + H is nonexpansive and if T: $\partial_H K$ + K ($\partial_H K$ is the relative boundary of H \cap K in H), then T has a fixed point in K.

Also we prove a common fixed point theorem for commuting linearly ordered semigroup of nonexpansive mappings having convex diminishing orbital diameter.

Theorem 1 generalizes results of De Marr [1], Mitchell [4] and Takahashi [5,6]. Theorem 2 and 3 extend the results of Kirk [2], [3] respectively.

References

- R. De Marr, Common fixed points for commuting contraction mappings, Pacific J. Math. 13 (1963), 1139-1141.
- W. A. Kirk, On mappings with diminishing orbital diameters, J. London Math. Soc. 44 (1969), 107-111.
- [3] _____, Fixed point theorems for nonlinear nonexpansive and generalized contraction mappings, Pacific J. Math. 38 (1971), 89-94.
- [4] T. Mitchell, Fixed points of reversible semigroups of nonexpansive mappings, Kodai Math. Sem. Rep. 22 (1972), 322-323.
- W. Takahashi, Fixed point theorems for amenable semigroups of nonexpansive mappings, Kodai Math. Sem. Rep. 21 (1969), 383-386.
- [6] _____, Convexity in metric spaces and nonexpansive mappings I, Kodai Math. Sem. Rep. 22 (1970), 142-149.

Lakehead University Thunder Bay, Ontario Canada P7B 5El