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Research Announcement: HOMOGENEOUS, HEREDITARILY INDECOMPOSABLE CONTINUA

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HOMOGENEOUS, HEREDITARILY INDECOMPOSABLE CONTINUA

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R. H. Bing [2] has proved that the pseudo-arc is homogeneous. This remarkable result stimulated the search to discover which continua are homogeneous. For a history of early work on this topic, see [3].

The pseudo-arc is a hereditarily indecomposable, arc-like (hence tree-like) continuum. No other homogeneous, tree-like continuum is known. Bing [1] has shown that no homogeneous, hereditarily indecomposable continuum is n-dimensional, $2 \leq n < \infty$.

In this announcement, we outline a proof of the following result.

Theorem 1. *Each homogeneous, hereditarily indecomposable continuum is tree-like.*

In 1955, F. B. Jones [6] classified homogeneous plane continua into three kinds: (1) nonseparating, (2) decomposable and separating, and (3) indecomposable and separating. Continua of types (1) and (3) must be hereditarily indecomposable [5], while continua of type (2) not homeomorphic to S^1 admit a continuous decomposition into elements of type (1) such that the resulting quotient space is homeomorphic to S^1 [6]. Hence the next theorem is a corollary to Theorem 1.

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Theorem 2. *Homogeneous, separating plane continua are decomposable.*

A result of E. G. Effros [4] has invigorated the search for homogeneous continua. We state the result in the following form.

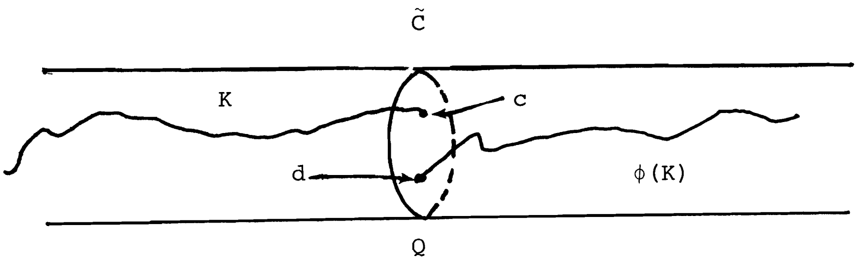
Theorem 3. *Let* M *be a homogeneous continuum. For each* $\varepsilon > 0$, *there exists* $\delta > 0$ *such that if* x *and* y *are points of* M *and* $d(x,y) < \delta$, *then some homeomorphism of* M *onto itself takes* x *to* y *and moves no point of* M *as much as* ε.

We say that a space satisfying the conclusion of Theorem 3 has the Effros property.

Outline of proof of Theorem 1. Let M be a homogeneous, hereditarily indecomposable continuum that is not tree-like. There exists an essential embedding of M into a certain space C, where C is a cube-with-handles if $\dim M = 1$, and the product of S^1 and the Hilbert cube, if $\dim M > 1$. Let $\sigma: \tilde{C} \to C$ be the universal covering space of C, and let $\tilde{M} = \sigma^{-1}(M)$. Each continuum of $\tilde{M}$ is indecomposable. No component of $\tilde{M}$ is compact. There exists a metric on $\tilde{C}$ with the property that σ is a local isometry and each deck transformation is an isometry. With this metric, $\tilde{M}$ has the Effros property.

We use the noncompact component of $\tilde{M}$ to find a continuum K in $\tilde{M}$, a cross-sectional disk Q in $\tilde{C}$, and a deck transformation $\phi: \tilde{C} \to \tilde{C}$ such that (1) some of K lies far to the left of Q and none lies far to the right, (2) some of $\phi(K)$ lies far to the right of Q and none lies far to the left, and (3) there exist points c in $K \cap Q$ and d in $\phi(K) \cap Q$ that are very close to each other.

Because $\tilde{M}$ has the Effros property, there exists a homeomorphism $h: \tilde{M} \to \tilde{M}$ such that $h(c) = d$. This implies that either the continuum $\phi(K) \cup h(K)$ is decomposable or h moves points of K too far to satisfy the Effros property. This contradiction completes the proof.



Proofs of Theorems 1 and 2 appear in [8] and [7], respectively.

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