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by

Thomas J. Peters

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Web:	http://topology.auburn.edu/tp/
Mail:	Topology Proceedings
	Department of Mathematics & Statistics
	Auburn University, Alabama 36849, USA
E-mail:	topolog@auburn.edu
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DENSE HOMEOMORPHIC SUBSPACES OF X* AND OF (EX)*

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1. Introduction and Motivation

The problem, "Characterize those spaces X for which X* and (EX)* have dense homeomorphic subspaces," was posed by R. G. Woods [Wo_3, p. 350]. A partial solution is given in this paper: "Let X be a nowhere locally compact space. Then X* and (EX)* have dense homeomorphic subspaces if and only if there exists a dense subspace of X* such that βX is extremally disconnected at each point of that dense subspace."

That βX is extremally disconnected at each remote point of X was demonstrated by E. K. van Douwen [vD, 5.2]. Thus it is often useful to determine when a space has the property that its set of remote points is dense in its remainder. It is shown that many of the G-spaces defined by Chae and Smith [CS] have this property. In particular the following theorem is proven: "Let X be a nearly realcompact G-space. If X is normal or extremally disconnected, then its set of remote points is dense in X*."

This work generalizes theorems of N. J. Fine and L. Gillman [FG], D. Plank [Pl], S. M. Robinson [R], E. K. van Douwen [vD], C. L. Gates [G], and R. G. Woods [Wo_3].

2. Basic Preliminaries

All spaces are assumed to be completely regular and Hausdorff. Let X be a space. The notation bX denotes a compactification of X; EX denotes the absolute of X [Wo_3, p. 326ff]; $T_{bX}(X)$ denotes the set of bX-remote points of X (defined below); $\tau^*(X)$ denotes the family of non-empty open subsets of X. The subspace $\beta X - X$ will be abbreviated as X*. A space X is said to be *nearly realcompact* if $\beta X - \nu X$ is dense in X* [B]; [vD, 2.7]. Other notation and terminology are standard as in [GJ] or [W], with the exceptions noted below.

2.1 *Definition.* Let X be a space. A point $p \in bX - X$ is bX-*remote* for X if there is no nowhere dense subset A of X such that $p \in cl_{bX}A$. When a point is βX-remote, it will simply be called a remote point (for X). Similarly, $T_{\beta X}(X)$ will simply be denoted as TX.

2.2 *Definition* [vD, 1.7]. A space X is said to be *extremally disconnected at the point* p if for every two disjoint open sets U and V in X

$$p \notin cl_X U \cap cl_X V$$

or, equivalently,

for each $U \in \tau^*(X)$, if $p \in cl_X U$, then $p \in int_X cl_X U$.

3. A Partial Response to Woods' Problem

Throughout this section let $k: EX \to X$ be a closed, perfect, irreducible continuous surjection and let $\bar{k}: \beta EX \to \beta X$ be its Stone extension.

3.1 *Lemma. Let* $p \in \beta X$. *Then* $|(\bar{k})^{-1}(p)| = 1$ *if and only if* βX *is extremally disconnected at* p.

Proof. (⇐) In [G, 2.3], it is shown that if $|(\overline{k})^{-1}(p)| > 1$, then there exists $W \in \tau^*(\beta X)$ such that $p \in cl_{\beta X}W \backslash int_{\beta X}cl_{\beta X}W$. Thus βX, is not extremally disconnected at p.

(⇒) If βX is not extremally disconnected at p, then there exist disjoint $U_1, U_2 \in \tau^*(\beta X)$ such that

$$p \in cl_{\beta X}U_1 \cap cl_{\beta X}U_2.$$

Since $\overline{k}$ is closed, it is clear that for i = 1,2,

$$p \in cl_{\beta X}U_i = \overline{k}(cl_{\beta EX}(\overline{k})^{-1}(U_i)).$$

But $(\overline{k})^{-1}(U_1) \cap (\overline{k})^{-1}(U_2) = \emptyset$. So, the extremal disconnectedness of βEX implies that

$$cl_{\beta EX}(\overline{k})^{-1}(U_1) \cap cl_{\beta EX}(\overline{k})^{-1}(U_2) = \emptyset \text{ [GJ, 1H4].}$$

Thus, $|(\overline{k})^{-1}(p)| > 1$.

3.2 *Remark.* Let S be a subspace of X^* and suppose that $(\overline{k})^{-1}(S)$ is dense in $(EX)^*$. Thus, in order for $\overline{k}|(\overline{k})^{-1}(S)$: $(\overline{k})^{-1}(S) \to S$ to be homeomorphism between dense subspaces of $(EX)^*$ and X^*, it is essential that βX be extremally disconnected at each point of S. (More generally, note that if T is any dense subset of $(EX)^*$ such that $\overline{k}|T: T \to \overline{k}[T]$ is a homeomorphism, then $\overline{k}[(EX)^* \backslash T] = X^* \backslash \overline{k}[T]$, [GJ, 6.11]. Thus, it is necessary that $|(\overline{k})^{-1}(p)| = 1$ for each $p \in \overline{k}[T]$.) In particular, whenever βX is not extremally disconnected at any point of X^*, the indicated mapping is not a homeomorphism. It is conceivable that some other unrelated homeomorphism between dense subspaces may exist, but its construction would necessarily be achieved via other methods. However, for the class of nowhere locally compact spaces, it can be shown that $(EX)^*$ and X^* have dense homeomorphic subspaces if and only if there exists a subspace S of X* such that $\bar{k}|(\bar{k})^{-1}(S)$ is a homeomorphism between dense subspaces.

3.3 *Remark.* Let X be a space and let bX be a compactification of X. Recall that a space is nowhere locally compact if and only if bX - X is dense in bX.

The easy proof of the following lemma is left as an exercise.

3.4 *Lemma. Let* S *be a dense subspace of a space* X. *Then* S *is an extremally disconnected space if and only if* X *is extremally disconnected at each point of* S.

3.5 *Theorem. Let* X *be a nowhere locally compact space and let* S *be a dense subspace of* X*. *Then the following are equivalent:*

(a) S *is an extremally disconnected space.*

(b) βX *is extremally disconnected at each point of* S.

(c) S *is a homeomorph of a dense subspace of* (EX)*.

Proof. (a) ⇒ (b). Remark 3.3 and Lemma 3.4.

(b) ⇒ (c). Since X is nowhere locally compact, the function $\bar{k}|(EX)^*$ is a perfect, irreducible, continuous function [Wo_1, 2.7, 3.1], [Wo_3, 5.3]. Hence, $(\bar{k})^{-1}(S)$ is a dense subspace of (EX)* [Wo_1, 1.5]. To see that $\bar{k}|(\bar{k})^{-1}(S)$ is closed, note that any closed subset of $(\bar{k})^{-1}(S)$ is of the form $C \cap (\bar{k})^{-1}(S)$ for some closed subset C of βEX. Then,

$(\bar{k}\,|(\bar{k})^{-1}(S))[C \cap (\bar{k})^{-1}(S)] = \bar{k}[C \cap (\bar{k})^{-1}(S)] = \bar{k}[C] \cap S$, where the last inequality follows since $\bar{k}\,|(\bar{k})^{-1}(S)$ is one-one by Lemma 3.1. But $\bar{k}[C] \cap S$ is a closed subset of S.

The other properties required for $\bar{k}\,|(\bar{k})^{-1}(S)$ to be a homeomorphism are easily deduced from its construction and from Lemma 3.1.

(c) ⇒ (a). Let $g: S_0 \to S$ be a homeomorphism of dense subspaces of (EX)* and X*. (Note that g is not assumed to be related to $\bar{k}$).

Since EX is nowhere locally compact, S_0 is actually a dense subspace of βEX, and thus, the space S_0 is extremally disconnected [GJ, 1H4]. As the homeomorphism g preserves extremal disconnectedness, it is clear that the space S is also extremally disconnected.

3.6 *Corollary. Let* X *be a nowhere locally compact space. Then* X* *and* (EX)* *have dense homeomorphic subspaces if and only if there exists a dense subspace of* X* *such that* βX *is extremally disconnected at each point of that dense subspace.*

There exist nowhere locally compact, realcompact spaces X such that X* and (EX)* fail to have dense homeomorphic subspaces. In particular, any nowhere locally compact realcompact space X such that βX is not extreamally disconnected at any point of X* has this property.

3.7 *Example.* Let $U(\omega_2)$ denote the space of uniform ultrafilters of the discrete space ω_2. That is,

$U(\omega_2) = \{p \in \beta(\omega_2): |A| = \omega_2 \text{ for all } A \in p\}$.

Let $\mathbb{Q}$ denote the rationals with the usual topology. Let $X = \mathbb{Q} \times U(\omega_2) \times U(\omega_2)$. Then X is a nowhere locally compact, realcompact space, but βX is not extremally disconnected at any point of X* [vDvM, p. 73].

3.8 *Remark.* The results of this section indicate that an initial attack on Woods' problem might include the characterization of all those spaces X such that X* has a dense subspace S where βX is extremally disconnected at each point of S. Such a characterization would not necessarily completely solve Woods' problem (other homeomorphisms unrelated to $\bar{k}$ may exist), but it would provide more information than in currently known. The difficulty of such a characterization led to the consideration of spaces X having the more tractable property that TX is dense in X*. Since βX is extremally disconnected at each remote point of X [vD, 5.2], it is clear that the density of TX in X* is sufficient to imply that X* and (EX)* have dense homeomorphic subspaces whenever the function $\bar{k}\,|(EX)^*$ is perfect and irreducible (see the proof of 3.5 (b) $\Rightarrow$ (c)). It is known that $\bar{k}|(EX)^*$ is a perfect irreducible continuous function if X is nearly realcompact [Wo_3, pp. 347-349], or nowhere locally compact [Wo_1, 2.7, 3.1]; [Wo_3, 5.3].

4. Density of TX in X*

The work of Chae and Smith [CS] established the existence of remote points for nonpseudocompact, normal, G-spaces. The next lemma shows that the hypothesis of normality may be replaced by extremal disconnectedness.

4.1 *Lemma. If* X *is a nonpseudocompact, extremally disconnected* G-*space, then* $|TX| \geq 2^c$.

Proof. Let $U \in \tau^*(X)$, $n < \omega$, and let $\mathcal{F}(U,n)$ be a G-family for U and n [P_2, 2.3]. Without loss of generality, each $F \in \mathcal{F}(U,n)$ is assumed to be regular closed [P_2, 7.15]. Since X is extremally disconnected, each such F is clopen. Using the construction of [CS, Theorem 1] it is clear that there exist remote families consisting of clopen subsets of X, where each such family has the finite intersection property. Let $\mathcal{E}$ be any such family and let D be a nowhere dense subset of X. Then there exists $E_D \in \mathcal{E}$ such that $D \cap E_D = \emptyset$. But, since E_D is clopen, it is clear that $cl_{\beta X}D \cap cl_{\beta X}E_D = \emptyset$. Hence, $\cap\{cl_{\beta X}E: E \in \mathcal{E}\}$ is a non-empty subset of TX. Clearly, $|TX| \geq 2^c$ [CS, Theorem 1].

4.2 *Remark.* Since EX is a G-space if and only if X is a G-space [P_2, 7.12], it is clear that if X is a nonpseudocompact G-space, then $TEX \neq \emptyset$. This observation prompts the following question.

4.3 *Question.* For a nonpseudocompact G-space X, does $TEX \neq \emptyset$ imply that $TX \neq \emptyset$? (For an equivalent problem see Remark 4.4 below.)

4.4 *Remark.* Note that a crucial element in the proof of Lemma 4.1 is the construction of a remote family $\mathcal{E}$ such that for each nowhere dense $D \subset X$, there exists an $E_D \in \mathcal{E}$ with the property that D and E_D are completely separated. In particular, if a nonpseudocompact G-space X is almost normal [SA, 2.1] (i.e., a space X is almost normal if every pair of disjoint closed sets, one of which is regular closed, can be completely separated [L, 3.5]), then a remote family $\mathcal{E}$ of regular closed sets can be constructed [CS, Theorem 1]; [P_2, 7.15] and TX will be non-empty. Thus, an equivalent formulation of Question 4.3 is: "Does there exist a non-pseudocompact G-space X (which necessarily cannot be almost normal) such that TX = $\emptyset$?"

The following lemma appears under a slightly different guise in [R, §3]. For the sake of clarity and completeness, its statement and a proof are included here.

4.5 *Lemma. Let* X *be a space which is extremally disconnected or normal and let* R *be a regular closed subset of* X. *If* p *is a remote point for* R, *then* p *is a remote point for* X.

Proof. If X is extremally disconnected, then R is clopen and the result is obvious.

If X is normal, then $cl_{\beta X}R = \beta R$ [GJ, 3D]. Since p is remote for R, $p \in (cl_{\beta X}R) - R$. Hence $p \in X^*$.

Suppose p is not a remote point for X. Then there exists a closed nowhere dense subset A of X such that $p \in cl_{\beta X}A$. Clearly,

$$p \in cl_{\beta X}R \cap cl_{\beta X}A = cl_{\beta X}(R \cap A),$$

where the set equality is due to the normality of X [Wi, 19K]. But, R ∩ A is a nowhere dense subset of R, contradicting the fact that p is a remote point for R.

The following lemma merely consolidates some results from the literature and is included for reference.

4.6 *Lemma. Let* X *be a space and* $W \in \tau^*(X)$. *Then* $cl_{\upsilon X}W$ *is compact if and only if* $cl_X W$ *is pseudocompact.*

Proof. (⇒) This implication appears in [HJ]. (See also [We, 11.24].)

(⇐) That $cl_{\upsilon X}W$ is pseudocompact may be seen via [CN, 2.5b(ii)]. Hence, $cl_{\upsilon X}W$ is compact [GJ, 8.10]; [W, 1.58].

The folloiwng definition appears in [C], where it is accredited to Frolík.

4.7 *Definition.* A space X is *locally pseudocompact at the point* $x \in X$ if x admits a pseudocompact neighborhood.

The terminology "X is nowhere locally pseudocompact" will have the obvious meaning.

4.8 *Lemma. Let* X *be a non-compact space and consider the following three conditions:*

(a) X *is nowhere locally pseudocompact.*

(b) X *is nearly realcompact.*

(c) *Each basis* $\mathcal{U}$ *of* X^* *has the property that if* $U \in \mathcal{U}$ *and if* $V \in \tau^*(\beta X)$ *such that* $U = V \cap X^*$, *then the space* $cl_X(V \cap X)$ *is nonpseudocompact.*

Conditions (b) *and* (c) *are equivalent and are implied by* (a). *Furthermore, condition* (b) *does not imply* (a).

Proof. (a) ⇒ (b). [JM, 6.1].

(b) ⇒ (c). Let $\mathcal{U}$ be a basis for X^*, let $U \in \mathcal{U}$ and let $V \in \tau^*(\beta X)$ such that $U = V \cap X^*$. Since X is nearly realcompact, it is clear that $V \cap \beta X - \upsilon X \neq \emptyset$. Hence, $cl_{\upsilon X}V = cl_{\upsilon X}(V \cap X)$ is not compact. So, $cl_X(V \cap X)$ is non-pseudocompact, by Lemma 4.6.

(c) ⇒ (b). Let $H \in \tau^*(X^*)$ and let $H_0 \in \tau^*(\beta X)$ such that $H = H_0 \cap X^*$. Let $U_0 \in \tau^*(\beta X)$ such that $U_0 \cap X^* \neq \emptyset$ and $cl_{\beta X}U_0 \subset H_0$.

Let $\mathcal{U}$ be a basis for X^*, let $U \in \mathcal{U}$ such that $U \subset U_0 \cap X^*$ and let $V \in \tau^*(\beta X)$ such that $U = V \cap X^*$. Without loss of generality, V may be chosen such that $V \subset U_0$.

Since $cl_X(V \cap X)$ is nonpseudocompact, it is clear from Lemma 4.6 that

$$\emptyset \neq cl_{\beta X}(V \cap X) \cap (\beta X - \upsilon X) = cl_{\beta X}V \cap (\beta X - \upsilon X).$$

But,

$$cl_{\beta X}V \cap (\beta X - \upsilon X) \subset cl_{\beta X}U_0 \cap (\beta X - \upsilon X) \subset H_0 \cap (\beta X - \upsilon X) \subset H_0 \cap X^* = H.$$

Thus, X is nearly realcompact.

(b) ⇏ (a). The relevant example is a noncompact locally compact metric space, because every metric space is nearly realcompact [R]; [Wo_3, p. 349].

4.9 *Theorem. Let* X *be a space which is extremally disconnected or normal. If* X *is a nearly realcompact* G*-space, then* TX *is dense in* X*. *Furthermore,* X* *and* (EX)* *have dense homeomorphic subspaces.*

Proof. Let $H \in \tau^*(X^*)$ and let $H_0 \in \tau^*(\beta X)$ such that $H = H_0 \cap X^*$. Let $U_0 \in \tau^*(\beta X)$ such that $U_0 \cap X^* \neq \emptyset$ and $cl_{\beta X}U_0 \subset H_0$.

Let $\mathcal{U}$ be the basis for X^*, let $U \in \mathcal{U}$ such that $U \subset U_0 \cap X^*$ and let $V \in \tau^*(\beta X)$ such that $U = V \cap X^*$ and $V \subset U_0$.

Then $cl_X(V \cap X)$ is a nonpseudocompact G-space [CS, p. 244]. Furthermore, $cl_X(V \cap X)$ is extremally disconnected or normal and, therefore, has at least 2^c remote points [CS]. Let p be a remote point for $cl_X(V \cap X)$. Then p is a remote point for X and

$$p \in cl_{\beta X}(V \cap X) - cl_X(V \cap X) = (cl_{\beta X}V) - X$$
$$\subset (cl_{\beta X}U_0) - X \subset H_0 - X = H.$$

The last statement of the theorem now follows from Remark 3.8.

4.10 *Corollary. If* X *is a metric space, then* TX *is dense in* X*.

Of course, Corollary 4.10 could be more directly deduced merely by citing Theorems 1 and 3 of [CS], the lemma of Sec. 3 of [R] and by observing that in the class of metric spaces, pseudocompactness is equivalent to compactness. However, Corollary 4.10 is not the primary result of the arguments given here. It is merely a pleasant by-product. In general, the class of G-spaces is much richer than the class of metric spaces. Thus, consideration of the class of G-spaces lends a much broader context to Woods' problem.

Further examples of the value of considering the class of G-spaces are given below.

Lemma 4.11 appears in the author's doctoral dissertation [P_1, 4.1]. Its statement and proof are included here for the sake of clarity and completeness.

4.11 *Lemma. If* bX *is a compactification of a space* X *and if* f *is the continuous function* $f: \beta X \to bX$ *such that* f *restricted to* X *is the identity function, then* $f^{-1}(T_{bX}(X)) \subset TX$.

Proof. Let $p \in T_{bX}(X)$. Suppose there exists $q \in f^{-1}(p)$ such that $q \notin TX$. Since f is perfect and $p \in bX - X$, it is clear that $q \in \beta X - X$ and, hence, there must exist some nowhere dense subset A of X such that $q \in cl_{\beta X}A$. But then,

$$p = f(q) \in f[cl_{\beta X}A] \subset cl_{bX}A,$$

which is a contradiction.

4.12 *Theorem. If* X *is a nowhere locally compact space and* bX *is a compactification of* X *such that* $T_{bX}(X)$ *is dense in* bX - X, *then* TX *is dense in* X*. *Furthermore,* X* *and* (EX)* *have dense homeomorphic subspaces.*

Proof. Let f be the continuous function $f: \beta X \to bX$ such that f restricted to X is the identity function. Since X is nowhere locally compact, it is clear that $T_{bX}(X)$ is dense in bX.

Furthermore, since f is a closed, irreducible function, the set $f^{-1}(T_{bX}X)$ is dense in βX [Wo_1, 1.5]. Hence, TX is dense in βX, by Lemma 4.11. Therefore, TX is dense in X*.

The second statement follows from Remark 3.8.

4.13 *Example.* Let α and γ be infinite cardinals, where $\gamma \geq \alpha$ and γ has the discrete topology. Let $X = \gamma^{\alpha}$. For each $\xi < \omega$, let $Y_\xi = \gamma$ and for $\omega \leq \xi < \alpha$, let Y_ξ be the one-point compactification of γ. Let $Y = \Pi_{\xi<\alpha}Y_\xi$. Then Y is a nonpseudocompact, normal [S], G-space [P_2, 6.2] and Y is nearly realcompact [Wo_3, p. 349]. So, by Theorem 4.9, TY is dense in Y*. Let $bX = \beta Y$. Then $TY \subset T_{bX}X$.

Let $V \in \tau^*(\beta Y)$. Then,

$$\emptyset \neq TY \cap (V\backslash Y) \subset T_{bX}(X) \cap (V\backslash X).$$

Hence, $T_{bX}(X)$ is dense in bX - X, which, by Theorem 4.12, suffices to show that TX is dense in X*.

Since the space X is nowhere locally compact, it is also obvious that TX is dense in βX.

The hypothesis of Theorem 4.9 that the space be nearly realcompact cannot be entirely eliminated as Example 4.14 demonstrates; that is, there exist nonpseudocompact normal G-spaces and nonpseudocompact extremally disconnected G-spaces, whose sets of remote points are not dense in their remainders.

4.14 *Example*. Let ω be the countable infinite discrete space. Let $W(\omega_1)$ and $W(\omega_1 + 1)$ be, respectively, the spaces ω_1 and $\omega_1 + 1$, each having the well-ordered topology. Let $\oplus$ denote disjoint topological union.

(a) For a normal space, let $X = \omega \oplus W(\omega_1)$. Then $\beta X = \beta\omega \oplus W(\omega_1 + 1)$ and $TX = \omega^*$, but $X^* = \omega^* \oplus \{\omega_1\}$.

(b) For an extremally disconnected space, let $Y = EX = \omega \oplus E(W(\omega_1))$. It is easy to see that $(E(W(\omega_1)))^* \neq \emptyset$. However, since $E(W(\omega_1))$ is pseudocompact with nonmeasurable cellularity, it has no remote points [T, p. 265]. So, $TY = \omega^*$, but $Y^* = \omega^* \oplus (E(W(\omega_1)))^*$.

5. A Related Theorem and Some Examples

The following theorem is related to previous results. It is a modest generalization of Corollary 8.3 of $[P_2]$, where the space X was assumed to be realcompact.

5.1 *Definition*. A space X is a *strong* G-*space* if both X and βX are G-spaces.

5.2 *Theorem. If* X *is nearly realcompact and* X *is a strong* G-*space, then* TX *is dense in* X*. *Furthermore,* X* *and* (EX)* *have dense homeomorphic subspaces.*

Proof. If X is pseudocompact, then the statement is vacuously true, so, without loss of generality, assume X is nonpseudocompact.

The space υX is a strong G-space [P_2, 7.17]. Hence $T(\upsilon X)$ is dense in $\beta X - \upsilon X$ [P_2, 8.3]. But $T(\upsilon X) \subset TX$. Since X is nearly realcompact, it is easily seen that TX is dense in X*.

The second statement follows from Remark 3.8.

5.3 *Examples.* (a) Let T_D be the "Dieudonne Plank" [Wo_3, p. 344]. That T_D is a strong G-space is seen by reference to [P_2, 4.2]. Furthermore, T_D is non-normal, but T_D satisfies the hypotheses of Theorem 5.2 above [Wo_3, p. 344, pp. 348-349].

(b) Let X_0 be Mrowka's example of an almost-realcompact, non-realcompact space [M] (see [Wo_2, 4.1] for a concise readable description). Let κ be any cardinal (finite or infinite). Then X_0^κ is a strong G-space [P_2, 8.7], satisfying the hypotheses of Theorem 5.2 above [Wo_3, pp. 348-349].

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University of Hartford

West Hartford, Connecticut 06117

Current Address: 16 Nicholas Ave.

Groton, Connecticut 06340