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PROBLEM SECTION

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PROBLEM SECTION

CONTRIBUTED PROBLEMS

The contributor's name is in parentheses immediately following the statement of the problem. In most cases, there is an article by the contributor in this volume containing material related to the problem.

B. Generalized Metric Spaces and Metrization

24. (*Peters*) Do there exist spaces X and Y such that neither X nor Y has a σ -discrete π -base (equivalently, a σ -locally finite π -base) but $X \times Y$ has one?

C. Compactness and Generalizations

42. (*Peters*) Is the class of G -spaces [defined in the article in this issue] finitely productive?

43. (*Peters*) Determine conditions on an infinite family of G -spaces which will ensure that their product is G . Specifically, if every countable partial product of some family $\langle X_\xi : \xi < \alpha \rangle$ of spaces is a G -space, then must their full product be one also?

44. (*Peters*) Do there exist non- G -spaces X and Y such that $X \times Y$ is a G -space?

45. (*van Douwen*) Is there a compact Fréchet-Urysohn space with a pseudocompact noncompact subspace? [Yes if $b = c$.]

46. (*van Douwen*) Suppose every pseudocompact subspace of a compact space X is compact. Must X be hereditarily realcompact? [No if \clubsuit .]

47. (*van Douwen*) Is there a regular Baire space X which has a 1-1 regular continuous image Y of smaller weight but no such image that is Baire?

48. (*Nyikos*) Is there a compact non-scattered space that is the union of a chain of compact scattered subspaces?

D. Paracompactness and Generalizations

35. (*Nyikos*) Does there exist a screenable anti-Dowker space? That is, does there exist a screenable space that is countably paracompact but not normal? [If PMEA, any example must be of character $\geq c$.]

E. Separation and Disconnectedness

11. (*Tall*) Levy collapse a supercompact cardinal to ω_2 . Are first countable (locally countable?) \aleph_1 -collectionwise normal spaces collectionwise normal?

See also M6.

F. Continua Theory

16. (*J. T. Rogers*) Is each non-degenerate, homogeneous, non-separating plane continuum a pseudo-arc?

In the following problems, a "curve" is a one-dimensional continuum and "Type 2" if it is aposyndetic but not locally connected, and homogeneous.

17. (*J. T. Rogers*) Is each Type 2 curve a bundle over the Menger universal curve with Cantor sets as the fibers?

18. (*J. T. Rogers*) Is each Type 2 curve an inverse limit of universal curves and maps? universal curves and fibrations? universal curves and covering maps?

19. (*J. T. Rogers*) Does each Type 2 curve contain an arc?

20. (*J. T. Rogers*) Does each Type 2 curve retract onto a solenoid?

21. (*J. T. Rogers*) Does each indecomposable cyclic homogeneous curve that is not a solenoid admit a continuous decomposition into tree-like curves so that the resulting quotient space is a solenoid?

22. (*J. T. Rogers*) Is every acyclic homogeneous curve tree-like? In other words, does trivial cohomology imply trivial shape for homogeneous curves?

23. (*J. T. Rogers*) Is every tree-like, homogeneous curve hereditarily indecomposable? a pseudo-arc? weakly chainable? does it have span zero? the fixed-point property?

24. (*J. T. Rogers*) Are any two tree-like, homogeneous curves hereditarily equivalent?

25. (*J. T. Rogers*) Is each decomposable, homogeneous continuum of dimension greater than one aposyndetic?

26. (*J. T. Rogers*) Is each indecomposable, nondegenerate, homogeneous continuum one-dimensional?

27. (*J. T. Rogers*) Must the elements of Jones's aposyndetic decomposition be hereditarily indecomposable?

28. (*J. T. Rogers*) Let X be a homogeneous curve, and let $H(X)$ be its homeomorphism group. Let \mathcal{G} be a

partition of X into proper, nondegenerate continua so that $H(X)$ respects \mathcal{G} (this means that either $h(G_1) = G_2$ or $h(G_1) \cap G_2 = \emptyset$, for all G_1 and G_2 in \mathcal{G} and all h in $H(X)$). Are the members of \mathcal{G} hereditarily indecomposable?

See also Sections G and I.

G. Mappings of Continua and Euclidean Spaces

13. (*J. Mayer*) Are there uncountably many inequivalent embeddings of the pseudo-arc in the plane with the same prime end structure?

14. (*J. Mayer*) Are there countably many inequivalent embeddings in the plane of every indecomposable chainable continuum (with the same prime end structure)?

15. (*J. T. Rogers*) Can Jones's aposyndetic decomposition raise dimension? lower dimension?

16. (*J. T. Rogers*) A homeomorphism is *primitively stable* if its restriction to some nonempty open set is the identity. Does each homogeneous continuum admit a nontrivial, primitively stable homeomorphism?

17. (*J. T. Rogers*) Is each homogeneous continuum bihomogeneous? That is, given points x and y in X , does there exist a homeomorphism h of X onto itself such that $h(x) = y$ and $h(y) = x$?

See also Sections F, H, I, and P.

H. Homogeneity and Mappings of Other Spaces

9. (*van Douwen*) For a linearly ordered set L define an equivalence relation $T_L = \{(x, y) \in L \times L : \text{there is an order-preserving bijection of } L \text{ taking } x \text{ to } y\}$.

- a. Does \mathbb{R} have a subset L such that T_L has only one equivalence class, but $\langle L, < \rangle$ is not isomorphic to $\langle L, \geq \rangle$?
- b. Does \mathbb{R} have a subset L such that T_L has exactly two equivalence classes, both dense [this much is possible] but of different cardinalities?

See also Sections F and G.

I. Infinite-Dimensional Topology and Shape Theory

7. (*M. Jani*) Is there a cell-like shape fibration $p: E \rightarrow B$ from a compactum E onto the dyadic solenoid B , which is not a shape equivalence?

8. (*J. T. Rogers*) Is any nondegenerate, homogeneous contractible continuum homeomorphic to the Hilbert cube?

See also F21, F22.

K. Connectedness

7. (*van Douwen*) Is there a connected (completely) regular space without disjoint dense subsets? [There are Hausdorff examples.]

See also X1.

M. Manifold Theory

6. (*Tall*) Can one prove the consistency of "normal manifolds are collectionwise normal" without assuming any large cardinal axioms?

P. Products, Hyperspaces, Remainders, and Similar Constructions

21. (*J. T. Rogers*) Let $f: X \rightarrow Y$ be a map between inverse limit spaces. When does there exist a map induced

from commuting diagrams on the inverse sequence that has the desired properties of f (such as being a homeomorphism taking the point x onto the point y)?

See also B24, C42, C43, C44, F18, and S14.

S. Problem Closely Related to Set Theory

13. (*van Douwen*) Let LN be the axiom that every linearly orderable space is normal. Does LN imply AC in ZF? [LN does not imply AC in ZF^- , i.e. without foundation. It is known that LN is equivalent to "for every complete linear order L there is a choice function for the collection of nonempty intervals of L ." From this $ZF \rightarrow LN$ follows easily; $AC \rightarrow LN$ is well known.]

14. (*Nyikos*) Call a point of $\omega^* = (\beta\omega - \omega)$ a *simple P-point* if it has a totally ordered clopen base.

- a. Does the existence of a simple P-point imply the existence of a scale, i.e. a cofinal well-ordered subset of $({}^\omega\omega, <^*)$?
- b. Is it consistent that there exist simple P-points p and q with bases of different cofinalities?

[The cofinality of any simple P-point is either b or d , so there can be at most two different cofinalities, and an affirmative answer to (a) implies a negative answer to (b).]

See also E11.

X. Special Constructions

1. (*G. Johnson*) If (M, S) is a G -system, is S connected?

2. (*G. Johnson*) If (M, S) is a G -system, m is a set in M which contains two points, $\{s\} = S \cap m$, and $p \in m \setminus \{s\}$, is $\{(1-t)s + tp: t \geq 0\}$ a subset of m ?

3. (*G. Johnson*) If (M, S) is a G -system for X and $\{w_i\}_{i=1}^{\infty}$ is a convergent sequence in X , must $\{s_i\}_{i=1}^{\infty}$ be a convergent sequence if s_i and w_i belong to the same set in M for all i ?

INFORMATION ON EARLIER PROBLEMS

Fl, vol. 1 (*Hagopian, attributed to Bing*) Is there a homogeneous tree-like continuum that contains an arc?

Solution. No, *Hagopian*, AMS Proceedings 88 (1983), 560-564.

Lectures on Set-Theoretic Topology 1st printing, C4

(*Hajnal*) Is there a (regular) Hausdorff hereditarily separable space X with $|X| > 2^{\omega_1}$? *Solution.* No if PFA even without "regular" (*S. Todorćević*, AMS transactions 280 (1983), 703-720) but it is also consistent that $2^{<c} = c$, c and 2^c are "arbitrarily large" and there is a regular (even zero-dimensional) hereditarily separable space of cardinality 2^c (*Juhász and Shelah*).

Classic Problem VI, vol. 2. Is there a compact space of countable tightness that is not sequential? *Note.* M. Rajagopalan has withdrawn his claim of an example assuming only CH. At present the weakest known hypothesis for constructing such a space is "CH + there exists a Souslin tree" (*Dahroug*).

Classic Problem VIII, vol. 2. Is every γ -space quasi-metrizable? *Solution.* No, R. Fox.

D21, vol. 3 (*Reed*) Does there exist in ZFC a normal space of cardinality ω_1 with a point-countable base that is not perfect? *Notes.* With c in place of ω_1 , there are many examples, such as the Michael line. Without "normal" there are examples also (*P. Davies*, AMS Proceedings 77 (1979), 276-278). If $MA + \neg CH$ there are no locally compact examples (*Balogh*).

D28, vol. 5 (*Wicke*) Is there a meta-Lindelöf space which is not weakly θ -refinable? *Solution.* Yes, *G. Gruenhage*.

D25, E8, #9, vol. 5 (*Watson*), least common refinement: Is there a locally compact, perfectly normal, collectionwise Hausdorff space that is not collectionwise normal? *Solution.* Yes if \diamond^* (*G. Gruenhage and P. Daniels*), no if $MA + \neg CH$ (*G. Gruenhage*, Russ. Math Surveys 35: 3 (1980), 49-55).

C39, vol. 6 (*van Douwen*) Let μ be the least cardinality of a compact space that is not sequentially compact. It is known that $2^{\aleph_1} \leq \mu \leq 2^{\aleph_2}$. What else can be said about μ ? *Consistency Result.* It is possible to have $\mu = \aleph_1 = c$, hence $\mu < 2^{\aleph_2}$ (*S. Shelah*).

D30, vol. 7 (*Nyikos*) Is there a first countable space (or even a space of countable pseudocharacter) that is weakly θ -refinable and countably metacompact, but not subparacompact? *Partial Solution.* Yes to the "countable pseudocharacter" version (*Nyikos*).