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# TOPOLOGY PROCEEDINGS



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## PROBLEM SECTION

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## PROBLEM SECTION

### CONTRIBUTED PROBLEMS

For the most part, the problems here listed are related to talks that were given at the 1984 Spring Topology Conference in Auburn, for which this volume of *Topology Proceedings* is the journal of record. In many cases, there is an article in this volume by the one posing the problem (in parentheses after the problem number) giving further information on the background of the problem.

#### A. Cardinal Invariants

19. (*van Douwen*) Is a first countable  $T_1$  space normal if every two disjoint closed sets of size  $\leq c$  can be put into disjoint open sets?

See also B26.

#### B. Generalized Metric Spaces and Metrization

25. (*Nyikos*) A space is *paranormal* if every countable discrete collection of closed sets  $\{F_n: n \in \omega\}$  can be expanded to a locally finite collection of open sets  $\{G_n: n \in \omega\}$ , i.e.  $F_n \subset G_n$  and  $G_n \cap F_m \neq \emptyset$  iff  $F_m = F_n$ . Is there a "real" example of a nonmetrizable paranormal Moore space?

26. (*Porter and Woods*) A space is *RC-perfect* if each of its open sets is a union of countably many regular closed subsets of the space. Is there a "real" example of a feebly compact, RC-perfect, regular space that is not separable? [A compact L-space is a "consistent" example.]

**D. Paracompactness and Generalizations**

36. (*Burke, Nyikos*) In a regular, first countable, countably metacompact space, must every closed discrete subspace be a  $G_\delta$ ? What if the space is countably paracompact? normal? [Yes to each question if PMEA. Yes to the second (Watson) and third (Fleissner) if  $V = L$ .]

**H. Homogeneity and Mappings of Other Spaces**

10. (*van Douwen*) Does every compact space without isolated points admit an irreducible map onto a continuum? Does  $\omega^*$ ? [Yes to the second part if CH.]

11. (*van Douwen*) Is every compact  $P'$ -space (i.e. nonempty  $G_\delta$ 's have nonempty interiors) an irreducible continuous image of a compact zero-dimensional  $P'$ -space (preferably of the same weight)?

12. (*van Douwen*) Let  $\kappa > \omega$ , let  $U(\kappa)$  denote the space of uniform ultrafilters on  $\kappa$  and let  $A(U(\kappa))$  be the group of autohomeomorphisms of  $U(\kappa)$ .

(a) Is every member of  $A(U(\kappa))$  induced by a permutation of  $\kappa$ ?

(b) Is  $A(U(\kappa))$  simple?

(c) Is there, for every  $h \in A(U(\kappa))$ , a nonempty proper clopen subset  $V$  of  $U(\kappa)$  with  $h \restriction V = V$ ? [Yes to (a) would imply yes to (b) and (c). Also, if yes to (c) then  $U(\kappa)$  and  $\omega^*$  are not homeomorphic.]

**P. Products, Hyperspaces, Remainders, and Similar Constructions**

22. (*Peters*) Must every non-pseudocompact  $G$ -space have remote points?

**INFORMATION ABOUT EARLIER PROBLEMS**

Problems listed here have appeared in Mary Ellen Rudin's *Lectures on Set-Theoretic Topology*, CBMS Reports #23, 1975 (Reprinted 1977), and in earlier issues of *Topology Proceedings*.

**From *Lectures on Set Theoretic Topology***

B11. (*Hager*) If  $X$  is a dense subset of a compact  $Y$  and every open set containing  $X$  is  $C^*$ -embedded in  $Y$ , then is  $X$   $C^*$ -embedded in  $Y$ ? *Solution.* No, *Mabruk Sola*. If one lets  $X = \Delta$  and  $Y = \zeta\Delta$ , then every open subspace  $U$  of  $\zeta\Delta$  containing  $\Delta$  is strongly zero-dimensional hence  $\beta U = \zeta\Delta$  and  $U$  is  $C^*$ -embedded in  $\zeta\Delta$ , but  $\Delta$  is not strongly zero-dimensional and so it is not  $C^*$ -embedded in  $\zeta\Delta$ .

D4. (*Reed*) Is every countably paracompact Moore space normal? *Solution.* Yes if PMEA (*Burke*) or PCEA (*Fleissner*) but no if there is no inner model with "many" measurable cardinals (*Fleissner*).

D5. (*Reed*) Is there a countably paracompact Moore space which is not paracompact (equivalently, metrizable)? *Solution.* Same as for D4, with "yes" and "no" switched.

E5. (*Green*) This problem was mis-stated in both printings. The closest nontrivial problem like it was:

(a) Does every noncompact Moore space which is closed in every Moore space in which it is embedded, have a dense subspace which is conditionally compact? (In other words, is every noncompact Moore-closed space  $e$ -countably compact?) *Solution.* No, *Stephenson, Topology*

*Proceedings*, volume 4, no. 2. However, the question Green seemed most interested in was:

(b) does every noncompact Moore-closed space have a noncompact,  $e$ -countably compact subspace? *Consistency results.* No if  $b = c$ , Zhou (in effect: see MR 85e: 54026) or if  $a = c$ , Nyikos, Berner, van Douwen.

A20, second printing (Weiss) If  $X$  is a compact scattered space such that  $X^\alpha - X^{\alpha+1}$  is countable for all  $\alpha$ , what are the bounds on order (minimal  $\alpha$  with  $X^\alpha$  finite) of  $X$ ? *Consistency results.* Baumgartner and Shelah have recently announced that  $\alpha = \omega_2$  is consistent. On the other hand, as Juhász and Weiss have pointed out [Coll. Math XL (1978), 63-68] if we assume CH use must have  $\alpha < \omega_2$ , but there exist ZFC examples for each  $\alpha < \omega_2$ .

B5, second printing (van Douwen) Is the continuous image of a supercompact space supercompact? *Solution.* No, van Mill and Mills. If one lets  $X$  = the subspace of  $(\omega_1 + 1)^2$  comprising the diagonal and everything below it, then  $X$  is supercompact, but the image under the map which identifies each  $(\alpha, \alpha)$  with  $(\alpha, \omega_1 + 1)$  is not supercompact.

D11a, second printing (Nyikos) Is every perfectly normal space with a point-countable base metrizable? *Consistency result.* No, unless there is an inner model with a proper class of measurable cardinals (Fleissner).

#### **From Topology Proceedings**

E1, vol. 1 (Wage) Does there exist an extremally disconnected Dowker space? *Solution.* Yes, Dow and van Mill. For information see *Handbook of Set Theoretic Topology*,

p. 772. This problem also appeared as part of B2 in the second printing of *Lectures on Set-Theoretic Topology*.

C24, vol. 3 (M. Pouzet) A space  $X$  is called *impartible* if for every partition  $\{A, B\}$  of  $X$ , there is a homeomorphism from  $X$  into  $A$  or  $B$ . Is there a compact Hausdorff impartible space? *Consistency results*. No if either  $\neg CH$  and  $A(\omega, 2^\omega)$ , or  $B(\omega, 2^\omega)$ ; in particular, if  $V$  satisfies the covering lemma wrt  $K$  (the core model) the answer is negative. (Hajnal, Juhász, Shelah, Splitting strongly almost disjoint families, preprint.) Hence if the answer is "yes" in any model, then there is an inner model with a proper class of measurable cardinals.

E5, vol. 3 (Archangel'skii) Does every zero-dimensional space have a strongly zero-dimensional subtopology? *Note*. All examples of zero-dimensional spaces which are known to the Problems Editor have strongly zero-dimensional subtopologies. This is clear in the locally compact examples, and has recently been shown for Prabir Roy's Space  $\Delta$ .

S2, vol. 3 (van Douwen and Rudin) In ZFC, are there two free ultrafilters on  $\omega$  with no common finite-to-one image? *Solution*. No, Shelah. *Note*. It was already known that under MA there are such ultrafilters.

B19, vol. 5 (Zhou) A space is said to have a small diagonal if any uncountable subset of  $X^{2-\Delta}$  has an uncountable subset with closure disjoint from the diagonal. Must a compact  $T_2$ -space with a small diagonal be metrizable? *Consistency result*. Yes if CH. Juhász, *Handbook of Set-Theoretic Topology*, Theorem 7.5.

A12, vol. 6 (*Nyikos*) Does there exist, for each cardinal  $\kappa$ , a first countable, locally compact, countably compact space of cardinality  $\geq \kappa$ ? *Consistency result (Shelah)*. An affirmative answer is compatible with any possible cardinal arithmetic.

D30, vol. 7 (*Nyikos*) Is there a first countable space (or even a space of countable pseudocharacter) that is weakly  $\theta$ -refinable and countably metacompact, but not subparacompact? *Consistency result*. Yes if  $\neg\text{CH}$ . In fact, Gruenhage and Balogh have shown that CH is equivalent to the statement that every locally compact, first countable,  $\theta$ -refinable (i.e. submetacompact) space is subparacompact. Gruenhage's  $\neg\text{CH}$  example is, in addition, metacompact.

C46, vol. 8 (*van Douwen*) Suppose every pseudocompact subspace of a compact space  $X$  is compact. Must  $X$  be hereditarily realcompact? *Solution*. No, *Nyikos*. The subspace  $T^+$  of the tangent bundle on the long line (v. 4, *Topology Proceedings*, pp. 271-276) is a Moore manifold in which every separable subspace is metrizable and so every pseudocompact subspace is compact, yet it is not realcompact. Its one-point compactification is the counterexample.

C48, vol. 8 (*Nyikos*) Is there a compact non-scattered space that is the union of a chain of compact scattered subspaces? *Solution*. No, *Juhász and van Douwen* have pointed out that a compact nonscattered space  $X$  has a separable nonscattered subspace, because  $X$  admits a continuous map onto  $[0,1]$  and any closed subspace  $Y$  to which the restriction is irreducible must be separable.

D35, vol. 8 (*Nyikos*) Does there exist a screenable anti-Dowker space? That is, does there exist a screenable space that is countably paracompact but not normal?

[If PMEA, any example must be of character  $\geq c$ .] *Solution.* Yes, applying the Wage machine [AMS Proceedings 57 (1976), 183-188] to *Bing's* example G gives a screenable space.

H9(a), vol. 8 (*van Douwen*) For a linearly ordered set  $L$  define an equivalence relation  $T_L = \{(x, y) \in L \times L : \text{there is an order-preserving bijection of } L \text{ taking } x \text{ to } y\}$ . Does  $\mathbb{R}$  have a subset  $L$  such that  $T_L$  has only one equivalence class, but  $(L, \leq)$  is not isomorphic to  $(L, \geq)$ ? *Solution.* Yes, *Baumgartner*, Ph.D. Thesis, Berkeley, 1970. Communicated by *Fred Galvin*.

S4, vol. 8 (*Nyikos*) Call a point of  $\omega^*$  a *simple P-point* if it has a totally ordered clopen base. (a) Does the existence of a simple P-point imply the existence of a scale, i.e. a cofinal well-ordered subset of  $(\omega, <^*)$ ? (b) Is it consistent that there exist simple P-points  $p$  and  $q$  with bases of different cofinalities? *Solution.* No to (a) and yes to (b) (*Shelah*). To be precise, there are models in which there are simple P-points and scales, but *Shelah* has a model in which the answer to (b) is yes, and such a model cannot contain a scale.