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CONTINUUM THEORY PROBLEMS—UPDATE

Wayne Lewis

A collection of continuum theory problems which I edited was published in Volume 8, Number 2 (1983) of *Topology Proceedings*. This article summarizes results on the problems as well as presenting a few new problems. While activity may not always justify yearly updates to this list, it is the intention of the editor to continue to issue updates at irregular intervals. Any comments or results on previously published problems, or any problems for inclusion in future updates should be sent to him at the address at the end of this update.

Information on Problems in Initial List
(Numbers refer to initial list.)

1. *(Brechner, Lewis, Toledo)* Can a chainable continuum admit two non-conjugate homeomorphisms of period n with the same fixed point set?

The question should be rephrased to require the sets of fundamental periods of points under the two homeomorphisms to be identical. Toledo has shown that for any sequence of positive integers $1 \le n_0 < n_1 < n_2 < \cdots < n_k$ where n_i divides n_j for $i < j$, there is a period n_k homeomorphism of the pseudo-arc with points of each of the fundamental periods n_i.

13. *(Fugate)* If X is circularly chainable and $f\colon X \to Y$ is open, then is Y either chainable or circularly chainable?

Krupski has shown that if X is a solenoid, then Y is either a point, a solenoid, or a Knaster continuum, i.e. an inverse limit of arcs with open bonding maps.

25b. *(Bellamy)* Allowing singletons as degenerate indecomposable continua, is the following statement true? Suppose X is an hereditarily unicoherent continuum and $f: X \to X$ is continuous. Then there is an indecomposable subcontinuum W of X such that $f(W) \subseteq W$.

Maćkowiak has described such a continuum X and map f so that the statement is false.

31. *(Bellamy)* Suppose X is triod-like (or K-like for some fixed tree K). Must X have the fixed point property?

Marsh has shown that an inverse limit of fans $\{F_i\}$, where each bonding map preserves ramification points and is except for one branch a homeomorphism of each branch of F_{i+1} onto a branch of F_i, has the fixed point property.

52. *(Bing)* Is a simple closed curve S in E^3 tame if it is isotopically homogeneous (i.e. for each p, q in S there is an ambient isotopy of E^3, leaving S invariant at each stage, with the 0th-level of the isotopy the identity and the last level a homeomorphism taking p to q)?

Shilepsky has conjectured that the answer is yes. Shilepsky and Bothe have independently constructed wild simple closed curves in E^3 which are homogeneously embedded in E^3 but not isotopically homogeneous.

56. *(Edwards)* If $f: S^3 \to S^2$ is a continuous surjection must there exist Σ^2 (an embedded copy of S^2 in S^3) such that $f|_{\Sigma^2}$ is a surjection?

Bestvina and Walsh have shown that the answer is no.

67. *(Lewis)* Is there a natural measure which can be put on the space M(P) of self-maps of the pseudo-arc? If so, what is the measure of the subspace $\hat{H}(P)$ of maps which are homeomorphisms onto their image? Is it the same as the measure of M(P)?

Kallman has shown that there is no standard Borel structure on H(P)--the full autohomeomorphism group of the pseudo-arc P--with respect to which H(P) is a Borel group and which admits a σ-finite Borel measure which is quasi-invariant under left translations. This seems to imply a negative answer to this question.

68. *(Lewis)* Does the pseudod-arc have uncountably many orbits under the action of its homeomorphism group? What about other non-chainable continua all of whose non-degenerate proper subcontinua are pseudo-arcs?

Lewis proved that no such continuum has a G_δ-orbit under the action of its homeomorphism group. Kennedy and Rogers observed that a version of Effros' theorem implies a positive answer to both questions.

78. *(Hagopian)* If a homogeneous continuum X contains an arc must it contain a solenoid or a simple closed curve? What if X contains no simple triod?

Maćkowiak and Tymchatyn have shown that the answer is yes if X is atriodic. Connor presented a candidate for a counter-example.

86. *(Bing)* Is every homogeneous tree-like continuum hereditarily indecomposable?

Krupski has shown that if X is a homogeneous continuum which contains a local endpoint, then either X is hereditarily indecomposable or X admits a continuous decomposition into mutually homeomorphic, nondegenerate, homogeneous, hereditarily indecomposable subcontinua with decomposition space a homogeneous continuum with no local endpoints.

119. *(Maćkowiak)* Does there exist a chainable continuum X such that if H and K are subcontinua of X then the only maps between H and K are the identity or constants?

Maćkowiak has constructed a nondegenerate chainable continuum with the desired property.

122. *(Lewis)* If h is a homeomorphism of $\Pi_{\alpha \in A} P_\alpha$, where each P_α is a pseudo-arc, is h necessarily of the form $h = \Pi_{\alpha \in A} h_{s(\alpha)}$, where s is a permutation of A and $h_{s(\alpha)}$ is a homeomorphism of P_α onto $P_{s(\alpha)}$?

Bellamy provided a positive answer if A is finite, and Bellamy and Kennedy provided a positive answer for arbitrary A.

131. *(Bellamy)* For countable non-limit ordinals α, what are the continuous images of $C(\alpha)$, the cone over α? For $\alpha \geq w^2 + 1$, what are the continuous pre-images of $C(\alpha)$?

Katsuura has characterized the continuous images of the harmonic fan.

Information on Problems From Other Lists

Each of the volumes of *Topology Proceedings* has contained a general problem section. Here we provide comments and results on the ones involving continuum theory in volumes 1 through 8. Letters and numbers are those from the relevant volumes.

Volume 1 (1976): F1. *(Bing)* Is there a homogeneous, tree-like continuum that contains an arc?

Hagopian has provided a negative answer. Of interest now is whether there exists a homogeneous, tree-like continuum with a decomposable subcontinuum.

F2. *(Ingram)* Is there an atriodic tree-like continuum which cannot be embedded in the plane?

Ingram has constructed an atriodic tree-like continuum which is not thought to be planar.

F4. *(Ingram)* What characterizes the tree-like continua which are in class w?

A continuum X is in class w if each continuous surjection from a continuum onto X is weakly confluent. Grispolakis and Tymchatyn have shown that a continuum X is in class w if and only if it has the covering property, i.e. for any Whitney map μ for $C(X)$ and any $t \in (0,\mu(X))$, no proper subcontinuum of $\mu^{-1}(t)$ covers X. They have also shown that a planar tree-like continuum is in class w if and only if it is atriodic.

Volume 2 (1977): This volume contained a dearth of problems in continuum theory.

Volume 3 (1978): F6. *(J. T. Rogers)* Suppose M and N are solenoids of pseudo-arcs that decompose to the same solenoid. Are M and N homeomorphic?

Lewis has provided a positive answer to this question and completed the classification of homogeneous, circle-like continua.

G9. *(Hagopian)* Is every continuous image of every λ-connected plane continuum λ-connected?

Outside of the plane the answer is no since Hagopian has shown that the product of two nondegenerate, hereditarily indecomposable continua is λ-connected.

G10. *(Heath and Fletcher)* Is there a Euclidean non-Galois homogeneous continuum?

W. Kuperberg has observed that the product of two or more copies of the Menger universal curve is homogeneous, but not Galois. An equivalent result is true for a product of pseudo-arcs.

Volume 4 (1979) through volume 6 (1981): Each of these volumes contained a dearth of new problems in continuum theory.

Volume 7 (1982): Though this volume contains new continuum theory problems, none has yet been solved. Several of the problems listed in it are also included in Continuum Theory Problems, *Topology Proceedings*, 8 (1983), 361-394.

Volume 8 (1983): Question F24 is meaningless as stated. It should read:

F24. *(Jones)* Is each tree-like, homogeneous curve hereditarily equivalent?

It should be kept in mind that in the general problem section of *Topology Proceedings* the name associated with a problem indicates only the person who submitted it to *Topology Proceedings* or mentioned it in a talk at the appropriate conference. It does not in many cases indicate who first posed the problem or who is most knowledgeable about or has worked most extensively on the problem.

Another good source of problems in continuum theory and other areas is the University of Houston Problem Book, though we do not have space here to provide a thorough review of its contents.

New Problems

Comparatively few new problems in continuum theory have been submitted to me since the original list went to press. Those which have been submitted and have not in the interim been solved are listed below. All readers are again encouraged to submit such problems, with appropriate commentary, for inclusion in future updates.

165. *(Bellamy)* Suppose X is a homogeneous, aposyndetic continuum which contains two disjoint subcontinua with interior. Is X mutually aposyndetic? What if X is also arcwise connected?

166. *(Bula)* Suppose $f: X \to Y$ is an open map, with each of X and Y compact metric and each $f^{-1}(y)$ infinite. Do there exist disjoint closed subsets F and H of X such that $f(F) = f(H) = Y$?

It is known that if each point inverse is perfect and Y is finite-dimensional then there exists a continuous surjection $g: X \to Y \times [0,1]$ such that $f = \pi_Y \circ g$, where π_Y is the projection of $Y \times [0,1]$ onto Y.

167. *(Lewis)* Under what conditions does there exist a wild embedding of the k-sphere S^k in E^n which is a homogeneous embedding?

Compare with questions 49 and 50.

168. *(Lewis)* Does there ever exist a wild embedding of S^k in E^n which is isotopically homogeneous?

Compare with question 52.

169. *(Lewis)* Does there exist a nondegenerate continuum K which can be embedded in E^n, $n \geq 3$, such that every embedding of K in E^n is a homogeneous embedding?

170. *(Minc)* Does there exist an hereditarily indecomposable continuum which is homogeneous with respect to continuous surjections but not homogeneous with respect to homeomorphisms?

The pseudo-circle and pseudo-solenoids are known not to have this property.

171. *(Bellamy)* Does there exist an hereditarily indecomposable non-metric continuum with only one composant?

Bellamy and Smith have independently constructed indecomposable, non-metric continua with only one or two composants. Smith has constructed an hereditarily indecomposable, non-metric continuum with only two composants.

172. *(Van Nall)* Is it true that an atriodic continuum in class w is hereditarily in class w if and only if each C-set in it is in class w?

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