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1. Introduction

In this paper we will include a brief historical account of the dimension theory of infinite-dimensional spaces especially as it was motivated by the Cell-Like Dimension Raising Mapping Problem (see [S]). We will construct the important example of R. Pol [P] and discuss why it indicates that the current "theory" is inadequate. We will introduce a new concept of dimension and will review an alternate definition of dimension given by Vainstein [V]. We will compare these and discuss how they correct for the problem in the current theory. Finally, several authors of current papers in this field, including Pol, have referred to a result implying that certain totally disconnected spaces can be chosen in such a way that they are G_δ-spaces. The references have been very obscure and consequently we conclude this paper with a proof that each map between compact metric spaces has a G_δ-section.

2. Background

The study of dimension theory discussed in this paper is motivated by the study of cell-like maps. R. H. Bing did much of the initial work with decompositions of $\mathcal{R}^3$ into cell-like sets. For these decompositions the quotient map is a cell-like map; a sample reference is [B1]. The cell-like notions were introduced by Lacher in 1968 [La]; by a *cell-like map* we mean a proper map where each point inverse has the shape of a point. In the work on cell-like maps since then, a standard assumption has been that the dimension of the range space is finite. It is still not known if a cell-like map on a finite-dimensional compactum can raise the dimension. This question has become known as the Cell-Like Diemnsion Raising Mapping Problem and is discussed at length in [S].

The real motivation for the research being discussed in this paper, assuming an interest in cell-like maps, came from the paper of George Kozlowski [K]. He proved many things about fine homotopy equivalences and reminded us of the earlier results from cohomology theory that if a cell-like map raises the dimension then it is raised to infinity and that any space containing subspaces of arbitrarily high finite dimension cannot be the image of a cell-like dimension raising map. This was the motivation for the authors of [RSW] to take a new look at the very interesting example of D. Henderson [H]. The assumption was that Henderson's example came the closest of anything in the literature to having the properties required of the image of a cell-like dimension raising map.

Henderson's example [H] in 1967 was the first infinite-dimensional compactum that contained no n-dimensional ($n \geq 1$) closed subsets. This answered an old problem that Tumarkin was credited with asking in 1926 (see [H]) and that first appeared in print in 1933 by Mazurkiewicz [M]. R. H. Bing studied the long and difficult paper of (his student) Henderson and in [B2] he published a simpler version. In 1971, Zarelua [Z1] gave an even simpler construction of a Henderson-type example and in 1974 [Z2] he constructed another example such that each non-degenerate subcontinuum was strongly infinite-dimensional.

In 1979, Rubin, Schori, and Walsh [RSW] developed a unified theory of dimension using essential families and continuum-wise separators for efficient construction of such examples. The same techniques also provided very easy constructions of n-dimensional and strongly infinite-dimensional totally disconnected spaces. Pol used the latter of these in his construction, as mentioned above. The techniques for constructing the Henderson-type examples were further refined in [SW] and in [S] and then Walsh [W1] made a major breakthrough with the first construction of a compactum containing no n-dimensional subsets of any kind ($n \geq 1$). This result closed off any hope of a quick negative solution to the dimension-raising problem.

Rubin in [RU1] and [Ru2] generalized Walsh's result in [W1] to obtain examples of hereditarily strongly infinite-dimensional compacta. In another direction, Walsh [W2], Bowers [Bo], and Rubin [Ru3], [Ru4] computed the cohomological dimension of most of the known hereditarily strongly infinite-dimensional examples to be infinity. The hope of course was to find an example with finite cohomological dimension since it was also well understood from Vietoris's theorems that the cohomological dimension of the image of a cell-like dimension raising map had to be finite. In fact,

R. D. Edwards proved and Walsh [W3] wrote up the beautiful result that there exists a compactum of infinite dimension and finite cohomological dimension iff there exists a cell-like map of a compactum of finite dimension onto a compactum of infinite dimension. This showed that two well-known old problems in dimension theory were equivalent.

3. Pol's Example

In this section we will present R. Pol's example [P] and discuss why it struck a shattering blow to the current theory of infinite dimensions. By *space* we will mean metric space. First, an n-family for a space X is a collection $C = \{(A_i,B_i): i = 1,\cdots,n\}$ of n pairs of disjoint closed sets in X. In this definition we allow n to be a positive integer or ω. The collection C is *essential* if whenever a closed set S_i separates A_i and B_i, $i = 1,\cdots,n$, then $\cap S_i \neq \emptyset$. A space is n-*dimensional* if it has an essential n-family and does not have an essential (n + 1)-family (see [RSW]). A space is *infinite-dimensional* if it is n dimensional for any positive integer n. A space is *strongly infinite-dimensional* (SID) if it has an essential ω-family and an infinite-dimensional space is *weakly infinite-dimensional* (WID) if it is not strongly infinite-dimensional. Finally, a space is *countable infinite-dimensional* (CID) if it is infinite-dimensional and the countable union of 0-dimensional spaces.

An example of a SID space is the Hilbert cube $Q = \prod_{i=1}^{\infty} I_i$, where each I_i is the closed unit interval. An essential ω-family for Q is $\{(A_i,B_i): i \geq 1\}$ where

$A_i = \pi_i^{-1}(0)$ and $B_i = \pi_i^{-1}(1)$ for the projection map $\pi_i: Q \to I_i$. An example of a CID space is the disjoint union of n-cells, $n = 1,2,\cdots$.

It is well known that a SID space is not CID, i.e. CID implies WID, but it was an old problem of Alexandrov's as to whether WID implies CID. Pol's example provides a counterexample. In particular, Pol constructed a compact metric WID space $X = Y \cup R$ where Y is SID and R is CID. The shattering blow to the general theory is that weak infinite dimensionality is not hereditary, i.e. X contains a subspace Y that is SID. More specifically, Y is a totally disconnected, SID, G_δ-space. Pol correctly stated that the existence of a SID totally disconnected space is implicit in section 4 of [RSW] and that, as remarked in the introduction, it can be chosen to be a G_δ-subset of Q. We will now explicitly do these constructions. A *section* for a map $f: X \to Y$ is any subset $S \subset X$ which intersects every non-empty $f^{-1}(y)$ in exactly one point. The following result follows from [RSW].

Lemma. If Y *is a subset of the Hilbert cube* Q *such that* Y *intersects each continuum in* Q *from* A_1 *to* B_1, *then* Y *is* SID.

Theorem 1. *There exists a* SID *totally disconnected* G_δ-*subset of* Q.

Proof. Let $\Delta \subset I_1$ be a Cantor set and let S be the collection of all continua in Q meeting A_1 and B_1. The set S with the Hausdorff metric is a compact metric space and hence we can take a continuous surjection $\alpha: \Delta \to S$. Letting $M = \cup\{\pi_1^{-1}(t) \cap \alpha(t): t \in \Delta\}$, the compactness of Δ and the continuity of α imply that M is compact and $\alpha|M$ is a continuous function from M onto Δ. Hence we can apply Theorem 3 to obtain a G_δ-section Y. Thus $\pi_1|Y$ is a 1-1 surjection onto the Cantor set Δ and thus Y is totally disconnected. Furthermore, Y is SID by the Lemma since it intersects each continuum from A_1 to B_1.

The next lemma follows directly from Corollary 3.2 of [Le]. For completeness, we include an alternate proof based on infinite dimensional topology. We thank the referee for this suggestion.

Lemma. Every separable metric space that is an absolute G_δ *can be compactified with a* CID *remainder.*

Proof. Let X be any complete separable metric space. Then X can be embedded as a closed subset of separable infinite-dimensional Hilbert space. To see this, embed X in the Hilbert cube $Q \subset l^2$. Then remove $\bar{X} \setminus X$ (which is σ-compact) from l^2. The result is homeomorphic to l^2 and X is a closed subset of this space. Now consider l^2 to be the complement of a CID fd-cap set in Q. This embeds X in Q in such a way that $\bar{X} \setminus X$ is a subset of the CID fd-cap set. This is the required compactification of X.

Pol's Example. There exists a WID *compactum* X *that is not* CID.

Proof. Take the space Y as constructed in Theorem 1 and by the above lemma let X be a compactification of Y with a CID remainder $X - Y = R$.

The space X is not CID since it contains a SID subset and the property of being CID is hereditary. We now prove that X is WID. Let $\{(A_i, B_i): i \geq 1\}$ be a potential essential ω-family for X. Let $R = \cup\{X_i: i \geq 2\}$ where each X_i is 0-dimensional and for each $i \geq 2$ let S_i be a separator of A_i and B_i in X where S_i and X_i are disjoint. It is a standard theorem in dimension theory that a separator can be chosen to miss a given 0-dimensional set. Then $S = \cap_{i=2}^{\infty} S_i$ is a compact subset of the totally disconnected space Y and hence is 0-dimensional. Thus, let S_1 be a separator in X of A_1 and B_1 that misses S. Then $\cap_{i=1}^{\infty} S_i = \emptyset$ and this completes the proof that X is WID.

Consequently we have the unfortunate situation of historically having three main classifications of infinite dimensional spaces, namely, CID, WID, and SID, where the WID and SID properties are not hereditary (or monotone) as we expect of a "dimension theory." For a discussion of the philosophy of dimension, see the Appendix of [HW].

4. Another definition for infinite dimensionality

We present a new definition of dimension directly motivated by Pol's example. This definition is equivalent to the usual one for finite dimensions, but leads to a type of weak infinite dimensionality that is hereditary. In fact the new concept of weak infinite dimensionality is equivalent to hereditarily weak infinite dimensionality in the usual sense. Our definition is closely tied to the essential family approach to dimension and clarifies the relationship between Pol's example and the concept of essential families.

Definitions. A collection $\{(A_i,B_i): i \in J\}$ is a *dimension family* for X (d-family) if each pair (A_i,B_i) of (not necessarily closed) subsets of X is separated in X and if $(A_i \cup B_i) \cap (\cup_{j\in J}(\overline{A}_j \cap \overline{B}_j)) = \emptyset$. Such a d-family is *essential* if whenever $\{S_i: i \in J\}$ is a collection of closed subsets of X separating A_i and B_i in X, then $\cap_{i\in J}S_i \not\subset \cup_{i\in J}(\overline{A}_i \cap \overline{B}_i)$. A space X has *dimension* $\geq$ n *with respect to dimension families* (d-dim(X) $\geq$ n) if X has an essential d-family of cardinality $\geq$ n. A space X is *strongly infinite-dimensional with respect to dimension families* (SID-d) if X has a denumerable essential d-family. Otherwise, X is *weakly infinite-dimensional with respect to dimension families* (WID-d).

Example. Let $X = I^2$ and let $Y = \{(x,y) \in X: y$ is irrational$\}$. Let π be the quotient map from X onto $X|G$ where G is the decomposition of X into points and the sets $I \times \{0\}$ and $I \times \{1\}$. Let $A = \{0\} \times I$ and $B = \{1\} \times I$. Then $\{\pi(A \cap Y),\pi(B \cap Y)\}$ is an essential family in $\pi(Y)$ and is an essential d-family for $\pi(X)$ even though it is not an essential family for $\pi(X)$.

The above example illustrates the following theorem.

Theorem 4. *A space* X *has an essential* d-*family if and only if* X *has a subspace* Y *with an essential family of the same cardinality.*

Proof. (⇒) Suppose $\{(A_i,B_i): i \in J\}$ is an essential d-family in X. Let $Y = X - \cup_{i\in J}(\bar{A}_i \cap \bar{B}_i)$. Then each $(\bar{A}_i \cap Y, \bar{B}_i \cap Y)$ is a pair of nonempty closed disjoint subsets of Y. Let $\{T_i: i \in J\}$ be a collection of closed subsets of Y separating $\bar{A}_i \cap Y$ and $\bar{B}_i \cap Y$ in Y. For each i, choose disjoint open (in X) sets W_i and Z_i with $\bar{A}_i \cap Y \subset W_i$, $\bar{B}_i \cap Y \subset Z_i$, and such that if $S_i = X - (W_i \cup Z_i)$, then $S_i - \cup_{i\in J}(\bar{A}_i \cap \bar{B}_i)$ is contained in T_i. Since $\cap_{i\in J}S_i \not\subset \cup_{i\in J}(\bar{A}_i \cap \bar{B}_i)$, we have $\cap_{i\in J}T_i \neq \emptyset$.

(⇐) Suppose Y is a subspace of X with an essential family $\{(C_i,D_i): i \in J\}$. It is straightforward to check that $\{(C_i,D_i): i \in J\}$ is an essential d-family for X.

This theorem yields the following corollaries:

Corollary 1. $\dim(X) \leq n$ *if and only if* $d\text{-}\dim(X) \leq n$.

Corollary 2. X *is* WID-d *if and only if* X *is* HWID.

Consequently, even though Pol's example is WID, it is SID-d.

It was recently pointed out to us that A. I. Vainstein [V] gave a definition of dimension in 1968 similar to the approach given above. Vainstein seemed to be motivated by the fact that weak infinite dimensionality could not be shown to be hereditary. (Pol's example shows that it is not.) It is clear that a space that is weakly infinite dimensional in Vainstein's sense is hereditarily weakly infinite dimensional. It is not clear whether the reverse implication holds. For completeness, we outline Vainstein's approach.

Definitions. Following Vainstein [V, p. 411], if A and B are closed subsets of a space X (A and B can intersect), then a closed set C is a *barrier between* A and B provided $C \cup (A \cap B)$ separates $A - B$ and $B - A$ in X. To simplify the statements of the following two theorems we introduce some terminology. A collection of pairs of closed subsets of X, $\{(A_i, B_i): i \in J\}$ is called a *Vainstein family*, or v-*family*, Such a v-family is *essential* if whenever $\{C_i: i \in J\}$ is a collection of barriers between A_i and B_i, then

$$\cap_{i\in J} C_i \not\subset \cup_{i\in J}(A_i \cap B_i).$$

A space X has *Vainstein-dimension* $\leq n$ (v-dim(X) $\leq n$) if it has no essential v-family of cardinality $\geq n + 1$. A space X is *strongly infinite-dimensional in the sense of Vainstein* (SID-v) if it has a countable essential v-family. Otherwise, X is *weakly infinite-dimensional in the sense of Vainstein* (WID-v).

Theorem [V, p. 411] 2. dim(X) $\leq n$ *if and only if* v-dim(X) $\leq n$.

Theorem [V, p. 411] 3. *If* X *is* WID-v *and* $Y \subset X$, *then* Y *is* WID-v.

The first theorem shows that Vainstein's dimension is equivalent to covering dimension for finite dimensional spaces. The second shows that WID-v is hereditary whereas Pol's example shows WID is not. Vainstein observes that any WID-v space is WID. This together with the second theorem above shows that WID-v implies hereditarily WID (HWID).

It is natural to ask whether WID-v and HWID are the same. If one could show that a space X has an essential v-family if and only if a subspace Y had an essential family of the same cardinality, then WID-v and HWID would be equivalent. One might attempt to produce an essential family by starting with an essential v-family $\{(A_i,B_i): i \in J\}$ by taking $\{(A_i - Y,B_i - Y): i \in J\}$ in $X - Y$ where $Y = \cup_{i\in J}(A_i \cap B_i)$. The following example shows this does not work.

Example. Let $X = [0,1] \times [0,1]$, $A_1 = \{0\} \times [0,1]$, $B_1 = \{1\} \times [0,1]$, $A_2 = A_1 \cup B_1 \cup [0,1] \times \{1\}$, and $B_2 = A_1 \cup B_1 \cup [0,1] \times \{0\}$. Then $\{(A_1,B_1),(A_2,B_2)\}$ is an essential v-family for X, but the above procedure does not yield an essential family for a subspace of X.

It should be clear that if $\{(A_i,B_i): i \in J\}$ is an essential d-family then $\{(\overline{A}_i,\overline{B}_i): i \in J\}$ is an essential v-family.

Question. Are WID-v and WID-d the same?

5. The existence of G_δ sections.

In this section we prove a theorem that was used in the construction of Pol's example. As stated in the Introduction the following theorem has been frequently quoted in the current literature but since the references to it seem to be quite obscure we include a proof.

Theorem 5 [Bu]. *If* f: X → Y *is a map between compact metric spaces, then there exists a* G_δ*-section.*

Proof. Construct a sequence $\{\mathcal{F}_n\}_{n\geq 0}$ of finite closed covers of X such that

1) mesh $\mathcal{F}_n < \frac{1}{2^n}$ and

2) for each $n \geq 0$, each F in $\mathcal{F}_n$ is equal to a union of elements of $\mathcal{F}_{n+1}$.

(Before proceeding, we pause to discuss the idea of the proof. First, it is helpful to view a section of f: X → Y as a subset S of X where $\{f(x): x \in S\}$ partitions Y (into points). We will achieve this as follows: For each of our closed covers $\mathcal{F}_n$ we will construct a refinement $\{h_n(F): F \in \mathcal{F}_n\}$ (in general many of the $h_n(F)$ will be empty) where 1) $\{f[h_n(F)]: F \in \mathcal{F}_n\}$ partitions Y, 2) if $F_n \subset F_{n-1}$, $(F_i \in \mathcal{F}_i)$, then $h_n(F_n) \subset h_{n-1}(F_{n-1})$, and 3) $S_n = \cup\{h_n(F): F \in \mathcal{F}_n\}$ is a G_δ. Our section will then be $S = \cap_{n=0}^{\infty} S_n$.)

We now continue with the proof. Construct a function $p_n: \mathcal{F}_{n+1} \to \mathcal{F}_n$ such that for $F \in \mathcal{F}_n$, $F = \cup\{F' \in \mathcal{F}_{n+1}: F' \in p_n^{-1}(F)\}$. Now, for each $F \in \mathcal{F}_n$, let $[F] = f^{-1}[f(F)]$. Linearly order (<) each $\mathcal{F}_n$ and for each $F \in \mathcal{F}_n$ define $h_n(F)$ as follows: For $F \in \mathcal{F}_0$,

$$h_0(F) = F - \cup\{[F']: F' \in \mathcal{F}_0, F' < F\}.$$

Clearly, each $h_0(F)$ is a G_δ since each closed set in a metric space is one. For $F \in \mathcal{F}_{n+1}$, let

$$h_{n+1}(F) = F \cap h_n(p_n(F)) - \cup\{[F']: F' \in \mathcal{J}_{n+1}, p_n(F') = p_n(F), F' < F\}$$

Each $h_{n+1}(F)$ is also a G_δ.

Claim. For each $n \geq 0$ and each fiber $H = f^{-1}(y)$ there exists a unique element $F \in \mathcal{J}_n$ such that $h_n(F)$ meets H, and we have $h_n(F) \cap H = F \cap H$, which is therefore a closed set.

We continue the proof of the theorem assuming the Claim. For each integer n, let $S_n = \cup\{h_n(F): F \in \mathcal{J}_n\}$. Then S_n is a G_δ since it is a finite union of G_δs, and we have $S_{n+1} \subset S_n$. Then $S = \cap\{S_n: n \geq 0\}$ is also a G_δ set since it is the countable intersection of G_δs. We shall show that S meets each $H = f^{-1}(y)$ in exactly one point. For each n, let $F_n(H)$ be the unique element $F \in \mathcal{J}_n$ such that $h_n(F)$ meets H. Then $S_n \cap H = F_n(H) \cap H$ and $S \cap H = \cap\{F_n(H) \cap H\}$. The sequence of closed sets $\{F_n(H)\}$ is nested and the diameters converge to 0; and the same is true for $\{F_n(H) \cap H\}$ and hence the intersection of the latter sequence, which is equal to $S \cap H$, is a single point. This completes the proof of the theorem.

Proof of Claim. For $n = 0$, consider the smallest (relative to the linear ordering $<$) element $F \in \mathcal{J}_0$ that meets H; then $F \cap H$ does not meet any set $[F']$ for which $F' \in \mathcal{J}_0$ and $F' < F$; hence it is contained in $h_0(F) \cap H$ and consequently is equal to this set. For uniqueness, we have $H \subset [F]$ and therefore $h_0(F') \cap H = \emptyset$ for $F' \in \mathcal{J}_0$ and $F' > F$. Thus the assertion is proved for $n = 0$. We continue by induction on n: by the inductive hypothesis there exists a unique element $D \in \mathcal{F}_n$ such that $h_n(D)$ meets H and this set is contianed in D which is the union of the sets F for which $F \in p_n^{-1}(D)$. There is a smallest element $F \in p_n^{-1}(D)$ such that D meets H. We therefore have

$$F \cap H \subset D \cap H = h_n(D) \cap H$$

by the inductive hypothesis. Hence

$$F \cap H \subset F \cap h_n(D)$$

and since by definition $F \cap H$ meets none of the sets [F'] for which $F' \in p_n^{-1}(D)$ and $F' < F$, it follows from the definition that

$$F \cap H \subset h_{n+1}(F)$$

but we clearly have $h_{n+1}(F) \cap H \subset F \cap H$ and hence $F \cap H = h_{n+1}(F) \cap H$. Moreover, for uniqueness we have $H \subset [F]$ and therefore if $F' \in p_n^{-1}(D)$ is such that $F' > F$, then $h_{n+1}(F') \cap H = \emptyset$. Hence the Claim is proved for all n.

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