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PROBLEM SECTION

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PROBLEM SECTION

CONTRIBUTED PROBLEMS

For the most part, the problems here listed are related to talks that were given at the 1985 Spring Topology Conference at Florida State University. In many cases, there is an article in this volume by the one posing the problem (in parentheses after the problem number) giving further information on the background of the problem.

A. Cardinal Invariants

20. *(A. García-Maynez)* Let X be a T_3-space and let λ be an infinite cardinal. Assume the pluming degree of X is $\leq \lambda$. Is it true that every compact subset of X lies in a compact set which has a local basis for its neighborhood system consisting of at most λ elements?

See also Y2.

B. Generalized Metric Spaces and Metrization

27. *(Ken-ichi Tamano)* Find an internal characterization of subspaces of the product of countably many Lašnev spaces.

28. *(Ken-ichi Tamano)* Does the product of countably many Lašnev spaces have a σ-hereditarily closure-preserving k-network?

29. *(Nyikos)* Is every locally compact, locally connected, countably paracompact Moore space metrizable? [Yes is consistent.]

See also P23 and P24.

C. Compactness and Generalizations

49. *(Porter)* Can each Hausdorff space be embedded in some CFC space (defined in an article in this issue)?

50. *(Porter)* Is the product of CFC spaces a CFC space?

See also A20, P23, and Y4.

G. Mappings of Continua and Euclidean Spaces

18. *(Beverly Brechner)* Let h be a regular homeomoprhism of B^3 onto itself, which is the identity on the boundary. Must h be the identity? (Recall that h is regular iff the family of all iterates of h forms an equicontinuous family of homeomorphisms.)

19. *(Beverly Brechner)* Is every regular, orientation preserving homeomorphism of S^3 either periodic, or a rotation, or a combination of rotations on two solid tori whose union is S^3?

See also T11.

M. Manifolds and Cell Complexes

7. *(Paul Latiolais)* Does there exist a pair of finite 2-dimensional CW-complexes which are homotopy equivalent but not simple homotopy equivalent?

8. *(Paul Latiolais)* Does there exist a finite 2-dimensional CW-complex K whose fundamental group is finite but not abelian, which is not simple homotopy equivalent to every n-dimensional complex homotopy equivalent to K?

9. *(Paul Latiolais)* Do the Whitehead torsions realized by self-equivalences of a finite 2-dimensional

CW-complex include all of the units of the Whitehead group of that complex?

10. *(Nyikos)* Is every normal, or countably paracompact, manifold collectionwise Hausdorff? [Yes if V = L or cMEA.] Is there a model of $MA(\omega_1)$ where the answer is yes?

See also B29, G18, G19, and T11.

P. Products, Hyperspaces, Remainders, and Similar Constructions

23. *(Isiwata)* Is there a pseudocompact κ-metric space X such that βX is not κ-metrizable?

24. *(Isiwata, attributed to Y. Tanaka)* Is there a κ-metric space X such that νX is not κ-metrizable where $|X|$ is nonmeasurable?

25. *(Mohan Tikoo)* Characterize all Hausdorff spaces for which $\sigma X = \mu X$.

See also S15.

S. Problems Closely Related to Set Theory

15. *(Shoulian Yang)* Let I be a subset of ω^*. If $|I| < 2^c$, does there exist $p \in \omega^*$ such that p is incomparable in the Rudin-Keisler order with all $q \in I$? [Yes is consistent.]

T. Algebraic and Geometric Topology

11. *(Pak)* Let $g: (M^n,x) \to (M^n,x)$ be a based homeomorphism on an n-dimensional manifold at $x \in M^n$. If the induced homomorphism $g_*: \Pi_k(M^n,x) \to \Pi_k(M^n,x)$ is the identity map for all k, is then g isotopic to the identity map? How about if M^n is an aspherical manifold?

See also M7, M8, and M9.

Y. Topological Games

Note. This class of problems appears in this issue for the first time. S9 and S10 were two problems on topological games that appeared earlier (v. 6).

1. *(I. Juhász)* Is there a neutral point-picking game in ZFC?

2. *(I. Juhász)* Is there a space X such that $\omega \cdot \omega < ow(X) < \omega_1$?

3. *(I. Juhász)* Does there exist, in ZFC, a T_3 space X for which the games $G_\omega^D(X)$ and/or $G_\omega^{SD}(X)$ are undecided?

4. *(I. Juhász)* Is it true, in ZFC, that for every compact Hausdorff space X and every cardinal κ the game G_κ^D is determined?

5. *(I. Juhász)* Is there a space X in ZFC such that $II \uparrow G_\alpha^D(X)$ for every $\alpha < \omega_1$, but $II \not\uparrow_M G_\omega^D(X)$?

INFORMATION ON EARLIER PROBLEMS

D6, vol. 1. *(Alster and Zenor)* Is every perfectly normal, locally Euclidean space collectionwise normal? *Solution.* No if $\diamondsuit^+$ *(Rudin)* but Yes if MA + $\neg$CH *(Rudin)*.

Classic Problem V, vol. 2. Does every infinite compact, hereditarily normal space of countable tightness contain a nontrivial converging sequence? *Consistency Results.* Yes if MM *(Fremlin and Nyikos)*, No if $\diamondsuit$ *(Fedorchuk)*. It is not yet known how much of the consistency strength of MM is required in the former result. [The full MM implies the consistency of a proper class of measurable cardinals.]

P5, vol. 3. *(Heath)* Is the Pixley-Roy hyperspace of $\mathbb{R}$ homogenous? *Solution.* Yes, *Wage.*

C32, vol. 5. *(Nyikos)* Is every separable, normal, first countable, countably compact space compact? *Consistency Results.* Yes if MM *(Fremlin and Nyikos)*, No if $p = \omega_1$ *(Franklin and Rajagopalan).* Same additional comments as for Classic Problem V.

M4, vol. 5. *(Nyikos)* Is every normal manifold collectionwise normal? *Consistency Results.* No if $\diamondsuit^+$ *(Rudin)*, Yes if cMEA *(Nyikos).* It is not known whether the consistency of yes requires the consistency of an inaccessible cardinal: M6, v. 8 *(Tall).*

A12, vol. 6. *(Nyikos)* Does there exist, for each cardinal κ, a first countable, locally compact, countably compact space of cardinality $\geq \kappa$? *Consistency Result.* Yes if $\square_\kappa$ and $\mathrm{cf}[\kappa]^\omega = \kappa^+$ for all singular cardinals of countable cofinality *(Nyikos)*, hence Yes if the Covering Lemma holds over the Core Model. A negative answer in some model would thus imply the presence of inner models with a proper class of measurable cardinals.

D33, vol. 7. *(Nyikos)* Does MA imply every locally compact Hausdorff space of weight $< \mathfrak{c}$ is either subparacompact or contains a countably compact noncompact subspace? *Solution.* No *(Nyikos)*: there is a ZFC example of a manifold of weight ω_1 which is quasi-developable but not even countably metacompact.