TOPOLOGY PROCEEDINGS
Volume 13, 1988
Pages 137–160

http://topology.auburn.edu/tp/

# CONTINUA ARBITRARY PRODUCTS OF WHICH DO NOT CONTAIN NONDEGENERATE HEREDITARILY INDECOMPOSABLE CONTINUA

by

Michel Smith

**Topology Proceedings**

**Web:** http://topology.auburn.edu/tp/
**Mail:** Topology Proceedings
Department of Mathematics & Statistics
Auburn University, Alabama 36849, USA
**E-mail:** topolog@auburn.edu
**ISSN:** 0146-4124

# CONTINUA ARBITRARY PRODUCTS OF WHICH DO NOT CONTAIN NONDEGENERATE HEREDITARILY INDECOMPOSABLE CONTINUA

**Michel Smith**

Let $X = [0,\infty)$, let $\beta X$ be the Stone-Čech compactification of X, and let $X^* = \beta X - X$. (See [W], [S1] for background information.) The author has shown [S2] that if $\kappa$ is a cardinal then the topological product $\prod_{\alpha\in\kappa} X^*$ does not contain a nondegenerate hereditarily indecomposable continuum. This result is surprising in view of Bellamy's result [Be] which implies that every nondegenerate subcontinuum of $X^*$ contains a nondegenerate indecomposable continuum. Also in the metric case Bing [Bi] showed that every two dimensional continuum contains a nondegenerate hereditarily indecomposable continuum. Therefore every product of two nondegenerate metric continua contains a nondegenerate hereditarily indecomposable continuum.

It is the purpose of this paper to generalize the author's original result. Let X denote a locally compact $\sigma$-compact metric space. We define the property of uniformly subdecomposable and show that if compact subsets of X have this property then $X^*$ does not contain a hereditarily indecomposable continuum. Futhermore if X is a locally compact $\sigma$-compact metric space so that $X^*$ does not contain a nondegenerate hereditarily indecomposable continuum then $\prod_{\alpha\in\kappa} X^*$ also does not contain a nondegenerate hereditarily indecomposable continuum.

**Definitions and Notation**

If X is a space and $H \subset X$ then $Cl_X(H)$ denotes the closure of H in X and $Bd_X(H)$ denotes the boundary of H in X. If $Y = \prod_{\alpha \in \kappa} X$ is a product space and $n \in \kappa$ then $\pi_n: Y \to X_n$ is the natural projection of Y onto the $n^{\underline{th}}$ coordinate space. The set of positive integers is denoted by N. If X is a metric space then the space $\beta X$ will be identified with the space of ultrafilters of closed subsets of X [W] and the points of X will be identified with the fixed ultrafilters in $\beta X$. If 0 is open in X then $Rgn_{\beta X}(0)$ denotes the open set in $\beta X$ defined by $Rgn_{\beta X}(0) = \{u \in \beta X | 0 \text{ contains a set in } u\}$. The subscript $\beta X$ may be omitted for notational convenience. If X is a locally compact metric space and H is a closed subset of X then the spaces $H^*$ and $X^* \cap Cl_{\beta X}H$ are homeomorphic and are sometimes identified. If G is a collection of subsets of X so that if G' is a finite subcollection of G then some member of g is contained in $\cap G'$ then G is called a filter base of subsets of X.

If X is a space then K(X) denotes the space of compact subsets of X with the standard hyperspace topology (see [N]), and C(X) denotes the subspace of K(X) consisting of the elements of K(X) which are subcontinua of X. If X is compact (metric) then so are K(X) and C(X).

We now wish to define a general class of metric spaces whose remainders will not contain hereditarily indecomposable continua.

*Definition.* The metric space X is uniformly subdecomposable if and only if in each closed (with respect to the Hausdorff metric) collection Z of subcontinua of X a closed collection W of subcontinua can be inscribed so that each member of Z contains a member of W and the members of W admit a decomposition into two subcontinua $A(I)$ and $B(I)$ so that non-empty subcompacta $a(I)$ can be chosen in $A(I) - B(I)$ with $a(I) \cap \cup\{B(J) \mid J \in W\} = \emptyset$ and analogous compacta $b(I)$ can be chosen in $B(I) - A(I)$.

*Definition.* Suppose that G is a set each element of which is a collection of subsets of the space X. Then let $Ls(G) = \cap\{Cl_{\beta X}(\cup g) \mid g \in G\}$. Note that $p \in Ls(G)$ if and only if $p \in \beta X$ and every open set in $\beta X$ containing p intersects a set in each element of G.

The following lemma follows easily from the definition.

*Lemma* 1.1. *If* G *is a set of collections of subsets of the space* X *then* $Ls(G)$ *is closed in* $\beta X$.

Assume in the following lemmas that X is a locally compact metric space.

*Lemma* 1.2. *Suppose that* K *is a subcontinuum of* $X^*$, p *and* q *are two points of* K *and* U *is an open set in* X *so that there is an element* $H_p \in p$ *and an element* $H_q \in q$ *so that* $H_p \subset U$ *and* $H_q \subset X - Cl_X U$. *Then* $Cl_{\beta X}(BdU) \cap K \neq \emptyset$.

*Proof.* Assume $Cl_{\beta X}(BdU) \cap K = \emptyset$. Let $V = X - Cl_X U$. Then $K \subset Rgn(U) \cup Rgn(V)$ because $Cl_{\beta X}(BdU) \cap K = \emptyset$. But $p \in Rgn(U)$ and $q \in Rgn(V)$, and $Rgn(U)$ and $Rgn(V)$ are disjoint open sets in $\beta X$. This contradicts the connectedness of K.

*Lemma* 1.3. *Suppose that* J *is a set of collections of subsets of* X *which is a filter base and for each* $j \in J$, $j$ *is a collection of continua. Suppose further that* M *is a collection of closed subsets of* X *which is a filter base such that if* $j \in J$ *then* $\cup j$ *contains an element of* M, *and that* u *is an ultrafilter in* $\beta X$ *that extends* M. *Furthermore for each* $j \in J$ *and* $H \in u$ *let* $f(j,H) = \{I \in j \mid I \cap H \neq \emptyset\}$ *and let* $F = \{f(j,H) \mid j \in J, H \in u\}$. *Then* $Ls(F)$ *is a continuum in* $\beta X$ *which contains* u.

*Proof*. Assume the hypothesis of the lemma and let $j \in J$ and $H \in u$. Then there is a set $\hat{H} \in M$ so that $\hat{H} \subset \cup j$. Then $\hat{H} \cap H \neq \emptyset$ and $\hat{H} \cap H \subset \cup j$. So some element I of j intersects $\hat{H} \cap H$. So $I \in f(j,H)$. Therefore for each $j \in J$ and $H \in u$ we have $f(j,H) \neq \emptyset$.

Assume that $u \notin Ls(F)$. Then there is an open set 0 containing u and an element $f(j,H) \in F$ so that $(\cup f(j,H)) \cap 0 = \emptyset$. Since $u \in 0$ there is an element $H' \in u$ such that $H' \subset 0 \cap X$. By hypothesis there is an element $H'' \in M$ so that $H'' \subset \cup j$. Therefore $H \cap H' \cap H'' \in u$. But $H \cap H' \cap H'' \subset 0 \cap X$ and $H \cap H' \cap H'' \subset \cup f(j,H)$ which is a contradiction. Therefore $u \in Ls(F)$.

Suppose that $Ls(F)$ is not a continuum. Then $Ls(F)$ is the union of two disjoint compact sets A and B, and assume $u \in A$. Let $U_A$ and $U_B$ be disjoint open sets containing A and B respectively. Then $Ls(F) \subset U_A \cup U_B$. For each point $x \in \beta X - U_A \cup U_B$ there is an open set $0_x$ and an element $f(j_x,H_x)$ of F so that $(\cup f(j_x,H_x)) \cap 0_x = \emptyset$. Some finite subcollection $0_{x_1}, 0_{x_2}, \cdots, 0_{x_n}$ of $\{0_x | x \in \beta X - U_A \cup U_B\}$ covers $\beta X - U_A \cup U_B$. Let $\hat{H}$ be an element of u with $\hat{H} \subset U_A$, let $\tilde{j}$ be an element of J which is a subset of $j_{x_1} \cap j_{x_2} \cap \cdots \cap j_{x_n}$, and let $\tilde{H} = \hat{H} \cap H_{x_1} \cap \cdots \cap H_{x_n}$. But then $(\cup f(\tilde{j},\tilde{H})) \cap (\cup_{i=1}^n 0_{x_n}) = \emptyset$ and so each element of $f(\tilde{j},\tilde{H})$ intersects $\hat{H}$ and cannot intersect $\cup_{i=1}^n 0_{x_i}$ and hence cannot intersect $\beta X - U_A \cup U_B$. Furthermore each element of $f(\tilde{j},\tilde{H})$ is connected and intersects $U_A$. Therefore $\cup f(\tilde{j},\tilde{H}) \subset U_A$. Therefore $Ls(F) \subset U_A$ which is a contradiction.

*Theorem* 1. *Let* X *be a locally compact σ-compact space so that every compact subspace of* X *is uniformly subdecomposable. Then* X* *does not contain a nondegenerate hereditarily indecomposable continuum.*

*Proof*. Let $K \subset X^*$ be a nondegenerate continuum. Let $p \in K$, U be an open set in X so that $p \in \mathrm{Rgn}(U)$, $K \not\subset Cl_{\beta X}U$, and let $H \in p$ be such that $H \subset U$. Since X is σ-compact there exists be a sequence of compact sets $H_1, H_2, \ldots$ with $H_i \cap H_j = \emptyset$ if $i \neq j$, $\cup_{i=1}^\infty H_i \subset H$, and $\cup_{i=1}^\infty H_i \in p$. Let $U_1, U_2, \cdots$ be a sequence of open sets in X so that $H_i \subset U_i \subset Cl_X U_i \subset U$ and $Cl_X(U_i)$ is compact for all i and $Cl_X U_i \cap Cl_X U_j = \emptyset$ if $i \neq j$. Let $\hat{U}$ denote $\cup_{i=1}^\infty U_i$ and let $\hat{H}$ denote $\cup_{i=1}^\infty H_i$.

Let $Q = Cl_{\beta X}Bd(\hat{U})$. By Lemma 2, $Q \cap K \neq \emptyset$.

Let G be the set to which the collection g belongs if and only if g is an open set in $\beta X$ which contains K. If $g \in G$ then let Tg be the collection to which I belongs if and only if I is a subcontinuum of $Cl_X(\hat{U})$ which intersects both Q and $\hat{H}$ and which lies in $Cl_X(X \cap g)$.

*Claim* 1. $Tg \neq \emptyset$ *for all* $g \in G$.

*Proof*. Suppose $g \in G$ and $Tg = \emptyset$. Let d be an open set in $\beta X$ such that $K \subset d \subset Cl_{\beta X}d \subset g$. Let $W = X \cap d$. Let $H' \subset \hat{H}$ be such that $H' \subset W$ and $H' \in p$. Let $H_i' = H_i \cap H'$. Then by assumption no subcontinuum of $Cl_X W \cap Cl_X U_i$ intersects both $H_i'$ and Q. So $Cl_X W \cap Cl_X U_i$ is the union of two disjoint compact sets $A_i$ and $B_i$ with $H_i \subset A_i$ and $Q \cap (Cl_X W \cap Cl_X U_i) \subset B_i$. Furthermore since $BdU_i \subset Q$ we have $A_i \subset U_i$. Then $A_i \cup B_i \subset g$ for all i. Let $U_A$ and $U_B$ be disjoint open sets in X so that $Cl_X U_A \cap Cl_X U_B = \emptyset$,

$$\bigcup_{i=1}^{\infty} A_i \subset U_A \subset Cl_X U_A \subset \hat{U} \text{ and}$$

$$\bigcup_{i=1}^{\infty} B_i \subset U_B \subset g.$$

So $Cl_X W \cap Cl_X U_i \subset U_A \cup U_B$, $Cl_{\beta X}(W \cap \hat{U}) \subset Cl_{\beta X}U_A \cup Cl_{\beta X}U_B$.

Let K' be the closure in $\beta X$ of the component of $K \cap Rgn(\hat{U})$ which contains p. By construction $Cl_{\beta X}U_A \subset Rgn(\hat{U})$ and $Cl_{\beta X}U_A \cap Q = \emptyset$. However $Q \cap K \subset d \subset g$ and K' must intersect $Bd(d \cap Rgn\ \hat{U})$. But $K' \subset d$ so K' intersects $Bd(Rgn\hat{U})$, hence $K' \cap Q \neq \emptyset$. By Lemma 1.2, $Cl_{\beta X}(BdU_A) \cap K' \neq \emptyset$. Let $q \in Bd_{\beta X}(Rgn(U_A)) \cap K'$. There exists $J \in q$ so that $J \subset \hat{U}$ and $J \subset W$, so $J \subset Cl_X\hat{U} \cap Cl_X W$, so $\bigcup_{i=1}^{\infty} A_i \cup \bigcup_{i=1}^{\infty} B_i \in q$, so either $J \cap (\bigcup_{i=1}^{\infty} A_i) \in q$ or

$J \cap (\cup_{i=1}^{\infty} B_i) \in q$. In either case $q \notin Bd(Rgn(U_A))$ which is a contradiction and Claim 1 is verified.

*Claim* 2. $\{Tg | g \in G\}$ *is a filter base.*

*Proof.* Let $g_1$ and $g_2$ be elements of G and let $g \in G$ be $g_1 \cap g_2$. Then

$$Tg \subset Tg_1 \cap Tg_2.$$

So the claim is easily verified by finite induction.

*Claim* 3. Tg *is a closed subset of* C(X) *for all* $g \in G$.

*Proof.* Suppose that Tg is not closed in C(X) and that $I_1, I_2, \cdots$ is a sequence of elements of Tg which converges to a point $I \in C(X)$ and $I \notin Tg$. Without loss of generality we may assume that there is an integer k so that $I_n \subset Cl_X U_k$ for all n. Therefore I must also lie in $Cl_X U_k$; furthermore since $I_n \subset Cl_X(X \cap g)$ for all n we have $I \subset Cl_X(X \cap g)$. Since $\hat{H}$ and Q are closed and $Q \cap I_n \neq \emptyset$ and $\hat{H} \cap I_n \neq \emptyset$ for all n then $I \cap \hat{H} \neq \emptyset$ and $I \cap Q \neq \emptyset$. Therefore $I \in Tg$ and so Tg is closed.

Note that since each element of Tg intersects both Q and $\hat{H}$ it follows that each element of Tg is nondegenerate.

Let $\tilde{g} \in G$. Let $T^n = \{I \in T\tilde{g} | I \subset Cl_X(U_n)\}$. Then $T\tilde{g} = \cup_{n=1}^{\infty} T^n$ and $T^n \subset C(Cl_X U_n)$. So by hypothesis for each n there exists a subset $W^n$ of $T^n$ and mappings $A^n, B^n: W^n \to C(Cl_X(U_n))$; $a^n, b^n: W^n \to K(Cl_X(U_n))$ which satisfy the definition of uniformly subdecomposable. Let $W = \cup_{n=1}^{\infty} W^n$ and let A, B: $W \to C(X)$; a, b: $W \to K(X)$ be defined by $A = \cup_{n=1}^{\infty} A^n$, $B = \cup_{n=1}^{\infty} B^n$, $a = \cup_{n=1}^{\infty} a^n$, and

$b = \cup_{n=1}^{\infty} b^n$. These are well-defined maps since the domains of the unioned maps are disjoint compact sets.

Let $S\colon W \to K(X)$ be defined by $S(I) = A(I) \cap B(I)$. It is not difficult to verify that $S$ is also continuous. Let $\tilde{G} = \{g \in G \mid g \subset \tilde{g}\}$. For each $g \in \tilde{G}$ let $Wg = \{J \in W \mid$ there exists $I \in Tg$ so that $J \subset I\}$ and let $Eg = \cup\{S(J) \mid J \in Wg\}$. Thus $Eg$ is a closed subset of $X$. Let $\mathcal{E} = \{Eg \mid g \in \tilde{G}\}$. Since $\{Tg \mid g \in \tilde{G}\}$ is a filter base then so is $\{Wg \mid g \in \tilde{G}\}$ and hence so is $\mathcal{E}$. Therefore there is an ultrafilter $u \in X^*$ which extends $\mathcal{E}$, so $\mathcal{E} \subset u$.

For each $g \in \tilde{G}$ and $L \in u$ let

$$\ell_A(g,L) = \{A(I) \mid I \in Wg \text{ and } I \cap L \neq \emptyset\}$$
$$\ell_B(g,L) = \{B(I) \mid I \in Wg \text{ and } I \cap L \neq \emptyset\}.$$

*Claim* 4. $\ell_A(g,L) \neq \emptyset$ *and* $\ell_B(g,L) \neq \emptyset$.

*Proof*. Let $g \in \tilde{G}$ and $L \in u$. Then since $u$ extends $\mathcal{E}$ we have $L \cap Eg \neq \emptyset$. So there is an element $I \in Wg$ so that $S(I) \cap L \neq \emptyset$, but $S(I) \subset A(I) \subset I$, so $I \cap L \neq \emptyset$. Thus $A(I) \in \ell_A(g,L)$. So $\ell_A(g,L) \neq \emptyset$ and similarly $\ell_B(g,L) \neq \emptyset$.

$$\text{Define } FA = \{\ell_A(g,L) \mid g \in \tilde{G},\ L \in u\}$$
$$FB = \{\ell_B(g,L) \mid g \in \tilde{G},\ L \in u\ .$$

Then by Lemma 1.3 $Ls(FA)$ and $Ls(FB)$ are continua in $\beta X$ which contain $u$.

*Claim* 5. $Ls(FA) \subset K$ *and* $Ls(FB) \subset K$.

*Proof.* If $z \notin K$ then there is an element $g \in G$ so that $z \notin Cl_{\beta X}(g)$. Let $L \in u$. But $Ls(FA) \subset Cl_{\beta X}(\cup \ell_A(g,L)) \subset Cl_{\beta X}(g)$ which is a contradiction. Similarly $Ls(FB) \subset K$.

Let $\tilde{a} = a(W) = \cup\{a(I) \mid I \in W\}$, and

$\tilde{b} = b(W) = \cup\{b(I) \mid I \in W\}$.

*Claim* 6. $Ls(FA) \cap Cl_{\beta X}(\tilde{a}) \neq \emptyset$ *and* $Ls(FB) \cap Cl_{\beta X}(\tilde{b}) \neq \emptyset$.

*Proof.* If $Ls(FA) \cap Cl_{\beta X}(\tilde{a}) = \emptyset$ then there is a covering $0_1, 0_2, \cdots, 0_n$ of $Cl_{\beta X}\tilde{a}$ and a set of elements of FA, $\{\ell_A(g_i,L_i)\}_{i=1}^n$ so that

$$\{0_i\}_{i=1}^n \text{ covers } Cl_{\beta X}\tilde{a} \text{ and}$$

$$0_i \cap (\cup \ell_A(g_i,L_i)) = \emptyset.$$

Let $\hat{g} = \cap_{i=1}^n g_i$ and let $\hat{L} = \cap_{i=1}^n L_i$. Then $\ell_A(\hat{g},\hat{L}) \cap (\cup_{i=1}^n 0_i) = \emptyset$ but this is a contradiction since by the definition $\ell_A(\hat{g},\hat{L})$ contains an element which intersects $\hat{a}$. Similarly $Ls(FQ) \cap Cl_{\beta X}(\tilde{b}) \neq \emptyset$.

*Claim* 7. $Ls(FA) \cap Cl_{\beta X}\tilde{b} = \emptyset$ and $Ls(FB) \cap Cl_{\beta X}\tilde{a} = \emptyset$.

*Proof.* If $\ell_A(g,L) \in FA$ then $\cup \ell_A(g,L) = \cup\{A(I) \mid I \in Wg\} \subset \{A(I) \mid I \in W\}$ and by condition 3 in the definition of uniformly subdecomposable we have $\tilde{b} \cap (\cup\{A(I) \mid I \in W\}) = \emptyset$. So $Ls(FA) \cap Cl_{\beta X}\tilde{b} = \emptyset$. Similarly $Ls(B) \cap Cl_{\beta X}\tilde{a} = \emptyset$.

So Claims 5, 6, and 7 show that $Ls(FP)$ and $Ls(FQ)$ are two intersecting continua neither one of which is a subset of the other. Therefore $Ls(FA) \cup Ls(FB)$ is a nondegenerate decomposable subcontinuum of K.

It is not difficult to verify that locally compact subspaces of some nice spaces such as the $\sin \frac{1}{x}$ continuum, the Knaster U continuum, or a solenoid satisfy the hypothesis of Theorem 1. For example if Y is a solenoid and X is a proper subspace of Y which is locally compact and Z is a compact subset of $C(X)$ then let $W = Z$ and define A, B, a, and b as follows. Let Y be embedded in $R^3$ with the standard embedding with a "clockwise" orientation assigned to it. Then for each $I \in W$, I is a rectifiable arc in $\mathbb{R}^3$, let $C_I$ denote the midpoint of I and let $P_I$ and $Q_I$ be the end points of I with $[P_I, Q_I]$ having a clockwise orientation. Then let $A(I)$ be the arc $[P_I, C_I]$, $B(I)$ be the arc $[C_j, Q_I]$, $a(I) = \{P_I\}$, and $b(I) = \{Q_I\}$. Then it can be seen that X is uniformly subdecomposable.

Also included among locally compact spaces which are uniformly subdecomposable are those which have finite rim type.

A theorem due to H. Cook [C] is needed for Theorem 2. Although the theorem was first proven in the metric case it is also true in the non-metric setting.

*Definition.* If X and Y are topological spaces and $F: X \to Y$ is a function then f is said to be confluent provided that if C is a continuum in Y then every component of $f^{-1}(C)$ is mapped onto C by f.

*Theorem* C [C]. *Suppose that* X *and* Y *are continua,* $f: X \to Y$ *is a mapping of* X *onto* Y *and* Y *is hereditarily indecomposable. Then* f *is confluent.*

*Definition.* The space X satisfies condition C means that for each nondegenerate subcontinuum E of X we have: $L = H \cup K$ is a subcontinuum of E where H and K are proper subcontinua of L, points $P \in H - K$ and $Q \in K - H$, open sets U, R, S, and V in X, a hereditarily indecomposable continuum M, an open set D in M, and mappings $h: X \to M$ and $g: X \to M$ such that:

1) $H \subset U$ and $K \subset V$,
2) $P \in R$ and $Cl_X R \subset U - V$,
3) $Q \in S$ and $Cl_X S \subset V - U$,
4) $h(P) \in D$ and $g(Q) \in D$,
5) $h^{-1}(D) \subset R$ and $g^{-1}(D) \subset S$,
6) $h(U) \cap h(S) = \emptyset$ and $g(V) \cap g(R) = \emptyset$.

Observe that if X is a nondegenerate continuum which satisfies condition C then X does not contain a nondegenerate hereditarily indecomposable continuum.

*Theorem* 2. *Suppose that* X *is a compact Hausdorff space which satisfies condition* C. *The if* $\kappa$ *is a cardinal, no nondegenerate subcontinuum of* $\prod_{\alpha \in \kappa} X$ *is hereditarily indecomposable.*

*Proof.* Suppose that the theorem is not true and that X is a compact Hausdorff space which satisfies the hypothesis of the theorem, that $Y = \prod_{j \in J} X$ and that I is a nondegenerate hereditarily indecomposable subcontinuum of Y. Let $n \in J$ be chosen so that $\pi_n(I)$ is nondegenerate. Let $E = \pi_n(I)$. Then there exists $L = H \cup K$ as described in they hypothesis and there exists points $P \in H - K$ and $Q \in K - H$, open sets U, V, R, and S in X, a hereditarily indecomposable continuum M, an open set D in M and mappings h and g satisfying conditions 1 - 6.

Let $a \in I$ be a point such that $a_n = \pi_n(a) \in H \cap K$. By Theorem C, $h|_{\pi_n(I)} \circ \pi|_I\colon I \to M$ and $g|_{\pi_n(I)} \circ \pi|_I\colon I \to M$ are confluent. Let $\hat{h} = h|_{\pi_n(I)}$, $\hat{g} = g|_{\pi_n(I)}$, and $\hat{\pi} = \pi_n|_I$. Let $C_H$ denote the component of $(\hat{h} \circ \hat{\pi})^{-1}(h(H))$ that contains a and let $C_K$ be the component of $(\hat{g} \circ \hat{\pi})^{-1}(g(K))$ that contains a.

*Claim.* $\hat{\pi}(C_H) \cap R \neq \emptyset$.

*Proof.* By condition 5, $h^{-1}(D) \subset R$ and by confluence $\hat{h} \circ \hat{\pi}(C_H) = \hat{h}(H)$. Since $h(P) \in D$ we have $h(H) \cap D \neq \emptyset$, so there is an element $x \in C_H$ so that $\hat{h} \circ \hat{\pi}(x) \in D$. But $h^{-1}(D) \subset R$ so $\hat{\pi}(x) \in R$.

Similarly we have $\hat{\pi}(C_K) \cap S \neq \emptyset$. Let $x_H \in C_H$ be a point so that $\hat{\pi}(x_H) \in R$ and let $x_K \in C_K$ be such that $\hat{\pi}(x_K) \in S$. Suppose $x_H \in C_K$ then $\hat{\pi}(x_H) \in \hat{\pi}(C_K)$ and since $\hat{g}(\hat{\pi}(C_K)) = g(K)$ we have $\hat{g}(\hat{\pi}(x_H)) \in g(K)$. By condition 6, $g(V) \cap g(R) = \emptyset$ and $K \subset V$. So $g(K) \cap g(R) = \emptyset$ which contradicts $\hat{\pi}(x_H) \in R$. Therefore $x_H \notin C_K$. Similarly $x_K \notin C_H$. Therefore $C_K \not\subset C_H$, $C_H \not\subset C_K$, and $C_K \cap C_H \neq \emptyset$; so $C_H \cup C_K$ is a decomposable subcontinuum of I. This establishes the theorem.

We need the following lemma which was proven in [S2]. We include the proof of the lemma for completeness.

*Lemma* 3.1. *Suppose* $X = [0,\infty)$, $M$ *is a pseudo-arc in the plane and* $A$ *is a piecewise linear ray disjoint from* $M$ *that limits down to* $M$, $H$ *is a nondegenerate subcontinuum of* $X^*$, $P \in H$ *and* $Q \in X^* - H$. *Suppose that* $U$, $R$, *and* $S$ *are open sets in* $\beta X$ *so that* $H \subset U$, $P \in R$, $Q \in S$, $Cl_{\beta X}R \subset U$, $Cl_{\beta X}U \cap Cl_{\beta X}S = \emptyset$. *Suppose further that* $p \in M$, $q \in M$, $D_p$ *and* $D_q$ *are closed circular discs in the plane with* $p$ *and* $q$ *in their respective interiors so that no vertex of* $A$ *intersects* $Bd(D_p \cup D_q)$, *and* $D_p \cap D_q = \emptyset$. *Then there exists a mapping* $h: X^* \to M$ *so that:*

1. $h(P) \in D_p$,
2. $h(S \cap X^*) \subset D_q$
3. $h^{-1}(D_p \cap M) \subset R$, *and*
4. $h(U \cap X^*) \cap h(S \cap X^*) = \emptyset$.

*Proof*. Suppose that $X$, $H$, $P$, etc. are as in the hypothesis. We will construct a mapping $h: X \to A$ so that the extension to $\beta X$ when restricted to $X^*$ will have the desired properties. Let $Y$ denote $M \cup A$. Since $A$ is piecewise linear, no vertex of $A$ intersects $Bd(D_p \cup D_q)$ and $D_p$ and $D_q$ are discs then no component of $A \cap (D_p \cup D_q)$ is degenerate and these components can be listed in order along $A$. Let $B_1, B_2, \cdots$ be the components of $A \cap D_p$ listed in order along the ray $A$. Let $C_1, C_2, \cdots$ be the components of $A \cap D_q$ listed in order along $A$. Since $Cl_{\beta X}U \cap Cl_{\beta X}S = \emptyset$, there exist countable sequences $\{V_i\}_{i=1}^{\infty}$ and $\{S_i\}_{i=1}^{\infty}$ of open intervals in $X$ so that

$Cl_X(X \cap U) \subset \cup_{i=1}^{\infty} V_i$, $Cl_X(X \cap S) \subset \cup_{i=1}^{\infty} S_i$, $Cl_X(\cup_{i=1}^{\infty} V_i) \cap Cl_X(\cup_{i=1}^{\infty} S_i) = \emptyset$, and $V_1 < S_1 < V_2 < S_2 <$ along X. Let $\{V_{n_i}\}_{i=1}^{\infty}$ denote the subsequence of $\{V_i\}_{i=1}^{\infty}$ each element of which contains a component of $R \cap X$. Without loss of generality we can assume $V_{n_1} = V_1$, and $B_1 < C_1$ along A.

Let $\{B_i\}_{i=1}^{\infty} \cup \{C_i\}_{i=1}^{\infty}$ be listed in order along A: $B_1, B_2, \ldots, B_{a_1}, C_1, C_2, \ldots, C_{b_1}, B_{a_1+1}, B_{a_1+2}, \ldots, B_{a_2}, C_{b_1+1}, C_{b_1+2}, \ldots, C_{b_2}, \ldots$. Thus $\{a_i\}_{i=1}^{\infty}$ and $\{b_i\}_{i=1}^{\infty}$ are increasing sequences of integers so that every element of $\{B_n\}_{n=a_i+1}^{a_{i+1}}$ follows $C_{b_i}$ and precedes $C_{b_i+1}$ along A and every element of $\{C_n\}_{n=b_{i-1}+1}^{b_i}$ follows $B_{a_i}$ and precedes $B_{a_i+1}$ along A. Let $L_P$ and $L_Q$ be elements of the ultrafilters P and Q respectively so that $L_P \subset R \cap X$ and $L_Q \subset S \cap X$. Thus $L_P \subset \cup_{i=1}^{\infty} V_{n_i}$ and $L_Q \subset \cup_{i=1}^{\infty} S_i$.

We wish to indroduce some notation. Let $M_1$ and $M_2$ be two closed intervals in X. Then let $[M_1,M_2]$ denote the set $\{x | y < x < z \text{ for all } y \in M_1 \text{ and } z \in M_2\} \cup M_1 \cup M_2$ and let $(M_1,M_2)$ denote the set $\{x | y < x < z \text{ for all } y \in M_1 \text{ and } z \in M_2\}$. Note that with this notation $[M_1,M_2]$ is a closed interval and $(M_1,M_2)$ is an open interval in X. The sets $[M_1,M_2]$ and $(M_1,M_2)$ are similarly defined for open intervals $M_1$ and $M_2$ in X, but in this case $[M_1,M_2]$ is open and $(M_1, M_2)$ is closed in X. We use the same notation if $M_1$ and $M_2$ are intervals in A.

We will construct $h: X \to A$ inductively. Suppose that h has been defined for all points of $V_{n_1} \cup (V_{n_1}, V_{n_k})$ so that:

1. $h(S_n) \subset \cup_{i=1}^{b_{k-1}} C_i$ for $n < n_k$,
2. $h(V_n) \cap \cup_{i=1}^{\infty} C_i = \emptyset$ for $n < n_k$,
3. $h^{-1}(\cup_{i=1}^{a_{k-1}} B_i) \subset R \cap (V_{n_1} \cup (V_{n_1}, V_{n_k}))$, and
4. $h(L_p \cap (V_{n_1} \cup (V_{n_1}, V_{n_k}))) \subset \cup_{i=1}^{a_{k-1}} B_i$.

Now we will construct h for $[V_{n_k}, V_{n_{k+1}}]$. Notice by construction that

$$(B_{a_i+1}, B_{a_{i+1}}) \cap D_q = \emptyset \text{ and}$$

$$(C_{b_i+1}, C_{b_{i+1}}) \cap D_p = \emptyset \text{ for all } i \in Z^+.$$

Map $V_{n_k}$ into $(C_{b_{k-1}}, C_{b_{k-1}+1})$ so that each $B_i$ for $a_{k-1} + 1 \leq i \leq a_k$ is the image of a subset of $R \cap X \cap V_{n_k}$, $[B_{a_{k-1}+1}, B_{a_k}]$ is in the image of $V_{n_k}$, and if $L_p \cap V_{n_k} \neq \emptyset$ then $h(L_p \cap V_{n_k}) \subset \cup_{i=a_{k-1}+1}^{a_k} B_i$. Map $(V_{n_k}, V_{n_k+1})$ onto $(B_{a_k}, B_{a_k+1})$ so that $h(S_{n_k}) \subset \cup_{i=b_{k-1}+1}^{b_k} C_i$.

Map $V_{n_k+1}$ into $(C_{b_k}, B_{a_{k+1}})$,

$S_{n_k+1}$ into $C_{b_k}$,

$\vdots$

$V_{n_k+\ell}$ into $(C_{b_k}, B_{a_{k+1}})$

$S_{n_k+\ell}$ into $C_{b_k}$

$\vdots$

$S_{n_{k+1}-1}$ into $C_{b_k}$.

Map $V_{n_{k+1}}$ into $(C_{b_k}, C_{b_k+1})$ so that each $B_i$ for $a_k +1 \leq i \leq a_{k+1}$ is the image of a subset of $R \cap X \cap V_{n_{k+1}}$, $[B_{a_k+1}, B_{a_{k+1}}]$ lies in the image of $V_{n_{k+1}}$, and if $L_p \cap V_{n_{k+1}} \neq \emptyset$ then $L_p \cap V_{n_{k+1}}$ is mapped into $\cup_{i=a_k+1}^{a_{k+1}} B_i$. Furthermore require that: $Cl_A(h(V_i)) \cap Cl_A(h(S_j)) = \emptyset$, $Cl_A(h(V_i)) \cap C_j = \emptyset$, and $B_i \cap Cl_A(h(S_j)) = \emptyset$ for all positive integers i and j. Then extend the map linearly over $[V_{n_k}, V_{n_k+1}] - \cup_{i=n_k}^{n_{k+1}} S_i \cup V_i$. It is not difficult to verify that h as defined above on $[V_{n_1}, V_{n_{k+1}}]$ satisfies conditions 1 - 4 with k replaced by k + 1.

Therefore by induction there exists a mapping $h\colon X \to A$ so that

I1: $h(\cup_{i=1}^{\infty} S_i) \subset \cup_{i=1}^{\infty} C_i$

I2: $h(\cup_{i=1}^{\infty} V_i) \cap (\cup_{i=1}^{\infty} C_i) = \emptyset$

I3: $h^{-1}(\cup_{i=1}^{\infty} B_i) \subset (R \cap X)$

I4: $h(L_p) \subset \cup_{i=1}^{\infty} B_i$

Thus h extends to a mapping $\hat{h}$: $\beta X \to Y$ so that $\hat{h}|_{X^*}$: $X^* \to M$ and from the above conditions we have:

J1: $\hat{h}(S \cap X^*) \subset \mathrm{Cl}_Y(\cup_{i=1}^{\infty} C_i) \cap M \subset D_q \cap M$,

J2: $\hat{h}(U \cap X^*) \subset Y - D_q$,

J3: $\hat{h}^{-1}(D_p) \subset R$, and

J4: $\hat{h}(L_p) \subset \mathrm{Cl}_Y(\cup_{i=1}^{\infty} B_i) = D_p \cap Y$.

Conditions J1, J2, and J4 follow easily from conditions I1, I2, and I4 respectively. Suppose that condition J3 is not satisfied. Then there exists a point $z \in \beta X - R$ so that $\hat{h}(z) \in D_p$. By condition I3, $z \notin X$.

So $z \in X^*$ and then there is a closed set L in z so that $L \cap (R \cap X) = \emptyset$. But then $h(L) \cap (\cup_{i=1}^{\infty} B_i) = \emptyset$ so $h(L) \cap (\mathrm{Cl}_Y(\cup_{i=1}^{\infty} B_i)) = \emptyset$ and hence $\hat{h}(z) \notin D_p$ which is a contradiction. Let $g = \hat{h}|_{X^*}$. By condition J4, $g(P) \in D_p$. By condition J1, $g(S \cap X^*) \subset D_p$. By condition J3, $g^{-1}(D_p \cap M) \subset R$. And $g(U \cap X^*) \cap g(S \cap X^*) = \emptyset$ follows from conditions J2 and J1. Then g is the required mapping and the lemma is established.

*Theorem* 3. *Let* X *be a locally compact* $\sigma$*-compact metric space so that* $X^*$ *does not contain a non-degenerate hereditarily indecomposable continuum. Then* $X^*$ *satisfies condition* C.

*Proof.* We wish to show that $X^*$ satisfies condition C. Let $z \in X^*$ be a point which lies in a subcontinuum $E$ of $X^*$. Let $B_1, B_2, \ldots$ be a sequence of open sets so that if $n \in N$,

1. $Cl_X(B_n)$ is compact,
2. $Cl_X(B_n) \subset B_{n+1}$, and
3. $\cup_{n=1}^{\infty} B_n = X$.

Let $C_n = Cl_X(B_n) - B_{n-1}$ for all $n \in N$. Either $\cup_{n=1}^{\infty} C_{2n} \in z$ or $\cup_{n=1}^{\infty} C_{2n-1} \in z$. Without loss of generality let us assume that $\cup_{n=1}^{\infty} C_{2n} \in z$. For each $n \in N$ let $L_n \subset C_{2n-1}$ be a closed separator so that $X - L_n$ is the union of two disjoint open sets $X_n^1$ and $X_n^2$ containing $\cup_{i=1}^{2n-2} C_i$ and $\cup_{i=2n}^{\infty} C_i$ respectively. Let $D_n = X_{n+1}^1 \cap X_n^2$, thus $C_{2n} \subset D_n$. Let $0_n$ be an open set in $X$ containing $C_{2n}$ such that

$$C_{2n} \subset 0_n \subset Cl_X 0_n \subset D_n.$$

Let $0 = \cup_{i=1}^{\infty} 0_n$ and let $\hat{0} = Rgn_{\beta X}(0)$. Note that $Cl_X 0_n \cap Cl_X 0_m = \emptyset$ for $n \neq m$, and $L_n \cap \cup_{i=1}^{\infty} CL_X 0_i = \emptyset$ for all $n$. Then $z \in \hat{0}$. Let $L$ be a decomposable subcontinuum of $\hat{0}$ which is a subset of $E$. Let $H$ and $K$ be proper subcontinua of $L$ so that $L = H \cup K$. Let $P \in H - K$ and let $Q \in K - H$. Let $U$, $V$, $R$, and $S$ be open sets in $\beta X$ so that

$$H \subset U \subset Cl_{\beta X} u \subset \hat{0},$$

$$K \subset V \subset Cl_{\beta X} V \subset \hat{0},$$

$$P \in R \subset Cl_{\beta X} R \subset U - Cl_{\beta X} V, \text{ and}$$

$$Q \in S \subset Cl_{\beta X} S \subset V - Cl_{\beta X} U.$$

For each $n \in N$ let

$$U_n = U \cap 0_n,$$
$$V_n = V \cap 0_n,$$
$$R_n = R \cap 0_n, \text{ and}$$
$$S_n = S \cap 0_n.$$

For each $n \in N$, let $h_n$ be a mapping $h_n\colon L_n \cup D_n \cup L_{n+1} \to [n,n+1]$ so that

1. $h_n^{-1}(n) = L_n$,
   $h_n^{-1}(n+1) = L_{n+1}$, and
2. $h_n(CL_X(U_n) \cup Cl_X(V_n)) = n + \frac{1}{2}$.

Let $J^P \in P$ be such that $J^P \subset R \subset X$ and let $J^Q \in Q$ be such that $J^Q \subset S \cap X$. Let $J_n^P = J^P \cap 0_n$ and $J_n^Q = J^Q \cap 0_n$ for all $n \in N$. If $n \in N$ let $g_n\colon Cl_X V_n \to [0,1]$ be a map such that

$$g_n^{-1}(0) = Bd_X V_n \cup Cl_X(V_n \cap U_n),$$
$$g_n^{-1}(\tfrac{1}{2}) = Bd_X S_n,$$
$$g_n^{-1}((\tfrac{1}{2}, 1]) = S_n, \text{ and}$$
$$g^{-1}(1) = J_n^Q$$

(where $g$ is not onto whenever $S_n = \emptyset$ or $J_n^Q = \emptyset$).

If $n \in N$ let $f_n\colon Cl_X U_n \to [0,1]$ be a map such that:

$$f_n^{-1}(0) = Bd(U_n) \cup Cl_X(V_n \cap U_n),$$
$$f_n^{-1}(\tfrac{1}{2}) = Bd(R_n),$$
$$f_n^{-1}((\tfrac{1}{2},1]) = R_n, \text{ and}$$
$$f_n^{-1}(1) = J_n^P.$$

Let $Z \subset E^2$ be defined as follows: $Z = \{(x,y)\,|\,y = 0$ and $x \geq 0$ or $x = n + \frac{1}{2}$ for some $n \in N$ and $-1 \leq y \leq 1\}$.
Let $h\colon X \to Z$ be defined as follows:

$$h(t) = (h_n(t), 0) \text{ if } t \in L_n \cup D_n \cup L_{n+1} \text{ and } t \notin U_n \cup V_n$$
$$= (h_n(t), f_n(t)) \text{ if } t \in U_n$$
$$= (h_n(t), -g_n(t)) \text{ if } t \in V_n.$$

Since $f_n(\mathrm{Cl}_X(U_n \cap V_n)) = g_n(\mathrm{Cl}_X(U_n \cap V_n)) = 0$ then $h\colon X \to Z$ is continuous. Note that $R \cap X = h^{-1}(\{(x,y) \in z\,|\,\frac{1}{2} < y\})$ and $S \cap X = h^{-1}(\{(x,y) \in z\,|\,y < -\frac{1}{2}\})$. Let $A = [0,\infty)$.

Let $j\colon Z \to A$ be defined by

$$j(x,y) = x \text{ if } y = 0$$
$$= n + \frac{1}{4} + (1-y)\frac{1}{4} \text{ if } x = n + \frac{1}{2} \text{ and } y \neq 0.$$

Then $j$ is continuous. Therefore $j \circ h\colon X \to A$ extends to a function $F\colon \beta X \to \beta A$ and since $(j \circ h)^{-1}([0,n])$ is compact, $F$ maps $X^*$ into $A^*$.

For each $n \in N$ let

$$U_n^A = \{x \in A\,|\,n + \tfrac{1}{16} < x < n + \tfrac{1}{2} + \tfrac{1}{16}\},$$
$$R_n^A = \{x \in A\,|\,n + \tfrac{1}{8} < x < n + \tfrac{3}{8}\},$$
$$V_n^A = \{x \in A\,|\,n - \tfrac{1}{16} < x < n + 1 - \tfrac{1}{16}\},$$
$$S_n^A = \{x \in A\,|\,n + \tfrac{5}{8} < x < n + \tfrac{7}{8}\}.$$

Let $U^A = \cup_{n=1}^{\infty} U_n^A$, $R^A = \cup_{n=1}^{\infty} R_n^A$, $V^A = \cup_{n=1}^{\infty} V_n^A$, $S^A = \cup_{n=1}^{\infty} S_n^A$, and

$$\hat{U} = F^{-1}(\mathrm{Rgn}_{\beta A} U^A),\ \hat{R} = F^{-1}(\mathrm{Rgn}_{\beta A} R^A),$$
$$\hat{V} = F^{-1}(\mathrm{Rgn}_{\beta A} V^A) \text{ and } \hat{S} = F^{-1}(\mathrm{Rgn}_{\beta A} S^A).$$

Then $\hat{U}$, $\hat{R}$, $\hat{V}$, and $\hat{S}$ satisfy properties 1 - 3 of condition C with respect to L, H, K, P, and Q.

Furthermore by Lemma 3.1 there exists a pseudo-arc $M$ in the plane and functions $\alpha$ and $\gamma$ with $\alpha$, $\gamma$: $A^* \to M$ and a disc $D$ in the plane so that:

i1. $\alpha(F(P)) \in D \cap M_A$

i2. $\alpha(\mathrm{Rgn}_{\beta A}(S^A) \cap A^*) \subset D$

i3. $\alpha^{-1}(D \cap M) \subset \mathrm{Rgn}_{\beta A}(R\ )$

i4. $\alpha(\mathrm{Rgn}_{\beta A}(U^A) \cap A^*) \cap \alpha(\mathrm{Rgn}_{\beta A}(S^A) \cap A^*) = \emptyset$.

j1. $\gamma(F(Q)) \in D \cap M$

j2. $\gamma(\mathrm{Rgn}_{\beta A}(R^A) \cap A^*) \subset D$

j3. $\gamma^{-1}(D \cap M) \subset \mathrm{Rgn}_{\beta A}(S^A)$

j4. $\gamma(\mathrm{Rgn}_{\beta A}(V^A) \cap A^*) \cap \gamma(\mathrm{Rgn}_{\beta A}(R^A) \cap A^*) = \emptyset$.

Then by letting $h = \alpha \circ F$ and $g = \gamma \circ F$ conditions 4 - 6 of condition C are satisfied.

*Corollary* 3.1. *If* $X$ *is a locally compact* $\sigma$-*compact metric space so that* $X^*$ *does not contain a nondegenerate hereditarily indecomposable continuum, then if* $\kappa$ *is a cardinal* $\prod_{\alpha\in\kappa} X^*$ *does not contain a nondegenerate hereditarily indecomposable continuum.*

*Theorem* 4. *Suppose that* $X$ *is a continuum which satisfies condition* C *and* $Y$ *is a continuum which does not contain a nondegenerate hereditarily indecomposable continuum. Then* $X \times Y$ *does not contain a nondegenerate hereditarily indecomposable continuum.*

*Proof.* Suppose that $X$ and $Y$ satisfy the hypothesis and that $I$ is a nondegenerate hereditarily indecomposable continuum which lies in $X \times Y$. Then since $Y$ does not contain a hereditarily indecomposable continuum $\pi_1(I)$ is nondegenerate. Let $\pi$ denote the projection $\pi_1: X \times Y \to X$ and let $E = \pi(I)$. Then by condition C there exists $L \subset E$ so that L is decomposable $L = H \cup K$ and there exist points $P \in H - K$ and $Q \in K - H$, open sets U, V, R, and S in X, a hereditarily indecomposable continuum M, and open set D in M, and mappings h and g satisfying 1-6 of condition C.

There is a point $a \in I$ such that $a = (a_1, a_2)$ and $a_1 = \pi(a) \in H \cap K$. From theorem C:

$$h|_{\pi(I)} \circ \pi|_I : I \to M \text{ and}$$

$$g|_{\pi(I)} \circ \pi|_I : I \to M$$

are confluent. Let $\hat{h}$, $\hat{g}$, and $\hat{\pi}$ denote $h|_{\pi(I)}$, $g|_{\pi(I)}$, and $\pi|_I$ respectively. Let $C_H$ and $C_K$ denote the components of $(\hat{h} \circ \hat{\pi})^{-1}(h(H))$ and $(\hat{g} \circ \hat{\pi})^{-1}(g(K))$ respectively that contain a.

By (5) of condition C, $h^{-1}(D) \subset R$ and by confluence we have $\hat{h} \circ \hat{\pi}(C_H) = \hat{h}(H)$. Since $h(P) \in D$ we have $h(H) \cap D \neq \emptyset$, so there is a point $x_H \in C_H$ so that $\hat{h} \circ \hat{\pi}(x_H) \in D$. Since $h^{-1}(D) \subset R$ we have $\hat{\pi}(x_H) \in R$. Similarly there is a point $x_K \in C_K$ so that $\hat{\pi}(x_K) \in S$. Suppose $x_H \in C_K$ the $\hat{\pi}(x_K) \in \hat{\pi}(C_K)$, so since $\hat{g}(\hat{\pi}(C_K)) = g(K)$ we have $\hat{g}(\hat{\pi}(x_K)) \in g(K)$. But by (6) of condition C, $g(V) \cap g(R) = \emptyset$ and $K \subset V$, so $g(K) \cap g(R) = \emptyset$ which contradicts the fact that

$\hat{\pi}(x_H) \in R$. Therefore the assumption that $x_H \in C_K$ is false so $x_H \notin C_K$. Similarly $x_K \notin C_H$. Therefore $C_K \not\subset C_H$, $C_H \not\subset C_K$, and $C_H \cap C_K \neq \emptyset$. So $C_H \cup C_K$ is a decomposable subcontinuum of I. This establishes the theorem.

*Corollary* 4.1. *If* $A = [0,\infty)$ *then* $\beta A \times \beta A$ *and* $\beta(A \times A)$ *are not homeomorphic.*

*Proof.* The space $\beta(A \times A)$ contains a nondegenerate hereditarily indecomposable continuum by Theorem 7 of [S3] which is non-metric. However $\beta A \times \beta A = A \times A \cup A^* \times A \cup A \times A^* \cup A^* \times A^*$ and by Theorem 4 none of these spaces contain a nondegenerate non-metric hereditarily indecomposable continuum.

*Question.* What conditions on the space X guarantee that X* does not contain nondegenerate hereditarily indecomposable continua for X locally compact metric.

**Bibliography**

[Be] D. P. Bellamy, *A non-metric indecômposable continuum*, Duke Math. J. 38, (1971), 15-20.

[Bi] R. H. Bing, *Higher-dimensional hereditarily indecomposable continua*, Trans. A.M.S. (1951), 267-273.

[C] H. Cook, *Continua which admit only the identify map onto nondegenerate subcontinua*, Fund. Math. LX (1967), 241-249.

[N] Nadler, *Hyperspace of sets*, Marcel Dekker, Inc., New York, NY 1978.

[S1] M. Smith, $\beta([0,\infty))$ *does not contain non-degenerate hereditarily indecomposable continua*, Proc. A.M.S. 101, No. 2 (1987), 377-384.

[S2] M. Smith, *No arbitrary product of* $\beta([0,\infty)) - [0,\infty)$ *contains a nondegenerate hereditarily indecomposable continuum*, Topology and its Applications, 28, (1988), 23-28.

[S3] M. Smith, $\beta(x - \{x\})$ *for* X *locally connected*, Topology and its Applications, 26 (1987), 239-250.

[W] R. C. Walker, *The Stone-Cech compactification*, Springer-Verlag, New York, Heidelberg, Berlin, 1974.

Auburn University

Auburn, Alabama 36849