EVEN HOMOGENEITY AND EFFROS' THEOREM

by

Kathryn F. Porter

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Introduction

In 1965, E. G. Effros [2] published an important result in the theory of transformation groups which has been used extensively in the study of homogeneity, function spaces, and continua theory. For example, G. Ungar [7] used Effros' Theorem to prove, among other things, that: Every 2-homogeneous metric continuum is locally connected. In this paper, the concept of even homogeneity will be introduced and used to extend a form of Effros' Theorem to allow its application to a larger collection of spaces. We first look at some definitions we shall be using.

If (X,T) is a topological space, then $H(X)$ is the collection of all self-homeomorphisms on X. A subgroup, G, of $H(X)$, is *transitive* provided that for each $x \in X$, the set $G(x) = X$, where $G(x) = \{y \in X:$ there exists $g \in G$ such that $g(x) = y\}$. A space, X, is *homogeneous* if for any $x, y \in X$ there is a homeomorphism, h, such that $h(x) = y$. A topology T' for G is called RMC [4] for G provided that, for all $g \in G$, the map, $m_g: G \to G$, defined by $m_g(f) = f \circ g$, is continuous.

1. ULH Spaces

In 1967, G. Ungar introduced the concept of uniform local homogeneity (ULH) [7] in order to generalize L. Ford, Jr.'s idea of strong local homogeneity (SLH) [3]. A completely regular space, (X,T), is called *uniformly locally homogeneous*, (ULH), provided there exists a uniformity, U, for X such that (1) $T = T_U$ and (2) for all $x \in X$ and $U \in U$, there exists an open neighborhood, O, of x such that if $y \in O$ there is some $g \in H(X)$ with $g(x) = y$ and graph$(g) \subseteq U$.

Note that graph$(g) \subseteq U$ if and only if $g(x) \in U[x]$ for each $x \in X$. Also, when we want to be more specific, we shall say "(X,T) is ULH w.r.t. U" which means that U satisfies the properties (1) and (2) above.

One of the nice properties of ULH spaces is that finite products of ULH spaces are ULH [7]. We extend this result to arbitrary products.

Theorem 1.1. Let Λ be an index set of arbitrary cardinality, and for all $\alpha \in \Lambda$, let X_α be a homogeneous ULH space. Let $X = \prod_{\alpha \in \Lambda} X_\alpha$. Then X is ULH.

Proof. Assume for all $\alpha \in \Lambda$, X_α is ULH w.r.t. U_α. Let P be the product uniformity on X. P has as a base the set $B = \{ \bigcap_{\alpha \in F} P_\alpha^{-1}(O_\alpha) :$ F is some finite subset of Λ and, for all $\alpha \in F$, $O_\alpha \in U_\alpha \}$ where the function, $P_\alpha : X \times X \to X_\alpha \times X_\alpha$, is defined by $P_\alpha(x,y) =$

$(\pi_\alpha(x), \pi_\alpha(y)) = (x_\alpha, y_\alpha)$. Let $p \in X$ and let $M \in P$, then

there exists a basis element, $B^* = \bigcap_{\alpha \in F} P_\alpha^{-1}(O_\alpha)$ such that

$(p,p) \in B^* \subseteq M$ and F is some finite subset of Λ. Now,

for each $\alpha \in \Lambda$, X_α is ULH w.r.t. U_α, therefore, for each

$\alpha \in F$, there exists a ULH neighborhood, V_α, in X_α, such

that $p_\alpha \in V_\alpha$. Set $V = \bigcap_{\alpha \in F} \pi_\alpha^{-1}(V_\alpha)$, then V is open in X.

If $z \in V$, then for each $\alpha \in F$, $z_\alpha \in V_\alpha$. So, for each

$\alpha \in F$, there is some $h_\alpha \in H(X_\alpha)$ such that $h_\alpha(p_\alpha) = z_\alpha$ and

graph $(h_\alpha) \subseteq O_\alpha$. For every $\alpha \in (\Lambda \backslash F)$, since X_α is homo-

geneous, there exists $\tilde{h}_\alpha \in H(X_\alpha)$ such that $\tilde{h}_\alpha(p_\alpha) = z_\alpha$.

Define $\hat{H}: X \to X$ by $\hat{H}(s) = <\hat{H}_\alpha(s_\alpha)>_{\alpha \in \Lambda}$ where:

$$\hat{H}_\alpha(s_\alpha) = \begin{cases} h_\alpha(s_\alpha) & \text{if } \alpha \in F \\ \tilde{h}_\alpha(s_\alpha) & \text{if } \alpha \in (\Lambda \backslash F) \end{cases}$$

$\hat{H} \in H(X)$ and $\hat{H}(p) = (z_\alpha)_{\alpha \in \Lambda} = z$. Show graph $(\hat{H}) \subseteq M$: Let

$t = (s, \hat{H}(s)) = ((s_\alpha)_{\alpha \in \Lambda}, (\hat{H}_\alpha(s_\alpha))_{\alpha \in \Lambda})$. For all $\alpha \in F$,

$(s_\alpha, \hat{H}_\alpha(s_\alpha)) = (s_\alpha, h_\alpha(s_\alpha)) \in$ graph $(h_\alpha) \subseteq O_\alpha$. Hence,

$t \in \bigcap_{\alpha \in F} P_\alpha^{-1}(O_\alpha) \subseteq M$. Therefore X is ULH w.r.t. P.

2. Even Homogeneity

Let (X,T) be a topological space and let G be a tran-

sitive subgroup of $H(X)$, with a topology T'. We say that

X is *evenly homogeneous w.r.t.* (G,T'), provided that for

all open sets $O \in T'$ such that e, the identity map on X,

is in O, then there exists an open cover $\hat{V}$ of X such that

if $x,y \in V \in \hat{V}$ then there exists $h \in O$ with $h(x) = y$. We

shall abbreviate this by E.H. w.r.t. (G,T'). The relation-

ship between even homogeneity and ULH is as follows.

Theorem 2.1. Let (X, U) *be a uniform space. Then,* X *is ULH w.r.t.* U *if and only if* X *is E.H. w.r.t.* $(H(X), T_U^\wedge)$ *where* $T_U^\wedge$ *is the topology induced on* $H(X)$ *by* U.

Proof. Let (X, U) be a uniform space.

($\Rightarrow$) Assume X is ULH w.r.t. U. Let $0 \in T_U^\wedge$ such that $e \in 0$. Then there exists $U \in U$ with $\hat{U}[e] \subset 0$, where $\hat{U} = \{(f,g): (f(x), g(x)) \in U$ for all $x \in X\}$. Now, there exists $V \in U$ such that $V = V^{-1}$ and $V \circ V \subset U$. So, for all $x \in X$, there exists an open neighborhood, 0_x, of x such that if $y \in 0_x$, there exists $g \in H(X)$ with $g(x) = y$ and graph$(g) \subset V$. $\{0_x\}_{x \in X}$ is an open cover of X, so if $p, q \in 0_x$ for some $x \in X$, there exist $h_1, h_2 \in H(X)$ with $h_1(x) = p$, $h_2(x) = q$ and for each $z \in X$, $(z, h_1(z))$, $(z, h_2(z)) \in V$. Hence, $h_2 \circ h_1^{-1}(p) = q$ and $(h_1(z), h_2(z)) \in V \circ V \subset U$, for all $z \in X$. Therefore, $(z, h_2 \circ h_1^{-1}(z)) \in U$, for all $z \in X$. Thus, $h_2 \circ h_1^{-1} \in \hat{U}[e] \subset 0$. So, X is E.H. w.r.t. $(H(X), T_U^\wedge)$.

($\Leftarrow$) Assume X is E.H. w.r.t. $(H(X), T_U^\wedge)$. Let $x \in X$ and $U \in U$. Then $e \in \hat{U}[e]$, so there exists an open cover, $\{0_\alpha\}_{\alpha \in \Lambda}$, of X such that if $p, q \in 0_\alpha$ for some $\alpha \in \Lambda$, there exists $h \in \hat{U}[e]$ with $h(p) = q$. $x \in X$, hence, there is some $\beta \in \Lambda$ such that $x \in 0_\beta$. If $y \in 0_\beta$, then there exists $h \in \hat{U}[e]$ such that $h(x) = y$. Recall that $h \in \hat{U}[e]$ if and only if graph$(h) \subset U$. Therefore X is ULH w.r.t. U.

As with ULH spaces, we have a result involving arbitrary products of evenly homogeneous spaces.

Theorem 2.2.   Let Λ be an index of arbitrary cardinality.   For all $\alpha \in \Lambda$, let (X, T_α) be a topological space and let G_α be a transitive subgroup of $H(X_\alpha)$.   For each $\alpha \in \Lambda$, let T_α' be a topology for G_α such that X_α is E.H. w.r.t. (G_α, T_α').   Set $X = \prod_{\alpha \in \Lambda} X_\alpha$ and $G = \prod_{\alpha \in \Lambda} G_\alpha$.   Let T and T_p be the product topologies on X and G respectively. Then X is E.H. w.r.t. (G, T_p).

Proof.   Let $O \in T_p$ with $e \in O$, where $e = \langle e_\alpha \rangle_{\alpha \in \Lambda}$ and e_α is the identity map on X_α.   Then there exists a basic open set, $B = \bigcap_{\alpha \in F} \pi_\alpha^{-1}(U_\alpha)$ where F is a finite subset of Λ, U_α is open in G_α for all $\alpha \in F$, and $e \in B \subset O$.   Now, for all $\alpha \in \Lambda$, X_α is E.H. w.r.t. (G_α, T_α').   So, for all $\alpha \in F$, there is some open cover, $\hat{W}_\alpha = \{W_\beta^\alpha\}_{\beta \in \Gamma_\alpha}$ such that if $p, q \in W_\beta^\alpha$ for some $\alpha \in F$ and some $\beta \in \Gamma_\alpha$, there exists some $h_{\alpha, \beta} \in U_\alpha$ with $h_{\alpha, \beta}(p) = q$.   Then, define $W = \{ \bigcap_{\alpha \in F} \pi_\alpha^{-1}(W_\beta^\alpha) : \beta \in \Gamma_\alpha \}$.   Note that W is an open cover of X.   If $s, t \in \bigcap_{\alpha \in F} \pi_\alpha^{-1}(W_\beta^\alpha)$ then for all $\alpha \in F$, $s_\alpha, t_\alpha \in W_\beta^\alpha$.   So, for each $\alpha \in F$, there exists some $h_{\alpha, \beta} \in U_\alpha$ such that $h_{\alpha, \beta}(s_\alpha) = t_\alpha$.   Also, since G_α is transitive for each $\alpha \in \Lambda$, we have that for each $\alpha \in (\Lambda \backslash F)$, there is an $h_\alpha \in G_\alpha$ such that $h_\alpha(s_\alpha) = t_\alpha$.   Define the function $\hat{H}: X \rightarrow X$ by $\hat{H}(x) = \langle g_\alpha(x_\alpha) \rangle_{\alpha \in \Lambda}$ where

$$g_\alpha = \begin{cases} h_{\alpha, \beta} & \text{if } \alpha \in F \\ h_\alpha & \text{if } \alpha \notin F \end{cases}$$

Thus, $\hat{H} \in G$, $\hat{H}(s) = t$, and $\hat{H} \in B \subset O$.   Therefore X is E.H. w.r.t. (G, T_p).

3. Evaluation Maps

Let (X,T) be a topological space and let G be a subgroup of $H(X)$ with a topology, T'. For each $x \in X$ we define the *evaluation map*, $E_x: (G,T') \to (X,T)$ by $E_x(g) = g(x)$. Ungar has shown the following about the evaluation map.

Theorem 3.1. [7] *Let* (X,U) *be a homogeneous uniform space and let* $H(X)$ *be given the induced uniform topology,* $T_U^\wedge$. *Then, for each* $x \in X$, *the evaluation map,* E_x, *is open if an only if* X *is ULH w.r.t.* U.

For even homogeneity we have the following results for the evaluation map.

Theorem 3.2. *Let* (X,T) *be a topological space and let* G *be a transitive subgroup of* $H(X)$. *Let* T' *be an RMC topology for* G. *If* X *is E.H. w.r.t.* (G,T') *then for each* $x \in X$, *the evaluation map,* $E_x: (G,T') \to X$, *is open.*

Proof. Assume X is E.H. w.r.t. (G,T'). Let $x \in X$ and let $O \in T'$. Suppose $y \in E_x(O)$, then there is some $h \in O$ such that $h(x) = y$. Then $h \in O$ implies that $e \in Oh^{-1} \in T'$, since T' is RMC. By even homogeneity there exists some open cover, $\{O_\alpha\}_{\alpha \in \Lambda}$, of X such that if $p,q \in O_\alpha$, for some $\alpha \in \Lambda$, then there is some $g \in Oh^{-1}$ with $g(p) = q$. But $y \in X$, so there exists a $\beta \in \Lambda$ such that $y \in O_\beta$ and if $r \in O_\beta$, there is an $f \in Oh^{-1}$ such that $f(y) = r$. Whence, $O_\beta \subset E_x(O)$ since $f \circ h \in O$ and

$f \circ h(x) = f(y) = r$.   Therefore, for all $x \in X$, the map

E_x: $(G,T') \to X$, is open.

   Corollary 3.1.   Let X be a topological space and let G be a transitive subgroup of $H(X)$.   Let T be an RMC topology for G such that X is E.H. w.r.t. (G,T).   Then for all topologies, $T*$, for G with $T* \subset T$, we have that for each $x \in X$, the evaluation map, E_x: $(G,T*) \to X$, is open.

   Corollary 3.2.   Let (X,T) be a topological space and let G be a transitive subgroup of $H(X)$.   Let T' be a topology for G such that (G,T') is a topological group. Then, X is E.H. w.r.t. (G,T') if and only if for each $x \in X$, the evaluation map, E_x: $(G,T') \to X$, is open.

   The following is an example of a topology for a function space for which the evaluation map is only open under extreme circumstances.

   Example 3.1.   Let (X,T) and $(Y,\tilde{T})$ be topological spaces.   For $O \subset X$ and $U \subset Y$, define the set $(O,U) =$ $\{f \in Y^X: f(O) \subset U\}$.   Let $S_{oo} = \{(O,U): O \in T \text{ and } U \in \tilde{T}\}$. Then S_{oo} is a subbasis for a topology, T_{oo}, on Y^X, called the *open-open topology* [5].

   We shall show that when X is a T_1 space, the evaluation map, E_x: $(H(X),T_{oo}) \to X$, is open for each $x \in X$, if and only if X is discrete: Assume that X is T_1.   Let $x \in X$.   Then $\{x\}$ is closed so that $(X \setminus \{x\})$ is open.   Note

that the set $V = ((X\backslash\{x\}), (X\backslash\{x\})) = (\{x\}, \{x\}) \in T_{oo}$.

Thus, $E_x(V) = \{x\}$. Since x was an arbitrary point in X,

hence $E_x(V)$ is open if and only if X is discrete.

The reason that the evaluation map, in the case when

X is not discrete, fails to be open, is that X is not

E.H. w.r.t. $(H(X), T_{oo})$: Fix an x in X. We have

$(\{x\},\{x\}) \in T_{oo}$ and $e \in (\{x\},\{x\})$. Suppose that V* is

an open cover of X. Therefore, there exists $V \in V*$ such

that $x \in V$. If $y \in V$, $y \neq x$, there is no $h \in (\{x\},\{x\})$

with $h(x) = y$, since if $h \in (\{x\},\{x\})$ then $h(x) = x$.

We now give an example of a space which is E.H. w.r.t.

(G, T), but which is not ULH. This example appeared in

Ford's paper [3] as an example of a space which possesses

a transitive homeomorphism group with no reasonable

topology.

Example 3.2. Let $X = R^2$ and let basic open neighbor-

hoods of a point $(x_0, y_0) \in X$ be of the form: $N_\varepsilon(x_0, y_0) =$

$\{(x,y): x_0 < x < x_0 + \varepsilon$ and $y_0 - \varepsilon < y < y_0 + \varepsilon\} \cup$

$\{(x_0, y_0)\}$ where $\varepsilon > 0$. Define $\hat{N}_\varepsilon(x_0, y_0) = N_\varepsilon(x_0, y_0)\backslash$

$\{(x_0, y_0)\}$.

X is T_2 but not regular since the point $(0,0)$ cannot

be separated from the set $C = \{(0,y): y \neq 0\}$ which is

closed in X. Hence X is not uniformizable and thus not

ULH.

Let G be the set of all translations, i.e., $G =$

$\{h: X \rightarrow X:$ there exists $a, b \in R$ such that for all

$(x,y) \in X$, $h(x,y) = (x + a, y + b)\}$.

Give G the topology, T, whose subbasic open sets are of the form $(\{(p,q)\}, \hat{N}_\varepsilon(s,t)) = \{f \in G: f(p,q) \in \hat{N}_\varepsilon(s,t)\}$, where $(p,q) \in R^2$.

Claim. X is E.H. w.r.t. (G,T): Let O be open in G such that $e \in O$. Then there exists a basic open set, say $B = \bigcap_{i=1}^{n}(\{(x_i,y_i)\}, \hat{N}_{\varepsilon_i}(p_i,q_i))$ with $e \in B \subset O$. Note that, without loss of generality, $(x_i,y_i) \neq (x_j,y_j)$ if $i \neq j$. Also we can choose the ε_i small enough so that the sets $\hat{N}_{\varepsilon_i}(p_i,q_i)$ do not intersect since X is T_2. Now $e \in B$, so we know that for all $i = 1,2,3,\ldots,n$, $(x_i,y_i) \in \hat{N}_{\varepsilon_i}(p_i,q_i)$. For each $i = 1,2,3,\ldots,n$, let $\delta_i = \min \{\varepsilon_i + p_i - x_i, \varepsilon_i - |y_i - q_i|\}$ and let $\varepsilon = \min_{i \in \{1,2,3,\ldots,n\}} \delta_i$.

Let $U = \{\hat{N}_\varepsilon(x,y): (x,y) \in X\}$. U is an open cover of X and let $(p,q), (s,t) \in \hat{N}_\varepsilon(\hat{x},\hat{y})$. Then define the function, $h \in G$ by $h(x,y) = (x - s + p, y - t + q)$. $h(s,t) = (p,q)$. So for each $i = 1,2,3,\ldots,n$, $h(x_i,y_i) = (x_i - s + p, y_i - t + q)$. Thus, $|p_i - (x_i - s + p)| \leq |p_i - x_i| + |p - s| < (x_i - p_i) + \varepsilon < \varepsilon_i$ also $|q_i - (y_i - t + q)| \leq |q_i - y_i| + \varepsilon < \varepsilon_i$. Hence, $h \in B \subset O$. Therefore X is E.H. w.r.t. (G,T).

4. Effros' Theorem

The following theorem is the form of Effros' Theorem [2] which is most often used. Recall that the *compact-open topology*, T_{co}, on $H(X)$, has subbasis elements of

the form $(C,U) = \{g \in H(X): g(C) \subset U\}$, where C is a
compact subset of X and U is an open set in X. When X is
a compact metric space, T_{co}, is the topology on $H(X)$ in-
duced by the sup metric.

   *Theorem 4.1. (Effros' Theorem) [1] Let (X,T) be a
non-degenerate, compact, homogeneous, metric space. Then
for all $x \in X$, the evaluation map, E_x: $(H(X), T_{co}) \to (X,T)$
is open.*

   We now extend Effros' result to uncountable products
of compact, homogeneous, metric spaces, by using the con-
cept of even homogeneity. Note that an uncountable pro-
duct of metrizable spaces is not metrizable, so that our
new result is not covered by Effros' Theorem. First,
though, we need the following theorem.

   *Theorem 4.2. Let X be a topological space and let G
be a transitive subgroup of $H(X)$ such that X is E.H. w.r.t.
(G,T). Let H be a subgroup of $H(X)$ such that $G \subset H$. Let
$\hat{T}$ be a topology on H and let T_S be the subspace topology
on G inherited from $(H,\hat{T})$. If $T_S \subset T$ then X is E.H. w.r.t.
$(H,\hat{T})$.*

   Proof. Let $O \in \tilde{T}$ such that $e \in O$. Then $e \in O \cap G \in$
$T_S \subset T$. Since X is E.H. w.r.t. (G,T), there exists an
open cover, $\hat{V}$, of X such that if $p,q \in V$ for some $V \in \hat{V}$,
then there is some $h \in O \cap G$ with $h(p) = q$. $O \cap G \subset O$,
hence X is E.H. w.r.t. $(H,\hat{T})$.

   Finally we are ready to prove the desired theorem.

Theorem 4.3. Let Λ be an uncountable index set.
For all $\alpha \in \Lambda$, let X_α be a nondegenerate, homogeneous,
compact, metric space. Let $X = \prod\limits_{\alpha \in \Lambda} X_\alpha$ and give X the
product topology, T. Then for each $x \in X$, the evaluation
map, $E_x: (H(X), T_{co}) \to X$ is open.

Proof. From Effros' Theorem, we have that for each
$\alpha \in \Lambda$ and for all $x_\alpha \in X_\alpha$, the evaluation map, E_{x_α} :

$(H(X_\alpha), T_{co}) \to X_\alpha$, is open. We know that $(H(X_\alpha), T_{co})$ is a
topological group, since X_α is compact Hausdorff. Hence,
Corollary 3.2 gives us that for each $\alpha \in \Lambda$, X_α is E.H.
w.r.t. $(H(X_\alpha), T_{co})$. Set $H = \prod\limits_{\alpha \in \Lambda} H(X_\alpha)$. Then by Theorem
2.2, we have that X is E.H. w.r.t. (H, T_p) where T_p is the
product topology on H. Let T_S be the subspace topology
on H as a subspace of $(H(X), T_{co})$.

Claim. $T_S \subset T_p$ on H: (This Claim is a known result.
However, the proof is given here for completeness.) Let
B be a subbasic open set in T_S. Without loss of generality,
we can choose $B = (C, V) \cap H$ where C is a compact subset of
X and V is open in X. Let $f \in B$. Then $f(C) \subset V$ and
$f = <f_\alpha>_{\alpha \in \Lambda}$. So for all $x \in C$, there exists a basic
open set, $O_x = \bigcap\limits_{\alpha \in F_x} \pi_\alpha^{-1}(O_\alpha)$ where F_x is a finite subset of
Λ and, for all $\alpha \in F_x$, O_α is open in X_α, and such that
$f(x) \in O_x \subset V$. By regularity, for each $x \in C$ and for
each $\alpha \in F_x$, there exists some $\varepsilon_{x_\alpha} > 0$ with

$f_\alpha(x_\alpha) \in B(f_\alpha(x_\alpha), \varepsilon_{x_\alpha}) \subset Cl_{X_\alpha} B(f_\alpha(x_\alpha), \varepsilon_{x_\alpha}) \subset O_\alpha$. Thus,

for all $x \in C$ and for all $\alpha \in F_x$, there exists some

$\delta_{x_\alpha} > 0$ such that $x_\alpha \in B(x_\alpha, \delta_{x_\alpha}) \subset Cl_{X_\alpha} B(x_\alpha, \delta_{x_\alpha}) \subset$

$f_\alpha^{-1}(B(f_\alpha(x_\alpha), \varepsilon_{x_\alpha})) \subset f_\alpha^{-1}(Cl_{X_\alpha} B(f_\alpha(x_\alpha), \varepsilon_{x_\alpha})) \subset f_\alpha^{-1}(O_\alpha)$.

Hence, for all $x \in C$ and for all $\alpha \in F_x$, $f_\alpha \in (Cl_{X_\alpha} B(x_\alpha, \delta_{x_\alpha})$,

$B(f_\alpha(x_\alpha), \varepsilon_{x_\alpha})) \in T_{co}$ on X_α.

Now $C \subset \bigcup\limits_{x \in C} \left[\bigcap\limits_{\alpha \in F_x} \pi_\alpha^{-1}(B(x_\alpha, \delta_{x_\alpha})) \right]$. C is compact, so

there exists a finite subcover, say

$$C \subset \bigcup\limits_{i=1}^{n} \left[\bigcap\limits_{\alpha \in F_{x_i}} \pi_\alpha^{-1}(B(x_\alpha^i, \delta_{x_\alpha^i})) \right]$$

$$\subset \bigcup\limits_{i=1}^{n} \left[\bigcap\limits_{\alpha \in F_{x_i}} \pi_\alpha^{-1}(Cl_{X_\alpha} B(x_\alpha^i, \delta_{x_\alpha^i})) \right]$$

$$\subset \bigcup\limits_{i=1}^{n} \left[\bigcap\limits_{\alpha \in F_{x_i}} \pi_\alpha^{-1} \circ f_\alpha^{-1}(B(f_\alpha(x_\alpha^i), \varepsilon_{x_\alpha^i})) \right]$$

$$\subset \bigcup\limits_{i=1}^{n} \left[\bigcap\limits_{\alpha \in F_{x_i}} \pi_\alpha^{-1} \circ f_\alpha^{-1}(Cl_{X_\alpha} B(f_\alpha(x_\alpha^i), \varepsilon_{x_\alpha^i})) \right]$$

$$\subset \bigcup\limits_{i=1}^{n} \left[\bigcap\limits_{\alpha \in F_{x_i}} \pi_\alpha^{-1} \circ f_\alpha^{-1}(O_\alpha) \right]$$

$$= \bigcup\limits_{i=1}^{n} \left[\bigcap\limits_{\alpha \in F_{x_i}} f^{-1} \circ \pi_\alpha^{-1}(O_\alpha) \right]$$

$$= \bigcup_{i=1}^{n} f^{-1}\left(\bigcap_{\alpha \in F_{x_i}} \pi_\alpha^{-1}(O_\alpha) \right)$$

$$= \bigcup_{i=1}^{n} f^{-1}(O_{x_i}) \subset f^{-1}(V)$$

Define $L = \bigcap_{\alpha \in F} \pi_\alpha^{-1}(Cl_{X_\alpha} B(x_\alpha, \delta_{x_\alpha}), B(f_\alpha(x_\alpha), \varepsilon_{x_\alpha}))$

where $F = \bigcup_{i=1}^{n} F_{x_i}$, then $f \in L \in T_p$. If $g \in L$ then for

all $\alpha \in F$, $g_\alpha(Cl_{X_\alpha} B(x_\alpha, \delta_{x_\alpha})) \subset B(f_\alpha(x_\alpha), \varepsilon_{x_\alpha})$. If $t \in C$,

there exists some k such that $t \in \bigcap_{\alpha \in F_{x_k}} \pi_\alpha^{-1}(B(x_\alpha^k, \delta_{x_\alpha^k})) \subset$

$\bigcap_{\alpha \in F_{x_k}} \pi_\alpha^{-1}(Cl_{X_\alpha} B(x_\alpha^k, \delta_{x_\alpha^k}))$. Thus, for all $\alpha \in F_{x_k} \subset F$,

$t_\alpha \in Cl_{X_\alpha} (B(x_\alpha^k, \delta_{x_\alpha^k}))$ which implies that for all

$\alpha \in F_{x_k}$, $g_\alpha(t_\alpha) \in B(f_\alpha(x_\alpha^k), \varepsilon_{x_\alpha^k})$. So, $g(t) \in$

$\bigcap_{\alpha \in F_{x_k}} \pi_\alpha^{-1}(B(f_\alpha(x_\alpha^k), \delta_{x_\alpha^k}) \subset O_{x_k} \subset V$. Thus, $g \in (C,V) \cap H$.

Hence, $T_s \subset T_p$.

Therefore, by Theorem 4.2, X is E.H. w.r.t. $(H(X), T_{co})$,

and so, Theorem 3.2 implies that $E_x: (H(X), T_{co}) \to X$, is

open, for each $x \in X$.

Some spaces which are included in the above case,

which were not covered by Effros' Theorem are uncountable

products of unit circles, S^1, and uncountable products of [0,1]'s. Note that [0,1] is not homogeneous, but the Hilbert Cube, Q, which is a countable product of [0,1]'s is homogeneous and an uncountable product of [0,1]'s can be written as an uncountable product of Q's.

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