TOPOLOGY PROCEEDINGS

Volume 13, 1988

Pages 403–406

http://topology.auburn.edu/tp/

PROBLEM SECTION

Topology Proceedings

Web: http://topology.auburn.edu/tp/
Mail: Topology Proceedings
Department of Mathematics & Statistics
Auburn University, Alabama 36849, USA
E-mail: topolog@auburn.edu
ISSN: 0146-4124

PROBLEM SECTION

CONTRIBUTED PROBLEMS

The problems designated [STC] are related to talks that were given at the 1988 Spring Topology Conference in Gainesville, Florida, for which this volume of TOPOLOGY PROCEEDINGS is the journal of record.

The Problems Editor invites anyone who has published a paper in TOPOLOGY PROCEEDINGS or attended a Spring Topology Conference to submit problems to this section. They need not be related to any articles, but if they are, please provide a reference. Please define any terms not in a general topology text or in referenced articles.

C. Compactness and Generalizations

57. (*Nyikos*) Is there an internal characterization of Rosenthal compacta?

58. (*Nyikos*) Is it consistent that every separable, hereditarily normal, countably compact space is compact?

59. (*Nyikos*) Is it consistent that every hereditarily normal, countably compact space is either compact or contains a copy of ω_1?

F. Continua Theory

30. (*E. Tymchatyn, attributed to Bellamy*) Let S_n be a solenoid and let K_n be the Knaster indecomposable continuum obtained by identifying in the topological group S_n each point with its inverse. Do there exist in K_2 two components without endpoints which are *not* homeomorphic? [STC]

31. (*E. Tymchatyn*) Are two homeomorphic composants in K_n [see preceding problem] in the same position? [STC]

32. (*E. Tymchatyn*) If S_n has a composant that is homeomorphic to one of S_m, is S_n homeomorphic to S_m? [STC]

See also G.23.

G. Mappings of Continua and Euclidean Spaces

23. (*E. Tymchatyn*) Is each homeomorphism $h: C \to \bar{C}$ of composants of solenoids homotopic to a linear homeomorphism $\bar{h}: C \to \bar{C}$ (i.e., $\bar{h}(x) = ax + b$ for each x)? A positive solution would imply positive solutions to the problems under heading F above. [STC]

O. Theory of Retracts: Extension of Continuous Functions

12. (*R. Levy*) Is there a ZFC example of a metric space having a subset that is 2-embedded (i.e., every continuous function into a two-point discrete space has an extension to a continuous function on the whole space) but not C*-embedded?

See also heading S.

P. Products, Hyperspaces, Remainders, and Similar Constructions

31. (*R. Levy*) Let X be either [0, 1) or a Euclidean space of dimension at least 2 (this is so $\beta X \setminus X$ will be connected). Is the set of weak P-points of $\beta X \setminus X$ connected?

32. (*R. Levy*) Is there a realcompact space X such that for some $p \in \beta X \setminus X$, the space $\beta X \setminus \{p\}$ is normal? We obtain an equivalent problem if "Lindelöf" is substituted for "realcompact."

S. Problems Closely Related to Set Theory

16. (*R. Levy*) Is it consistent that there is a version of ψ such that every subset of $\aleph_1$ nonisolated points is 2-embedded [defined in O-12], or C*-embedded? The first question is equivalent to asking for a MAD family M of subsets of ω such that, given disjoint subfamilies S and T of cardinality $\aleph_1$, there is $A \subset \omega$ such that A almost contains each member of S and almost misses each member of T.

INFORMATION ON EARLIER PROBLEMS

From Arkhangel'skii's Survey Paper

[Uspehii. Mat. Nauk. 33:6 (1978) 29-84 (=Russian Math. Surveys 33:6 (1978) 33-96)].

8. For which cardinals τ does there exist "naïvely" a regular space for which $hl(X^n) \leq \tau$ for all $n \in \mathbf{N}$ and $d(X) > \tau$? *Partial Solution.* Yes, for any singular cardinal τ satisfying $2^{cf\tau} < \tau$ (*Todorčević*). See also partial solutions in Volume 11 and Volume 12 of TOPOLOGY PROCEEDINGS.

From TOPOLOGY PROCEEDINGS

Classic Problem V, Vol. 2, related problem: Is every separable hereditarily normal compact space countably tight? *Consistency results.* (*Nyikos*) Yes if there are no uncountable Q-sets, and no if $\mathbf{p} > \omega_1$ and the club filter on ω_1 has a base of cardinality $< \mathbf{b}$. In any model of set theory, the question is equivalent to asking whether it is impossible for the Franklin-Rajagopalan space $\gamma\mathbf{N}$ to be hereditarily normal. If this holds in some model of PFA, then this would give an affirmative solution to Problem C.58 above, as well as solving the other problems related to Classic Problem V in Vol. 2.

B.30, Vol. 11 (*M. E. Rudin*) A Collins-Roscoe-Reed space is one in which each point x has a special countable open base $\mathbf{W}_x$ with the property that, if U is a neighborhood of a point y, there is a neighborhood V of y such that, for all $x \in V$ there is a $W \in \mathbf{W}_x$ with $y \in V \subset U$. [Recall that such a space is metrizable precisely if $\mathbf{W}_x$ can be made a nested decreasing sequence for each x]. It is easy to see that every space with a point countable base is a Collins-Roscoe-Reed space. Is the converse true? *Note*. W. S. Watson has withdrawn his claim of a counterexample under V=L, and the problem is still completely open.

C.53, Vol. 11 (*Uspensky*) Is every Eberlein compact space of nonmeasurable cardinal bisequential? *Solution*. No (*Nyikos*). The result does hold, however, for uniform Eberlein compacta.

D.37, Vol. 11 (*Nyikos*) Is there a "real" example of a locally compact, realcompact, first countable space of cardinality $\aleph_1$ that is not normal? *Solution*. Yes, there is a Moore space obtained by splitting nonisolated points of the Cantor tree, which has all the desired properties. See Theorem II.4.2 in Shelah's *Proper Forcing*, Springer-Verlag Lecture Notes #940, 1982.