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ČECH-COMPLETENESS AND COUNTABLY SUBCOMPACTNESS

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In [6] J. De Groot introduced the notion of (countably) subcompact spaces and proved a remarkable result that a metric space X is subcompact if and only if X is Čech-complete. Aarts and Lutzer [1] proved that a Moore space X is subcompact if X is Čech-complete, and asked whether every Čech-complete space is countably subcompact. Recently, M. C. Rayburn [8] remarked that it is not even known whether every paracompact, Čech-complete space is countably subcompact.

In this paper, we investigate relations between Čech-completeness and countably subcompactness. In Section 2, as a generalization of the above result obtained by Aarts and Lutzer, we show that a quasi-Moore space X is subcompact if X is Čech-complete, and moreover, that a peripherally compact Moore space X is Čech-complete, if X is countably \mathcal{B} -subcompact for a special subcollection \mathcal{B} of the regular open sets. In Section 3, we prove that if X is \mathcal{B} -subcompact for an open base \mathcal{B} , and X has a normal sequence of \mathcal{B} -covers, then the metric space \tilde{X} associated with this normal sequence is complete. From this result, if X is a paracompact, countably \mathcal{B} -subcompact space, then the metric space \tilde{X} associated with any normal sequence of \mathcal{B} -covers is complete, and hence, we show that

a countably subcompact, paracompact p -space is Čech-complete. In Section 4, we give some examples.

All spaces considered in this paper are assumed to be completely regular Hausdorff. N denotes the set of positive integers, and βX denotes the Stone-Čech compactification of a space X . The notions and terminologies can be found in [4].

1. Preliminaries

Let \mathcal{B} be an open base of a space X . A non-empty subset F of \mathcal{B} is called *regular \mathcal{B} -filter base* if every finite intersection of elements of F contains an element of F whose closure is contained in this intersection [6]. A regular \mathcal{B} -filter base F is countable, then F is called *countably regular \mathcal{B} -filter base*, especially, $\{U_n \in \mathcal{B} \mid n \in N, \text{cl}_X U_{n+1} \subset U_n\}$ is called a *regular \mathcal{B} -sequence*. A space X is called *\mathcal{B} -subcompact* (resp. *countably \mathcal{B} -subcompact*) if every regular \mathcal{B} -filter base (resp. countably regular \mathcal{B} -filter base) has the non-empty intersection. Every \mathcal{B} -subcompact space is countably \mathcal{B} -subcompact. A space X is *subcompact* (resp. *countably subcompact*) if there exists an open base \mathcal{B} such that X is \mathcal{B} -subcompact (resp. countably \mathcal{B} -subcompact) [6].

Now we mention a few facts about subcompactness. If \mathcal{B}_1 and \mathcal{B}_2 are open base of X and $\mathcal{B}_1 \subset \mathcal{B}_2$, then \mathcal{B}_2 -subcompactness (resp. countably \mathcal{B}_2 -subcompactness) of X implies \mathcal{B}_1 -subcompactness (resp. countably

\mathcal{B}_1 -subcompactness). A space X is \mathcal{B} -subcompact (resp. countably \mathcal{B} -subcompact) relative to any open base \mathcal{B} of X iff X is compact (resp. pseudo-compact) [6, 8]. If X is locally compact and $\mathcal{B} = \{U \mid U \text{ is open and } \text{cl}_X U \text{ is compact}\}$, then X is clearly \mathcal{B} -subcompact. Therefore if X is a locally compact and not compact (resp. not pseudo-compact), then there exist two open bases \mathcal{B}_1 and \mathcal{B}_2 such that X is \mathcal{B}_1 -subcompact (resp. countably \mathcal{B}_1 -subcompact) but not \mathcal{B}_2 -subcompact (resp. not countably \mathcal{B}_2 -subcompact). If X is pseudo-compact and not compact, then there exists an open base \mathcal{B} such that X is countably \mathcal{B} -subcompact but not \mathcal{B} -subcompact.

Let T be a collection of subsets of X . We recall that T have the "descending chain condition" provided that every sequence of elements of T which consists of properly decreasing sets, is finite [6]. We need the following lemmas.

Lemma 1.1. Let \mathcal{B} be an open base of a space X . Then X is countably \mathcal{B} -subcompact if and only if every regular \mathcal{B} -sequence has the non-empty intersection.

Proof. The "only if" part is obvious, we prove the "if" part. Let F be a countably regular \mathcal{B} -filter base and let $F = \{F_n \mid n \in \mathbb{N}\}$. Put $U_1 = F_1$ and put $n_1 = \min\{k \mid \text{cl}_X F_k \subset U_1\}$. There exists a $U_2 \in F$ such that $U_2 \subset F_1 \cap F_2 \cap \dots \cap F_{n_1}$. Put $n_2 = \min\{k \mid \text{cl}_X F_k \subset U_2\}$, then $n_1 < n_2$. Repeating this process, we obtain a regular

\mathcal{B} -sequence $\{U_n \mid n \in \mathbb{N}\}$ such that $\bigcap_{n=1}^{\infty} F_n = \bigcap_{n=1}^{\infty} U_n \neq \emptyset$.

Therefore X is countably \mathcal{B} -subcompact.

The proof of the following is similar to one of the set theoretic lemma of Groot [6].

Lemma 1.2. Every collection T of subsets of X has a subcollection T' satisfying the descending chain condition such that

$$\cup\{F \mid F \in T\} = \cup\{F' \mid F' \in T'\}.$$

2. \check{C} ech-completeness and Countably Subcompactness of a Moore Space

A *development* (resp. *quasi-development* [2]) for a space X is a sequence $\{U_n \mid n \in \mathbb{N}\}$ of open covers (resp. families of open sets) such that for every point $x \in X$, $\{\text{st}(x, U_n) \mid n \in \mathbb{N}\}$ is a base at x , where $\text{st}(x, U_n) = \cup\{U \mid U \in U_n, x \in U\}$. In this paper, a *Moore space* (resp. *quasi-Moore space*) is a Tychonoff space which has a development (resp. quasi-development).

As relations between \check{C} ech-completeness and subcompactness, the following are known: (1) \check{C} ech-completeness \Leftrightarrow subcompactness \Leftrightarrow countably subcompactness for a metric space [6], and (2) \check{C} ech-completeness \Rightarrow subcompactness \Leftrightarrow countably subcompactness for a Moore space [1]. We first give a generalization of (2).

Theorem 2.1. Suppose X is a quasi-Moore space. Then:

- (1) *if X is \check{C} ech-complete, then X is subcompact.*

(2) *X is subcompact if and only if X is countably subcompact.*

Proof. (1) Let (W_n) be a quasi-development of X and $\{G_n \mid n \in \mathbb{N}\}$ a family of open subsets of βX such that $X = \bigcap_{n=1}^{\infty} G_n$. For each $x \in X$ and $n \in \mathbb{N}$, there exists an

open subset $W_n(x)$ of βX such that $x \in W_n(x) \subset \text{cl}_{\beta X} W_n(x) \subset G_n$. Let $U_n(x) = \{U \cap W_n(x) \mid x \in U \text{ and } U \in W_n\}$. Put $U_n = \{V \in U_n(x) \mid x \in X\}$, and $\mathcal{B} = \{V \in U_n \mid n \in \mathbb{N}\}$. It is evident that $\{U_n \mid n \in \mathbb{N}\}$ is a quasi-development and \mathcal{B} is an open base of X . By Lemma 1.2 we may assume that each U_n satisfies the descending chain condition. Let F be a regular \mathcal{B} -filter base and $\{U_n \in F \mid n \in \mathbb{N}\}$ a regular \mathcal{B} -sequence. Now we may suppose that $U_n \in U_n$, because each U_n satisfies the descending chain condition, and hence we may assume $U_n \in U_n$ for each $n \in \mathbb{N}$. Therefore $\phi \neq \bigcap \{\text{cl}_{\beta X} F \mid F \in F\} \subset \bigcap_{n=1}^{\infty} \text{cl}_{\beta X} U_n \subset \bigcap_{n=1}^{\infty} G_n = X$, and hence X is

\mathcal{B} -subcompact.

(2) It suffices to show that countable \mathcal{B} -subcompactness implies \mathcal{B} -compactness. Suppose that X is countably \mathcal{B} -subcompact for some open base \mathcal{B} . Without loss of generality, we can take a quasi-development $\{U_n \mid n \in \mathbb{N}\}$ with $U_n \subset \mathcal{B}$ such that every U_n satisfies the descending chain condition. Let F be a regular \mathcal{B} -filter base and $\bigcap F = \phi$. Then there is no minimal element in F , and hence F contains a regular \mathcal{B} -sequence $\{U_n \mid n \in \mathbb{N}\}$. Now we may assume that $U_n \in U_n$, because each U_n satisfies the

descending chain condition. Since X is countably \mathcal{B} -subcompact, we take a point $x \in \bigcap_{n=1}^{\infty} U_n$. Then

$\{\text{st}(x, U_n) \mid n \in \mathbb{N}\}$ is a base at x , and $\omega = \{U_n \mid n \in \mathbb{N}\}$ is also a base at x . Thus $\{x\} = \bigcap \{U_n \mid U_n \in \omega\}$. On the other hand, since $\bigcap F = \emptyset$, there is a $U_a \in F$ such that $x \notin U_a$. Take $U_b \in F$ such that $\text{cl}_X U_b \subset U_a$. Since $x \notin \text{cl}_X U_b$, there exists $U_j \in \omega$ such that $U_j \cap U_b = \emptyset$. This is a contradiction, and hence X is \mathcal{B} -subcompact.

Let \mathcal{B} be an open base in a space X . We use the notion of π -bases in the sense of Nagami-Kodama [7], and then we shall say that \mathcal{B} is a *weak π -base* if for any $U_1, U_2, \dots, U_n \in \mathcal{B}$, following three conditions are satisfied,

$$(1) \quad X - \text{cl}_X U_i \in \mathcal{B} \text{ for } i = 1, 2, \dots, n.$$

$$(2) \quad \bigcap_{i=1}^{\infty} U_i \in \mathcal{B}.$$

$$(3) \quad U_i - \text{cl}_X U_j \in \mathcal{B} \text{ for } i, j = 1, 2, \dots, n.$$

We recall that a space X is *peripherally compact* if for each $x \in X$ and any neighborhood V of x , there exists an open neighborhood U of x such that $U \subset V$ and the boundary ∂U of U is compact. If X is peripherally compact, then it is easy to see that $\mathcal{B}_{\text{ro}}(X) = \{U \mid U \text{ is regular open and } \partial U \text{ is compact}\}$ is a weak π -base.

Theorem 2.2. Suppose X is a peripherally compact, Moore space. Then X is \check{C} ech-complete if X is countably \mathcal{B} -subcompact for some weak π -base $\mathcal{B} \subset \mathcal{B}_{\text{ro}}(X)$.

Proof. Let $\{U_n \mid n \in \mathbb{N}\}$ be a development of X . We may assume that $U_n \subset B$ for each $n \in \mathbb{N}$. Put $G_i = \bigcup \{ \text{int}_{\beta X} \text{cl}_{\beta X} W \mid W \in U_i \}$, and take $p \in \bigcap_{i=1}^{\infty} G_i - X$. For each $i \in \mathbb{N}$, there is a $W_i \in U_i$ such that $p \in \text{int}_{\beta X} \text{cl}_{\beta X} W_i$. Since ∂W_1 is compact, there are $V_1, \dots, V_n \in B$ such that $\partial W_1 \subset \bigcup_{i=1}^n V_i$ and $p \notin \text{cl}_{\beta X} V_i$ for each $i = 1, \dots, n$. Put $W_1' = W_1 - \bigcup_{i=1}^n \text{cl}_X V_i$. Since B is a weak π -base, $W_1' \in B$ and $p \in \text{int}_{\beta X} \text{cl}_{\beta X} W_1'$. Put $U_1 = W_1$ and $U_2 = W_2 \cap W_1'$. Then $U_2 \in B$, $p \in \text{int}_{\beta X} \text{cl}_{\beta X} U_2$ and $\text{cl}_X U_2 \subset U_1$.

Repeating this process, we have a B -regular sequence $\{U_n \mid n \in \mathbb{N}\}$ such that $U_n \subset W_n$ and $p \in \text{int}_{\beta X} \text{cl}_{\beta X} U_n$ for each $n \in \mathbb{N}$. From the B -countably subcompactness of X , there exists a point $x \in \bigcap_{n=1}^{\infty} U_n$. Since $x \in \bigcap_{n=1}^{\infty} W_n$ and $\{W_n \mid n \in \mathbb{N}\}$ is a neighborhood base at x , $\{U_n \mid n \in \mathbb{N}\}$ is also a neighborhood base at x . Since X is a first countable space, $\{ \text{int}_{\beta X} \text{cl}_{\beta X} U_n \mid n \in \mathbb{N} \}$ is a neighborhood base at x in βX , and then $\bigcap_{n=1}^{\infty} \text{int}_{\beta X} \text{cl}_{\beta X} U_n = \{x\}$. But $p \neq x$ and $p \in \bigcap_{n=1}^{\infty} \text{int}_{\beta X} \text{cl}_{\beta X} U_n$. This is a contradiction. Therefore $X = \bigcap_{i=1}^{\infty} G_i$, and hence X is \check{C} ech-complete.

3. \check{C} ech-completeness and countably subcompactness of a paracompact p -space

A sequence $\{U_n \mid n \in \mathbb{N}\}$ of open covers of X is said to be a *normal sequence* if $U_{n+1}^* < U_n$ for every $n \in \mathbb{N}$,

where $U_{n+1}^* = \{st(U, U_{n+1}) \mid U \in U_{n+1}\}$. For an open base \mathcal{B} of X , we call $\{U_n \mid n \in \mathbb{N}\}$ a *normal sequence of \mathcal{B} -covers* if $U_n \subset \mathcal{B}$ ($n \in \mathbb{N}$). As is well-known, for a normal sequence $\{U_n \mid n \in \mathbb{N}\}$ of open covers of a space X , there exists a pseudo-metric function ρ on $X \times X$ such that

$$(1) \quad st(x, U_n) \subset B(x, 1/2^{n-2}),$$

$$(2) \quad B(x, 1/2^n) \subset st(x, U_n),$$

where $B(x, 1/2^n) = \{y \in X \mid \rho(x, y) < 1/2^n\}$ (cf. [7]).

Moreover, by the equivalence relation " $x \sim y$ iff $\rho(x, y) = 0$ ", equivalently $y \in \bigcap_{n=1}^{\infty} st(x, U_n)$ " $(\tilde{X}, \tilde{\rho})$ is a metric

space, and $\varphi: X \rightarrow \tilde{X}$ defined by $\varphi(x) = \tilde{x} = \bigcap_{n=1}^{\infty} st(x, U_n)$

is a continuous mapping of X onto \tilde{X} , where

$$\tilde{X} = \{\tilde{x} \mid x \in X, \bigcap_{n=1}^{\infty} st(x, U_n) = \tilde{x}\} \text{ and } \tilde{\rho}(\tilde{x}, \tilde{y}) = \rho(x, y).$$

$(\tilde{X}, \tilde{\rho})$ is called a *metric space associated with $\{U_n \mid n \in \mathbb{N}\}$* .

It is obvious that the conditions (1) and (2) above imply the following.

$$(1^*) \quad \varphi(st(x, U_n)) \subset B(\varphi(x), 1/2^{n-2})$$

$$(2^*) \quad \varphi^{-1}(B(\tilde{x}, 1/2^n)) \subset st(x, U_n).$$

Theorem 3.1. Let X be \mathcal{B} -subcompact for an open base \mathcal{B} . If X has a normal sequence of \mathcal{B} -covers $\{U_n \mid n \in \mathbb{N}\}$, then the metric space $(\tilde{X}, \tilde{\rho})$ associated with $\{U_n \mid n \in \mathbb{N}\}$ is complete.

Proof. We shall use the notations above.

Let us take a Cauchy sequence $\{\tilde{x}_n \mid n \in \mathbb{N}\}$ in \tilde{X} , and put $T_n = \{x_i \mid i \geq n\}$. For every $n \geq 2$, there is a

$k(n) \in \mathbb{N}$ such that if $i, j \geq k(n)$ then $\tilde{\rho}(\tilde{x}_i, \tilde{x}_j) < 1/2^n$.

Thus by (2*),

$$T_{k(n)} \subset \varphi^{-1}(B(\tilde{x}_{k(n)}, 1/2^n)) \subset \text{st}(x_{k(n)}, U_n) \subset \text{cl}_X \text{st}(x_{k(n)}, U_n) \subset \text{st}^2(x_{k(n)}, U_n).$$

Since $x_i \in \text{st}(x_{k(n)}, U_n)$ for each $x_i \in T_{k(n)}$,

$$\text{st}(x_i, U_n) \subset \text{cl}_X \text{st}(x_i, U_n) \subset \text{st}^2(x_{k(n)}, U_n)$$

There is a $U_{n-1} \in \mathcal{U}_{n-1}$ such that $\text{st}^2(x_{k(n)}, U_n) \subset U_{n-1}$,

because $U_n^* < U_{n-1}$. Therefore every U in \mathcal{U}_n which contains x_i for any $i \geq k(n)$ satisfies that $\text{cl}_X U \subset U_{n-1}$.

Similarly we show that there is a $U_n \in \mathcal{U}_n$, and every V in \mathcal{U}_{n+1} which contains x_i for any $i \geq k(n+1) \geq k(n)$

satisfies $\text{cl}_X V \subset U_n$. Thus we have a regular \mathcal{B} -sequence

$\{U_n \mid n \in \mathbb{N}\}$, and then $\bigcap_{n=1}^{\infty} U_n \neq \emptyset$. Take $x \in \bigcap_{n=1}^{\infty} U_n$ and

let $\varphi(x) = \tilde{x}$. Since $\bigcap_{n=1}^{\infty} U_n = \bigcap_{n=1}^{\infty} \text{cl}_X U_n$, for every neigh-

borhood W of x and $n \in \mathbb{N}$, $W \cap U_n \neq \emptyset$. Taking this W as

V_n which is contained in U_n and $x \in V_n$ for each $n \in \mathbb{N}$,

$U_n \subset \text{st}^2(x, U_n)$. Therefore $U_n \subset \text{st}(x, U_{n-1})$. From (1*)

$\varphi(\text{st}(x, U_{n-1})) \subset \tilde{B}(\varphi(x), 1/2^{n-3})$. Since there is $m_0 \in \mathbb{N}$

such that $m \geq m_0$ implies $x_m \in U_{n-1}$, we have $\tilde{\rho}(\tilde{x}, \tilde{x}_m) <$

$1/2^{n-3}$ for every $m \geq m_0$. This shows that a Cauchy

sequence $\{\tilde{x}_n \mid n \in \mathbb{N}\}$ converges to \tilde{x} , and then \tilde{X} is com-

plete.

From this theorem, we have the following.

Theorem 3.2. If X is a paracompact countably \mathcal{B} -subcompact space, then for any normal sequence of \mathcal{B} -covers

$\{U_n \mid n \in \mathbb{N}\}$, a metric space associated with $\{U_n \mid n \in \mathbb{N}\}$ is complete.

We recall that a space X is a *paracompact p -space* if there exists a normal sequence $\{U_n \mid n \in \mathbb{N}\}$ of open covers of X such that $\{\text{st}(x, U_n) \mid x \in X, n \in \mathbb{N}\}$ has a strict p -structure, that is, $K(x) = \bigcap_{n=1}^{\infty} \text{st}(x, U_n)$ is compact for every $x \in X$ and for any open set $U \supset K(x)$ there exists an $n \in \mathbb{N}$ with $\text{st}(x, U_n) \subset U$ (cf. [3]).

Lemma 3.3. Let $\{U_n \mid n \in \mathbb{N}\}$ and $\{W_n \mid n \in \mathbb{N}\}$ be two normal sequences of open covers of X with $W_n^* \subset U_n$ ($n \in \mathbb{N}$). If $\{\text{st}(x, U_n) \mid x \in X, n \in \mathbb{N}\}$ has a strict p -structure, so has $\{\text{st}(x, W_n) \mid x \in X, n \in \mathbb{N}\}$.

Proof. Since $\{W_n \mid n \in \mathbb{N}\}$ is a normal sequence of covers, it is easily showed that $Q(x) = \bigcap_{n=1}^{\infty} \text{st}(x, W_n) = \bigcap_{n=1}^{\infty} \text{cl}_X(\text{st}(x, W_n))$, and then $Q(x)$ is compact by compactness of $K(x)$. Moreover, if $x_n \in \text{st}(x, W_n)$ for every $n \in \mathbb{N}$, then by the assumption, the sequence $\{x_n \mid n \in \mathbb{N}\}$ has a cluster point in $Q(x)$. Thus $\{\text{st}(x, W_n) \mid x \in X, n \in \mathbb{N}\}$ has a strict p -structure.

Theorem 3.4. A countably subcompact, paracompact p -space is Čech-complete.

Proof. Let X be a countably \mathcal{B} -subcompact, paracompact p -space where \mathcal{B} is an open base of X . From 3.3 there is a normal sequence of \mathcal{B} -covers $\{U_n \mid n \in \mathbb{N}\}$ such that

$\{st(x, U_n) \mid x \in X, n \in N\}$ is a strict p -structure. We consider $\varphi: X \rightarrow \tilde{X}$. $\varphi^{-1}(\tilde{x}) = \bigcap_{n=1}^{\infty} st(x, U_n)$ ($=K(x)$) is

compact. To see that φ is a closed mapping, we suppose that F is closed in X and $\tilde{x} \notin \varphi(F)$. Since $K(x) \cap F = \emptyset$, there exists an $n \in N$ such that $st(x, U_n) \cap F = \emptyset$, and hence $\varphi^{-1}(\tilde{B}(\tilde{x}, 1/2^n)) \cap F = \emptyset$ by (2*). Thus $\tilde{B}(\tilde{x}, 1/2^n) \cap \varphi(F) = \emptyset$. This implies $\tilde{x} \notin cl_X \varphi(F)$, i.e., $\varphi(F)$ is closed, so φ is closed. Thus X is Čech-complete by Theorem 3.2 and Frolik's theorem [5]; a paracompact space X is Čech-complete iff X admits a perfect mapping onto a complete metric space.

4. Examples

In this section, we give examples related to Theorems 3.1 and 3.5.

Example 4.1. The Sorgenfrey line S is paracompact but not Čech-complete (for example see [4, p. 270] or [1, 2.4.6]). On the other hand, S is \mathcal{B} -subcompact where \mathcal{B} is an open base of S which consists of bounded, half-open intervals and has the property that if I_1 and I_2 are distinct members of \mathcal{B} , then $\sup I_1 \neq \sup I_2$ [1; 5.1.4]. Thus, Theorem 3.5 is not true if we drop the assumption "p-ness" of space.

Next, let $U_n = \{[x, 1/2^n) \mid x \in S\}$ ($n \in N$). Then $\{U_n \mid n \in N\}$ is a normal sequence of open covers of S , but $U_n \not\subset \mathcal{B}$ for any $n \in N$.

Let $(\tilde{X}, \tilde{\rho})$ be a metric space associated with $\{U_n \mid n \in \mathbb{N}\}$. Then \tilde{X} is the set of real numbers. Moreover, $\tilde{\rho}(x, y) = 1$ if $|x - y| \geq 1$ and $\tilde{\rho}(x, y) = |x - y|$ if $|x - y| < 1$. Thus $(\tilde{X}, \tilde{\rho})$ is complete. This shows that there exists a normal sequence of open covers $\{U_n \mid n \in \mathbb{N}\}$, $U_n \not\subset B$, such that the metric space associated with $\{U_n \mid n \in \mathbb{N}\}$ is complete even if S is paracompact B -subcompact.

Example 4.2. Let Q be the set of all rational numbers, and let $P = Q \cap [0, 1]$, where $[0, 1]$ be a closed unit interval in the Euclidean space R . For every $r \in P$, put $X_r = [0, 1]$. Let X be the topological sum of X_r ($r \in P$), and let φ be a mapping from X onto P such that $\varphi(x) = r$ if $x \in X_r$. It is obvious that X is σ -compact and locally compact. Thus X is Čech-complete and subcompact.

Let $W_n = \{(p - 1/2^n, p + 1/2^n) \cap P \mid p \in P\}$ for any $n \in \mathbb{N}$, and let $U_n = \varphi^{-1}(W_n)$. Since $\{W_n \mid n \in \mathbb{N}\}$ is a normal sequence of open covers of a subspace P in R , $\{U_n \mid n \in \mathbb{N}\}$ is a normal sequence of open covers of X . Let $(\tilde{X}, \tilde{\rho})$ be a metric space associated with $\{U_n \mid n \in \mathbb{N}\}$. Since $(\tilde{X}, \tilde{\rho}) = (Q, d)$, where d is usual metric, $(\tilde{X}, \tilde{\rho})$ is not complete. This shows that it does not always follow that the completeness of the metric space associated with a normal sequence $\{U_n \mid n \in \mathbb{N}\}$ is complete if we drop the assumption $U_n \subset B$ in Theorem 3.1 even if a space X is σ -compact, locally compact.

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