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PROBLEM SECTION

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PROBLEM SECTION

CONTRIBUTED PROBLEMS

Problems which were stated in, or are relevant to, a paper in this volume of TOPOLOGY PROCEEDINGS are followed by a [v. 14]. Those relevant to papers in earlier volumes have both volume and page numbers listed.

The Problems Editor invites anyone who has published a paper in TOPOLOGY PROCEEDINGS or attended a Spring Topology Conference to submit problems to this section. They need not be related to any articles, but if they are, please provide a reference. Please define any terms not in a general topology text or in referenced articles.

A Cardinal Invariants

22. (*Shakhmatov*) Assume that τ is a Tychonoff [resp. Hausdorff, regular, T_1, etc.] homogeneous topology on a set X. Are there Tychonoff [resp. Hausdorff, regular, T_1, etc.] homogeneous topologies τ_* and τ^* on X such that $\tau_* \subset \tau \subset \tau^*$ and $w(X,\tau_*) \leq nw(X,\tau)$ and $w(X,\tau^*) \leq nw(X,\tau)$?

For background on this problem for the case of topological groups and other topological algebras, see papers by Arkhangel'skii in Dokl. Akad. Nauk SSSR 247 (1979) 779-782 = Soviet Math. Doklady 20 (1979) 783-787, where the "left half" is achieved in the category of topological groups and continuous homeomorphisms; by Shakhmatov in Vestnik Moskov. Univ. Ser. I Matem. Mekh. 39, no. 2 (1984) 42-45 = Moscow Univ. Math. Bull. 39 (no. 2) (1984) 57-60, where this is extended to many other categories; and by Pestov and Shakhmatov in the Vestnik v. 42, no. 3 (1986) 98-101 (pp. 92-95 in the English translation) where the "right half" is shown to fail in the categories of topological groups and topological vector spaces, for countable net weight; in the latter case, R^∞ provides a counterexample.

See also the problems under heading L.

G. Mappings of Continua and Euclidean Spaces

In this heading and the next, "homeomorphism" means "autohomeomorphism".

24. (*J. Kennedy*) If X is a continuum and $x \in X$, let G_x denote the set of all points of X to which x can be taken by a homeomorphism of X. It is known that G_x is a Borel set, even if connectedness of X is dropped. If α is a countable ordinal, does there exist a continuum X for which some G_x is a Borel set in class F_α but not in F_γ for $\gamma < \alpha$? [v. 14]

25. (*J. Kennedy*, attributed to Marcy Barge) Does there exist a weakly homogeneous planar continuum X with the property that each homeomorphism it admits possesses a dense set of periodic points? [v. 14]

26. (*J. Kennedy*) Does there exist a homogeneous continuum X with the property that each of its homeomorphisms, except the identity, is transitive? [v. 14]

27. (*J. Kennedy*) Does there exist a homogeneous continuum X that admits a transitive homeomorphism and that has the property that each of its homeomorphisms admits a dense set of periodic points? [v. 14]

28. (*J. Kennedy*) Does there exist a homogeneous continuum X with the property that for each nonidentity homeomorphism h of X, there is some nonempty proper open set U with $h(U) \subset U$? [v. 14]

J. Group Actions

5. (*J. Mashburn*) Will a space X acted upon by a finite group of homeomorphisms necessarily have a countable closed migrant cover it it is subparacompact? What if X is a Moore space or a σ-space? [v. 14]

6. (*J. Mashburn*) If a paracompact space X with finite Ind is acted upon freely by a finite group of homeomorphisms, must X have a finite open or closed migrant cover? What if dim X is finite? If the answer to either one is yes, what is the optimal bound on the size of the cover? [v. 14]

L. Topological Algebra

8. (*Shakhmatov*) Let G be a countably compact (Hausdorff) topological group. Is then $t(G \times G) = t(G)$ (here t denotes tightness)? What if $t(G)$ is countable?

If countable compactness is dropped, there are counterexamples under various set-theoretic hypotheses as shown by Malykhin under CH, Malykhin and Shakhmatov in the model obtained by adding one Cohen real to a model of MA + $\neg$ CH, and Shakhmatov in the model obtained by first adding ω_2 Cohen reals to a model of GCH, then uses the Martin-Solovay poset for obtaining MA + $\neg$ CH; in the last case, examples were found of dense pseudocompact subgroups G of 2^{ω_1} for which G^n is hereditarily separable and Fréchet-Urysohn but G^{n+1} has uncountable tightness.

9. (*Shakhmatov*) Let G be a countably compact Fréchet-Urysohn topological group. Is G $\times$ G Fréchet-Urysohn? Is G^n Fréchet-Urysohn for all n? (A counterexample could not be α_3 since the product of a Fréchet-Urysohn α_3-space and a countably compact Fréchet-Urysohn space is Fréchet-Urysohn: A. V. Arkhangel'skii, The frequency spectrum of a topological space and the product operation, Trudy Mosk. Mat. Obs. 40(1979) = Trans. Moscow Math. Soc. 1981, Issue 2, 163-200).

10. (*Shakhmatov*) Let G be a topological group so that G^n is Fréchet-Urysohn for every natural number n. (a) Is G^ω Fréchet-Urysohn? (b) What if one assumes also that G is countably compact? (A counterexample could not be α_3: T. Nogura, Top. Appl. 20 (1985) 59-66, Corollary 3.8).

11. (*Shakhmatov*) Is every countably compact sequential topological group Fréchet-Urysohn? An affirmative answer would imply one for 9 and also 10 (b): the product of a countably compact sequential space and a sequential space is sequential.

See also the background references for A22, and the information on Arkhangel'skii's survey paper in the next subsection.

O. Theory of Retracts; Extension of Continuous Functions

13. (*R. Levy*) Is there a ZFC example of a metric space having a subset that is 2-embedded but not ω-embedded? (A subset S of a space X is κ-*embedded* if every continuous function from S into a discrete space of cardinality κ has an extension to a continuous function on X).

See also the subsection on earlier problems.

Q. Generalizations of Topological Spaces

5. (*Rhonda McKee*) Let (X,μ) be a nearness space and let K(X) denote the group (under composition) of all near-homeomorphisms from (X,μ) to itself. When is it true that if K(X) and K(Y) are isomorphic, then X and Y are near-homeomorphic? [v. 14]

T. Algebraic and Geometric Topology

12. (*Ning Lu*) In Topology Proceedings, v. 13, No. 2, the presentations of the groups M_g $g \geq 3$ in "On the mapping class groups of the closed orientable surfaces" (pp. 293-323) are not so simple as that of M_2 given in the preceding article (pp. 249-291). The main reason is the extra Lantern law. Is there a simpler equivalent form

from the Lantern law in the generators L, N, and T, or a more useful presentation of M_g for $g \geq 3$?

13. *(Ning Lu)* D. Johnson, Ann. Math. 118 (1983) 423-442, showed the Torelli groups M_g are finitely generated for $g \geq 3$. Is there a way to write Johnson's generators in terms of the generators L, N, and T [see articles referenced in preceeding problem] of the surface mapping class groups which will be useful in studying the fundamental group of homology spheres?

W. Algebraic Problems

1. *(Ning Lu)* Call a group G *balanced* if it admits a finite set s of generators, so that any two elements of s can be mapped to each other by some automorphism of G which leaves s invariant. [An example is the group M_2 described in Problem T12 above, with $s = \{\Gamma_0, \ldots, \Gamma_5\}$ the set of six Dehn twists given in Section 3 of the paper referenced second in T12].

Characterize the balanced groups.

INFORMATION ON EARLIER PROBLEMS

From Arkhangel'skii's Survey Paper

Uspehii. Mat. Nauk. 33:6 (1978) 29-84 (= Russian Math. Surveys 33:6 (1978) 33-96).

24. Is $t(X \times X) = t(X)$ for each countably compact Tychonoff space?

Correction. In v. 11 it was erroneously announced that Malyhin and Shakhmatov had found a model in which there is a counterexample. However, the example the Problems Editor had in mind was not countably compact; moreover, that example was Fréchet-Urysohn, and (See Problem L11 above) a counterexample cannot be sequential.

From TOPOLOGY PROCEEDINGS

C.20. (v. 3) (*van Douwen*) Consider the following statements about an infinite compact Hausdorff space X:

(a) there are $Y \subset X$ and $y \in Y$ such that $\chi(y,Y) \in \{\omega,\omega_1\}$.

(b) there is a decreasing family $\mathcal{F}$ of closed sets with $|\mathcal{F}| \in \{\omega,\omega_1\}$ and $|\cap\mathcal{F}| = 1$.

Without loss of generality, X is separable, hence CH $\Rightarrow$ (a). Clearly (a) $\Rightarrow$ (b). What happens under $\neg$CH?

Partial answer. Juhász and Szentmiklóssy have shown that if X is of uncountable tightness, then X has a convergent free ω_1-sequence, providing a closed Y as in part (a). Hence PFA implies (a), hence (b), by Balogh's theorem that PFA implies every compact Hausdorff space of countable tightness is sequential. Also, (a) holds in a model obtained by adding uncountably many Cohen reals in any model of set theory since Juhász has shown that every compact Hausdorff space of countable tightness has a point of character $\leq \omega_1$ in that model. Juhász and Szentmiklóssy have also shown that (a) has an affirmative solution under ♣

Remark: A late note in vol. 3 that Hušek had solved (b) completely was in error.

012. (v. 13) Is there a ZFC example of a metric space having a subset that is 2-embedded but not C*-embedded?

Remark. The answer is trivially, Yes, e.g., the open unit interval is trivially 2-embedded in the closed unit interval. This question was a bad transcription of a question of R. Levy, whose correct statement is 013 in this issue.