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Research Announcement:

SOME NEW TWO-DIMENSIONAL HOMOGENEOUS CONTINUA

by

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SOME NEW TWO-DIMENSIONAL HOMOGENEOUS CONTINUA

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A continuum is a compact, connected metric space. A continuum is homogeneous if, for any pair of points x, y in X, there exists a homeomorphism $h : (X, x) \to (X, y)$. A continuum Y is aposyndetic if for each pair of distinct points x and y in Y, there is a subcontinuum S of Y such that $x \in int(S)$ and $y \in Y - S$.

A continuous decomposition of a continuum is a partition of the continuum into subcontinua such that the quotient map of the partition is both open and closed. Let X be a continuum which has a continuous decomposition into continua, and let $f : X \to Q$ be the quotient map where Q is a homogeneous continuum. We call the following property of X, with respect to f, Property H:

> If h is any homeomorphism of Q, and if $h(f(x)) = f(y)$, then there is a homeomorphism $\widehat{h} : (X, x) \to (X, y)$ such that $f \circ \widehat{h} = h \circ f$.

We announce the proof of the following theorem:

Theorem. *Let X be a continuum with a continuous decomposition into nondegenerate continua, and let $f : X \to Q$ be the quotient map, where Q is a homogeneous continuum and X has Property H with respect to f. Let $\widetilde{X} = \{(x, y) \in X \times X : f(x) = f(y)\} = \cup\{f^{-1}(q) \times f^{-1}(q) : q \in Q\}$. Then $\widetilde{X}$ is an aposyndetic, homogeneous continuum such that for each $q \in Q$,*

$$dim(f^{-1}(q) \times f^{-1}(q)) \leq dim\widetilde{X} \leq dim(X \times X).$$

*It is acknowledged that this research was a dissertation done at Tulane University under the direction of Professor James T. Rogers, Jr..

Three continua which satisfy the hypothesis of the above theorem are the pseudo-arc, the circle of pseudo-arcs, and a solenoid of pseudo-arcs.

A pseudo-arc, which we will denote P, is a chainable, hereditarily indecomposable continuum. A chainable continuum is a continuum which is homeomorphic to an inverse limit of arcs, and an indecomposable continuum is a continuum which is not the union of two of its proper subcontinua. A continuum is hereditarily indecomposable if every subcontinuum is indecomposable. The pseudo-arc was first constructed by Knaster [3] in 1922. It was shown to be homogeneous by Bing [1] in 1948.

A circle of pseudo-arcs (CP) is a circle-like continuum with a continuous decomposition into pseudo-arcs, such that the quotient space is a circle. Bing and Jones[2] constructed a circle of pseudo-arcs in 1954, and showed that the circle of pseudo-arcs is homogeneous.

A solenoid is an inverse limit of circles with covering maps as the bonding maps. In 1977, for each solenoid S, J.T. Rogers [6] constructed a solenoid of pseudo-arcs (SP), that is, a homogeneous continuum with a continuous decomposition into pseudo-arcs, such that the quotient space is S.

In 1984, W. Lewis [4] generalized the above results by showing that the points of any homogeneous one-dimensional continuum Q can be "blown up" into pseudo-arcs, so that the resulting continuum is a homogeneous continuum with a continuous decomposition into pseudo-arcs such that the quotient space is Q. The continuum obtained by blowing up the points of the pseudo-arc into pseudo-arcs is known to be homeomorphic to the pseudo-arc [5]. The continua constructed in Lewis' paper satisfy Property H with respect to the quotient map of the continuous decomposition into pseudo-arcs. Hence, for each of these continua, the construction in the theorem above yields a two-dimensional, aposyndetic, homogeneous continuum.

In a paper to be published elsewhere, we will show that $\widetilde{P}, \widetilde{CP}$, and $\widetilde{SP}$ are not homeomorphic to any known homogeneous continua. J.T. Rogers has informed the author that, in the case of $\widetilde{CP}$, this solves a problem posed by the late Andrew Conner about 10 years ago.

References

1. R. H. Bing, *A homogeneous indecomposable plane continuum*, Duke Math. J. **15** (1984), 729-742.
2. —— and F. B. Jones, *Another homogeneous plane continuum*, Trans. Amer. Math. Soc. **90** (1959), 171-192.
3. B. Knaster, *Un continua dont tout sous-continu est indecomposable*, Fund. Math. **3** (1922), 247.
4. W. Lewis, *Continuous curves of pseudo-arcs,* Houston J. Math. **11** (1985), 91-99.
5. ——, *The pseudo-arc of pseudo-arcs is unique*, Houston J. Math. **10** (1984), 227-233.
6. ——, *Solenoids of pseudo-arcs*, Houston J. Math. **3** (1977), 531-537.

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