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Research Announcement

ENDPOINTS OF CHAINABLE CONTINUA

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A *continuum* is a nonempty compact connected metric space. The *trivial* continuum consists of one point and a *nontrivial* continuum consists of at least two points. Hence, a nontrivial continuum has the same cardinality as the reals.

Let (X, d) be a metric space. A *chain* is a finite sequence $C : L_1, L_2, \ldots, L_n$ of open sets in X such that $L_j \cap L_k \neq \emptyset$ iff $| j - k | \leq 1$. Each L_j is called a *link*.

A chain C is an *ϵ-chain* if $\epsilon > 0$ and the diameter of each link of C is less than ϵ, i.e., mesh $C < \epsilon$. A continuum X is *chainable* if, for each $\epsilon > 0$, there is an ϵ-chain of open sets of X covering X. A point x of a chainable continuum X is an *endpoint* of X if there exists a sequence $\{C_j\}_{j=1}^{\infty}$ of chains covering X such that:

(a) mesh $C_j < 2^{-j}$ for each j and

(b) x is in the first link of each chain C_j.

We denote the set of endpoints of a chainable continuum X by $Ep(X)$.

A continnum is *decomposable* if it is the union of two proper subcontinua. It is *indecomposable* if it is not decomposable.

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A metric space X is *complete* if each Cauchy sequence in X converges in X. It is *incomplete* if it is not complete.

We announce the proofs of the following theorems:

Theorem 1. *There exist (in)decomposable continua with (exactly) n endpoints for each nonnegative integer n.*

Theorem 2. *There exist (in)decomposable continua X such that $Ep(X)$ is complete and countably infinite.*

Theorem 3. *There exist (in)decomposable continua X such that the set $Ep(X)$ is incomplete and countably infinite.*

Theorem 4. *There exist (in)decomposable continua X such that $Ep(X)$ is complete and uncountable.*

Theorem 5. *There exist (in)decomposable continua X such that the set $Ep(X)$ is incomplete and uncountable.*

The above results were presented at the *Twenty-sixth Annual Spring Topology Conference* held during April 2-4, 1992 in Charlotte, North Carolina.

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