Topology Proceedings

Web:	http://topology.auburn.edu/tp/
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E-mail:	topolog@auburn.edu
ISSN:	0146-4124

Topology Proceedings
Vol 17, 1992

Research Announcement

COVERING SPACES OF HOMOGENEOUS CONTINUA

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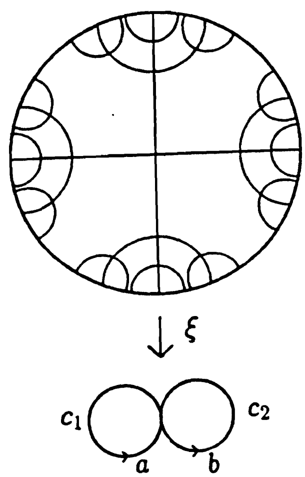
A *continuum* is a compact, connected metric space. A continuum X is said to be *homogeneous* if, for any pair of points x and y of X, there is a homeomorphism $h : X \to X$ with the property that $h(x) = y$.

Let $\mathcal{S}^1$ be the unit circle, and let $\mathcal{W} = \mathcal{S}^1 \vee \mathcal{S}^1$, the one-point union of two circles. It is known that the universal covering space $\widetilde{\mathcal{W}}$ of $\mathcal{W}$ can be embedded in the Poincaré model of the hyperbolic plane in such a way that $\widetilde{\mathcal{W}}$ is the "infinite snowflake" pictured below [2].



*The author wishes very much to thank Professor James T. Rogers Jr. for his guidance, patience and frinedship during the realization of this research.

Let us denote the Hilbert cube by $\mathbb{Q}$. Let $\Gamma = \mathcal{S}^1 \times \mathbb{Q}$ or $\Gamma = \mathcal{W} \times \mathbb{Q}$, and let $\sigma : \widetilde{\Gamma} \to \Gamma$ be the universal covering space Γ. Professor James T. Rogers Jr. [3] has shown that if X is a homogeneous contunuum essentially embedded in Γ, then $\widetilde{X} = \sigma^{-1}(X)$ is homogeneous, Thus $\widetilde{X}$ is a homogeneous, locally compact, separable metric space. Aarts and Oversteegen [1] proved that such a space is homeomorphic to $\widetilde{\mathcal{K}} \times \widetilde{\mathcal{B}}$, where $\widetilde{\mathcal{K}}$ is a component of $\widetilde{X}$, and $\widetilde{\mathcal{B}}$ is a locally compact, totally disconnected, homogeneous, metric space.

Given two points $\tilde{x}$ and $\tilde{y}$ of $\widetilde{X}$, we say that $\tilde{x} \sim \tilde{y}$ if there is a continuum $\widetilde{Y}$ in $\widetilde{X}$ containing $\tilde{x}$ and $\tilde{y}$. It is clear that " $\sim$" is an equivalence relation. We call the equivalence classes the *continuum components* of $\widetilde{X}$.

We announce the proofs of the following theorems:

Theorem 1. *Let X be a homogeneous continuum essentially embedded in Γ. If $\widetilde{\mathcal{K}}$ is a component of $\widetilde{X}$ and if $\widetilde{\mathcal{K}}_C$ is a continuum component of $\widetilde{X}$, then $\widetilde{\mathcal{K}}, \widetilde{\mathcal{K}}_C, \sigma(\widetilde{\mathcal{K}})$, and $\sigma(\widetilde{\mathcal{K}}_C)$ are homogeneous.*

Theorem 2. *Assume the hypothesis of Theorem 1. If X is also one-dimensional, then $\sigma(\widetilde{\mathcal{K}})$ and $\sigma(\widetilde{\mathcal{K}}_C)$ are dense in X.*

Theorem 3. *There exists a homogeneous continuum X and distinct embeddings of X in Γ such that the components of $\widetilde{X}$ of the corresponding embeddings are nonhomeomorphic. The images of such components under the covering maps are not homeomorphic either.*

References

[1] Aarts, J. M. and L. G. Oversteegen, *The Product Structure of Homogeneous Spaces*, Indag. Math. **1** (1990), 1-5.

[2] J. T. Rogers Jr., *Decomposition of Continua Over the Hyperbolic Plane*, Trans. Amer. Math. Soc. **310** (1988), 277-291.

[3] ——, *Homogeneous Separating Plane Continua are Decomposable*, Mich. J. Math. **28** (1981), 317-322.

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