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PROBLEM SECTION

CONTRIBUTED PROBLEMS

Each problem is related to a paper by the contributor in either vol. 15 or vol. 16, and this is indicated along with the contributor's name in parentheses after each problem.

B. Generalized Metric Spaces and Metrization

34. (*Hoshina, v.15, communicated by T. Goto*) Can every Lašnev space be embedded in a Lašnev space that is the closed continuous image of a complete metric space?

35. (*Okuyama, v.16*) Is every Lindelöf Hausdorff space a weak $P(\aleph_0)$-space? [*No if MA or* $\mathfrak{b} = \omega_1$.]

Remarks. For a paracompact [*resp.* Lindelöf] regular space X, the product $X \times \mathbb{P}$ is paracompact [*resp.* Lindelöf] iff X is a weak $P(\aleph_0)$-space. For further information see:

B. Lawrence, The influence of a small cardinal on the product of a Lindelöf space and irrationals, Proc. Amer. Math. Soc. 110 (1990) 535-542.

K. Alster, The product of a Lindelöf space with the space of irrationals under Martin's axiom, Proc. Amer. Math. Soc. 110 (1990) 543-547.

K. Alster, Some remarks concerning the Lindelöf property of the product of a Lindelöf spaace with the irrationals, Top. Appl. 44 (1992) 19-25.

See also problems D40, L14, P33 and R7.

D. Paracompactness and Generalizations

38. (*Borges, v.16*) Are subparacompact spaces D-spaces?

39. (*Borges, v.16*) Are monotonically normal paracompact spaces D-spaces?

40. (*Borges, v.16*) Is the countable product of Sorgenfrey lines a D-space?

Remark. In the article by Borges and Wehrly, it was also asked whether the finite product of irrational Sorgenfrey lines is a D-space, but this was answered affirmatively by Peter de Caux back in vol. 6 of **Topology Proceedings** where he showed [pp. 31-43] that each subspace of each finite power of the Sorgenfrey line is a D-space.

See also L24 and P33.

G. Mappings of Continua and Euclidean Spaces

28. (*N. Grace, v.15*) If X is a θ_n-continuum and $f : X \twoheadrightarrow Y$ is proximately refinable, must Y be a θ_{2n}-continuum?

H. Homogeneity and Mappings of General Spaces

17. (*T. Wilson, v.16*) Let A be a compact metric space and let $g : A \to A$ be a continuous surjection. The sequence $S = \{x_n\}_{n=0}^{\infty} \subset A \times [0,1]$ is a *generating sequence for* g if (i) A is the derived set of S, (ii) the function T_0 defined by $T_0 x_n = x_{n+1}$ is continuous on S and (iii) T_0 has a continuous extension to $c\ell(S)$ such that $T \restriction A = g$. [We are identifying $A \times \{0\}$ with A.] When do generating sequences exist?

18. (*T. Wilson, v.16*) Suppose A is countable and S is a generating sequence for $g : A \to A$. Let A^α denote the α^{th} derived set of A, and let α_0 be the least ordinal such that A^{α_0} is finite. If p is a fixed point of g, is $p \in A^{\alpha_0}$? More generally, is $A^\alpha \subset g(A^\alpha)$?

See also P33.

L. Topological Algebra

12. (*Shakhmatov, v.15*) Is there a (countable) Fréchet- Urysohn group which is an α_3-space without being an α_2-space?

13. (*Shakhmatov, v.15*) Is it consistent with ZFC to have a Fréchet-Urysohn $\alpha_{1,5}$-group which is not a v-group?

14. (*Shakhmatov, v.15*) Is there a "real" (= requiring no additional set-theoretic assumptions beyond ZFC) example of a countable nonmetrizable w-group?

15. (*Shakhmatov, v.15*) Is there a "real" example of a Fréchet-Urysohn topological group that is not an α_3-space?

16. (*Shakhmatov, v.15*) Do the convergence properties α^i ($i = 0, 1, \ldots, \infty$) coincide for Fréchet-Urysohn topological groups?

17. (*Shakhmatov, v.15*) Is every Fréchet-Urysohn group an α^∞-space?

18. (*Shakhmatov, v.15*) Do some new implications between α_i-properties, $i \in \{1; 1,5; 2; 3; 4\}$, and α^k-properties, $k \in \omega \cup \{\infty\}$, appear in Fréchet-Urysohn groups belonging to one of the following classes:

(i) countably compact groups,
(ii) pseudocompact groups,
(iii) precompact groups (= subgroups of compact groups), and
(iv) groups complete in their two-sided uniformity?

19. (*Shakhmatov, v.15*) Is every Fréchet-Urysohn group having a base of open neighborhoods of its neutral element consisting of subgroups a w-space?

In the following problems, recall that a group G is said to be $\mathbb{R}$-factorizable if for any continuous real-valued function f on G there exist a continuous homomorphism π of G onto a group H of countable weight and a continuous function h on H such that $f = h \circ \pi$.

20. (*Tkačenko, v.16*) Is every c.c.c. topological group $\mathbb{R}$-factorizable? what if it is separable?

21. (*Tkačenko, v.16*) [a very special case of the preceding problem:] Let S be the Sorgenfrey line and $A(S)$ the free Abelian topological group over S. Is $A(S)$ $\mathbb{R}$-factorizable?

22. (*Tkačenko, v.16*) Let g be a continuous real-valued function on an $\aleph_0$-bounded group G. Are there a continuous homomorphism π of G onto a group H of weight at most $2^{\aleph_0}$ and a continuous function h on H such that $g = h \circ \pi$?

Note. Not every $\aleph_0$-bounded group is $\mathbb{R}$-factorizable; but the contributor conjectures that the above weakening of $\mathbb{R}$-factorizability holds for it.

23. (*Tkačenko, v.16*) Is every subgroup of $\mathbb{Z}^\tau$ $\mathbb{R}$-factorizable?

Note. Every subgroup of $\mathbb{Z}^\tau$, for each τ, is $\aleph_0$-bounded but, by a recent result of Uspenskiĭ, is not necessarily c.c.c.

24. (*Tkačenko, v.16*) (a) Must every locally finite family of open subsets of an $\mathbb{R}$-factorizable group be countable? (b) Is every $\mathbb{R}$-factorizable group G weakly Lindelöf? that is, is it true that every open cover of G has a countable subfamilly a union of which is dense in G?

25. (*Tkačenko, v.16*) Does a continuous homomorphic image of an $\mathbb{R}$-factorizable group inherit the $\mathbb{R}$-factorization property? [If the homomorphism is open as well, the answer is Yes, as shown in Theorem 3.1 of Tkačenko's article.]

26. (*Tkačenko, v.16*) Is every $\aleph_0$-bounded group a continuous image of an $\mathbb{R}$-factorizable group?

Remark. Yes to L26 implies No to L25.

27. (*Tkačenko, v.16*) Is the $\mathbb{R}$-factorization property inherited by finite products?

28. (*Tkačenko, v.16*) [A special case of the preceding problem:] Is the product of an $\mathbb{R}$-factorizable group with a compact group $\mathbb{R}$-factorizable?

Note. An affirmative answer to Problem L24 (a) would imply one to Problem L28.

29. (*Tkačenko, v.16*) Suppose G is an $\mathbb{R}$-factorizable group of countable o-tightness and K is a compact group. Is the product $G \times K$ $\mathbb{R}$-factorizable? what if G is a k-group?

30. (*Tkačenko, v.16*) Must the product of a Lindelöf group with a totally bounded group be $\mathbb{R}$-factorizable? *Note.* We

may assume without loss of generality that the totally bounded factor is second countable.

31. (*Tkačenko, v.16*) Is it true that the closure of a $G_{\delta,\Sigma}$ set in a k-group H is a G_δ-set? What if H is sequential or Fréchet-Urysohn?

P. Products, Hyperspaces, Remainders and Similar Constructions

33. (*Okuyama, v. 16*) Let X be a paracompact Hausdorff space and Y a K-analytic space. If $X \times Y$ is normal, then is $X \times Y$ paracompact?

Comment by the contributor. It seems that this question concerns the property of a mapping such as $id_X \times \varphi$, where φ is an upper semi-continuous, compact-valued mapping from the space $\mathbb{P}$ of irrationals to the power set of Y.

See also L27 through L30, and R7.

R. Dimension Theory

7. (*Hoshina, v.15*) Suppose $X \times Y$ is normal T_1, where Y is a Lašnev space. Does

$$dim(X \times Y) \leq dimX + dimY$$

hold for the covering dimension dim? [Yes if X is paracompact.]

S. Problems Closely Related to Set Theory

In the following problems, definitions are as in the Steprāns article in vol.16, except that here we will refer to maximal antichains of monotone paths in $\mathcal{P}(\mathbb{N}^n)/\mathcal{B}_n$ as Cook sets for all n, not just for $n = 2$.

17. Does there exist a Cook set in $\mathbb{N}^3$? [Yes if MA.]

18. Does the existence of a Cook set in $\mathbb{N}^3$ imply the existence of a Cook set in $\mathbb{N}^4$?

19. For each $N \in \omega \setminus \{0,1\}$, does there exist a model of set theory in which there is a Cook set in $\mathbb{N}^{N+1}$ but not in $\mathbb{N}^N$?

20. Call a family of monotone paths in $\mathbb{N}^k$ *weakly maximal* if any two paths are separated and the family cannot be extended to a larger family with this property. Let $\mathfrak{a}_k^-$ [*resp.* $\mathfrak{a}_k$] be the least cardinality of a weakly maximal [it resp. maximal, assuming one exists] family of monotone paths in $\mathbb{N}^k$. Does $\mathfrak{a}_k^-$ equal $\mathfrak{a}_k$ when the latter exists?

21. Recall that $\mathfrak{a}$ represents the least cardinality of an infinite maximal almost disjoint family in $\mathcal{P}(\omega)$. What are the relationships between the cardinal $\mathfrak{a}$, the cardinals $\mathfrak{a}_k$, and the cardinals $\mathfrak{a}_k^-$?

INFORMATION ON EARLIER PROBLEMS

Volume 2

Classic Problem V, related problems. Is every separable compact T_5 space of countable tightness? of cardinality $\leq \mathfrak{c}$? sequentially compact? sequential? *Solution.* Yes to all if PFA: see the research announcement in this volume by Nyikos, Shapirovskiĭ, Szentmiklóssy, and Veličković. Negative answers to all these questions are consistent. For the first, this was shown by Nyikos in Top. Appl. 44 (1992) 271–292. For the others, this was already pointed out in Volume 2.

Classic Problem VI, related problem B. (*Efimov*) Does a compact space of countable tightness have a dense set of points of first countability? *Solution.* No is consistent (*Malykhin*), and Yes follows from PFA (*Dow*).

Volume 7

H10 (*van Douwen*) One can show that a compact zero-dimensional space X is the continuous image of a compact orderable space if X has a clopen family $\mathcal{S}$ which is T_0-point-separating (i.e. if $x \neq y$ then there is $S \in \mathcal{S}$ such that $|S \cap \{x,y\}| = 1$) and of rank 1 (i.e. two members are either disjoint or comparable). Is the converse false? *Solution.* No, the converse is also true. (*S. Purisch*)

R4 (*Rubin*) Does there exist a separable metric space, compact or not, which has finite cohomological dimension and infinite topological dimension? *Solution.* Yes, there is even a compact example. (*A. Dranishnikov*)

Volume 12

C55 (*Nyikos*) Is $2^{\mathfrak{s}}$ always the smallest cardinality of an infinite compact Hausdorff space with no nontrivial convergent sequences? *Remarks.* In v.12 it was erroneously stated that Fedorchuk had shown, in effect, that there is always one of cardinality $2^{\mathfrak{s}}$. This is incorrect: the proof works only when $\mathfrak{s} = \aleph_1$.

C56 (*Nyikos*) Is it consistent that every separable, hereditarily normal, countably compact space is compact? *Solution.* Yes. See the research announcement by Nyikos, Shapirovskiĭ, Szentmiklóssy, and Veličković in this issue.

Volume 14

J5 (*Balogh, Mashburn, and Nyikos*) Will a space X freely acted upon by a finite group of autohomeomorphisms necessarily have a countable closed migrant cover if it is subparacompact? What if X is a Moore space or a σ-space? *Partial Solution.* Yes in the case of semistratifiable spaces, hence Yes in the case of Moore spaces or σ-spaces. (*Peter von Rosenberg*)

J6 (*Balogh, Mashburn, and Nyikos*) If a paracompact space X with finite Ind is acted upon freely by a finite group of autohomeomorphisms, must X have a finite open or closed migrant cover? What if dim X is finite? If the answer to either one is Yes, what is the optimal bound on the size of the cover? *Partial Solution.* If dim X is finite, then X does have finite open migrant covers and hence finite migrant closed covers. (*Peter von Rosenberg*)