Topology Proceedings

Web:	http://topology.auburn.edu/tp/
Mail:	Topology Proceedings Department of Mathematics & Statistics Auburn University, Alabama 36849, USA
E-mail:	topolog@auburn.edu
ISSN:	0146-4124

Topology Proceedings
Vol 18, 1993

NONCONTRACTIBLE HYPERSPACE WITHOUT R^i-CONTINUA

CHOON JAI RHEE[1] AND KUL HUR

ABSTRACT. We resolve the following question of Charatonik in [1,Question 21, p.214] affirmatively: Does there exist a metric continuum X such that its hyperspace $C(X)$ of subcontinua is not contractible and $C(X)$ contains no R^i-continuum?

1. PRELIMINARY.

Let X be a metric continuum with metric d. Denoted by $C(X)$ the hyperspace of all nonempty subcontinua of X endowed with the Hausdorff metric H induced by d which is defined by $H(A,B) = \inf\{\epsilon > 0 : A \subset N(\epsilon, B)\ and\ B \subset N(\epsilon, A)\}$, where $N(\epsilon, A) = \{x \in X : d(x,a) < \epsilon$ for some $a \in A\}$. And if $A = \{x\}$ then we agree that $N(\epsilon, \{x\}) = N(\epsilon, x)$. We shall also be considering the hyperspace $C(C(X)) = C^2(X)$ with the Hausdorff metric H^2 induced by H and $C^3(X)$ with its metric H^3 induced by H^2.

For each point $x \in X$, let $T(x) = \{A \in C(X) : x \in A\}$. $T(x)$ is called the total fiber of X at x. $T(x)$ is always closed and arcwise connected subset of $C(X)$. An element $A \in T(x)$ is said to be admissible at x in X if, for each $\epsilon > 0$, there is a $\delta > 0$ such that each point y in the δ-neighborhood of x has an element $B \in T(y)$ such that $H(A,B) < \epsilon$. Let $\mathcal{A}(x)$ be the set of all elements of $T(x)$ which are admissible at x in X. $\mathcal{A}(x)$ is said to be the admissible fiber at x in X. We denote the

[1]The author expresses his gratitude to Korean Science and Engineering Foundation and Won Kwang University for the support.

total fiber and the admissible fiber at $B \in C(X)$ in $C(X)$ by $\mathcal{A}(B)$ and $\mathcal{T}(B)$ respectively. Let M be the set of all $x \in X$ at which $T(x) \neq \mathcal{A}(x)$. M is called the $\mathcal{M}$-set of X [9]. Correspondingly we denote the $\mathcal{M}$-set of $C(X)$ by $\mathcal{N}$. The points in the complement of the $\mathcal{M}$-set of a metric continuum are called k-points of the space. For more about the admissibility and $\mathcal{M}$-set, we refer [7, 8, 9]. And the concepts of R^i-continua are given in [2].

An order arc in $C(X)$ is an arc α in $C(X)$ such that if $A, B \in \alpha$, then $A \subset B$ or $B \subset A$. If $C, D \in C(X)$ and $C \subset D$, $C \neq D$, then there is a nondegenerate order arc in $C(X)$ with end points C and D [6]. We call such arc an order arc in $C(X)$ from C to D.

We enlist a few known facts on connectedness im kleinen and local connectedness in $C(X)$ and several new results relating to order arcs.

(1.1)Lemma [7, Propositions 1.5 and 1.6]. *Let X be a metric continuum. (1). For each $x \in X$, $\{x\}, X \in \mathcal{A}(x)$ and $\mathcal{A}(x)$ is closed. (2). If $A, B \in C(X)$ such that $A \in \mathcal{A}(a)$, $B \in \mathcal{A}(x)$, and $x \in A \cap B$, then $A \cup B \in \mathcal{A}(a)$.*

(1.2) Lemma. *Let X be a metric continuum. If X is connected im kleinen at $x \in X$ then x is a k-point of X.*

Proof: Let $\epsilon > 0$ and $A \in T(x)$. Since X is connected im kleinen at x, let U be the $\frac{\epsilon}{2}$-neighborhood of x and let V be δ-neighborhood of x, $0 < \delta < \frac{\epsilon}{2}$, such that if $y \in V$ then x and y lie in a connected subset C of U. Let $B = \bar{C} \cup A$. Then $H(A, B) < \epsilon$. Therefore $A \in \mathcal{A}(x)$.

(1.2.1) Lemma. *Let X be a metric continuum and $x \in X$.* Then the total fiber $T(x)$ is a closed and path-connected subset of $C(X)$. Thus if x is a k-point, then $\mathcal{A}(x)$ is path-connected.

Proof: $T(x)$ is clearly closed and $X \in T(x)$. If $A \in T(x)$, then there is an order arc from A to X. If x is a k-point, then $\mathcal{A}(x) = T(x)$ so that $\mathcal{A}(x)$ is path-connected.

(1.3) Lemma [10, Proposition 2]. *If X is a metric continuum such that $\mathcal{A}(x)$ contains an order arc from $\{x\}$ to X for each $x \in X$, then $\mathcal{A}(x)$ is path-connected.*

Proof: Let $A \in \mathcal{A}(x)$. Let α be an order arc in $\mathcal{A}(x)$ from $\{x\}$ to X. Then the set $\beta = \{A \cup A_t : A_t \in \alpha\}$ is contained in $\mathcal{A}(x)$ by part (2) of (1.1), and it is easy to see that β is an order arc.

(1.4) Lemma [1, Corollary 16]. *Let X be a metric continuum. If, for each $x \in X$, the admissible fiber $\mathcal{A}(x)$ is path-connected, then X does not contain any R^i-continuum for $i \in \{1, 2, 3\}$.*

Eberhart's lemma can be restated as follows.

(1.5) Lemma [3, Lemma 2.1.2]. *Suppose an element $A \in C(X)$ contains a point at which X is connected im kleinen. Then $C(X)$ is connected im kleinen at A.*

(1.6) Corollary. *If X is locally arcwise connected at $a \in A$ and A is a point of $C(X)$, then $C(X)$ is locally arcwise connected at A.*

Proof: Let V be a connected ϵ-neighborhood of a in X and let $\mathcal{O}$ be the ϵ-neighborhood of A in $C(X)$ and $B \in \mathcal{O}$. Let $y \in V \cap B$ and C be a subcontinuum of V which contains both a and y. Let α and β be order arcs in $C(X)$ respectively from A and B to $A \cup B \cup C$. Then $\alpha \cup \beta \subset \mathcal{O}$. Therefore there is an arc in $\mathcal{O}$ joining A to B.

(1.7) Corollary. *Let $\mathcal{N}$ be the $\mathcal{M}$-set of $C(X)$. Then, for each $A \in \mathcal{N}$, X is not connected im kleinen at any point of A.*

Thus $\bigcup\mathcal{N}$ is entirely contained in the set N of all points x at which X is not connected im kleinen.

Let D be a subset of a metric space X. Let $C(D) = \{B \in C(X) : B \subset D\}$. For $A \in C(X)$, let $H(A, C(D)) = \inf\{H(A,B) : B \in C(D)\}$. The next lemma is contained in [5, Theorem 2].

(1.8) Lemma. *Let $A \in C(X)$. If, for each open set U in X containing A, there is a $\delta > 0$ such that $H(A, C(D)) \geq \delta$ for all components D of U not containing A, then $C(X)$ is connected im kleinen at A.*

Proof: Let $\mathcal{O}_\epsilon$ be the ϵ-neighborhood in $C(X)$ of A. Let $U_{\frac{\epsilon}{2}}$ be the $\frac{\epsilon}{2}$-ball about A in X, and let C be the component of $U_{\frac{\epsilon}{2}}$ containing A. Let $0 < \delta < \frac{\epsilon}{2}$ such that $H(A, C(D)) \geq \delta$ for all components D of $U_{\frac{\epsilon}{2}}$ different from C. Let $\mathcal{V}$ be the δ-neighborhood of A in $C(X)$ and let $B \in \mathcal{V}$. Since $H(A,B) < \delta$, $B \cap D = \emptyset$ for all components D of $U_{\frac{\epsilon}{2}}$ different from C. Hence $B \subset C$. Let α_A and α_B be order arcs respectively from A and B to $\bar{C}$. Then $\alpha_A \cup \alpha_B \subset \mathcal{O}_\epsilon$ so that there is an arc in $\mathcal{O}_\epsilon$ between A and B.

(1.8.1) Lemma. *Let $A \in C(X)$. If $C(X)$ is connected im kleinen at A, then the admissible fiber $\mathcal{A}(A)$ at A in $C(X)$ is path-connected.*

The proof is similar to that of Lemma (1.2).

Suppose α is an order arc in $C(X)$. When we say α is parametrized we mean $\alpha = \{A_t\}$, $t \in [0,1]$, is parametrized in such a way that $A_s \subset A_t$ whenever $s < t$. We call A_0 the initial element of α and A_1 the terminal element of α. Define $\alpha_t = \{A_s \in \alpha : 0 \leq s \leq t\}$ for each $t \in [0,1]$, and let $\hat{\alpha} = \{\alpha_t\}_{t \in [0,1]}$. Then each α_t is an order arc in $C(X)$ and $\hat{\alpha}$ is an order arc in $C^2(X)$. We say that $\hat{\alpha}$ is the order arc in $C^2(X)$ induced by α.

(1.9) Theorem. *Let $\alpha = \{A_t\}_{t\in[0,1]}$ and $\beta = \{B_t\}_{t\in[0,1]}$ be parametrized order arcs in $C(X)$. Let $\hat{\alpha} = \{\alpha_t\}_{t\in I}$ and $\hat{\beta} =$*

$\{\beta_s\}_{s\in I}$ be the induced order arcs by α and β respectively. Then $H^2(\alpha,\beta) < \epsilon$ if and only if $H^3(\hat{\alpha},\hat{\beta}) < \epsilon$.

Proof: Suppose $H^3(\hat{\alpha},\hat{\beta}) < \epsilon$. Since $\cup\hat{\alpha} = \alpha$ and $\cup\hat{\beta} = \beta$ and $H^2(\cup\hat{\alpha},\cup\hat{\beta}) \leq H^3(\hat{\alpha},\hat{\beta}) < \epsilon$, we have $H^2(\alpha,\beta) < \epsilon$.
Suppose $H^2(\alpha,\beta) < \epsilon$. Let $\alpha_{t_0} \in \hat{\alpha}$. Then α_{t_0} is an order arc from A_0 to A_{t_0}. Then there is an element $B_{s_0} \in \beta$ such that $H(A_{t_0}, B_{s_0}) < \epsilon$. Let $\beta_{s_0} \in \hat{\beta}$ be an order arc from B_0 to B_{s_0} such that $H(A_{t_0}, B_{s_0}) < \epsilon$. We show that $H^2(\alpha_{t_0}, \beta_{s_0}) < \epsilon$ by contradiction. Suppose there is an element $A_t \in \alpha_{t_0}$ such that $H(A_t, B_s) \geq \epsilon$ for each $0 \leq s \leq s_0$. In particular, $H(A_t, B_{s_0}) \geq \epsilon$. Let $B_{s'} \in \beta$ such that $H(A_t, B_{s'}) < \epsilon$. Then $s_0 < s'$ so that $B_{s_0} \subset B_{s'}$. But then (i). $B_{s_0} \subset B_{s'}$ and $H(A_t, B_{s'}) < \epsilon$ imply that $B_{s_0} \subset B_{s'} \subset N(\epsilon, A_t)$, and (ii). $A_t \subset A_{t_0}$ and $H(A_{t_0}, B_{s_0}) < \epsilon$ imply that $A_t \subset A_{t_0} \subset N(\epsilon, B_{s_0})$. Combining (i) and (ii) we have $H(A_t, B_{s_0}) < \epsilon$ which is a contradiction. Thus for each $A_t \in \alpha_{t_0}$ there is an element $B_s \in \beta_{s_0}$ such that $H(A_t, B_s) < \epsilon$. Similarly one can show that, for each $B_s \in \beta_{s_0}$, there is an element $A_t \in \alpha_{t_0}$ such that $H(A_t, B_s) < \epsilon$. Therefore we have $H^2(\alpha_{t_0}, \beta_{s_0}) < \epsilon$. Since α_{t_0} and β_{s_0} are arbitrary, we have $H^3(\hat{\alpha},\hat{\beta}) < \epsilon$.

(1.9.1) Corollary. *Let $\alpha = \{A_t\}_{t\in[0,1]}$ be a parametrized order arc. Let $\hat{\alpha} = \{\alpha_t\}_{t\in[0,1]}$ be the induced order arc by α. If, for each $\epsilon > 0$, there is $0 < \delta < \epsilon$ such that whenever $B_0 \in C(X)$ with $H(A_0, B_0) < \delta$ there is an order arc β with the initial element B_0 such that $H^2(\alpha,\beta) < \epsilon$, then each $\alpha_t \in \mathcal{A}(A_0)$.*

(1.10) Lemma. *Let X be a metric conitinuum. Let $\alpha = \{A_t\}_{t\in[0,1]}$ be an order arc in $C(X)$ from A_0 to A_1. Let $C \in C(X)$. (a). If $A_0 \cap C \neq \emptyset$ and $A_1 \setminus C \neq \emptyset$, then the set $\beta = \{C \cup A_t : A_t \in \alpha\}$ is an order arc in $C(X)$. Furthermore, if, in addition, $H(C, A_0) < \epsilon$ then $H^2(\alpha,\beta) < \epsilon$. (b). If $C \cap A_1 \neq \emptyset$, $C \setminus A_1 \neq \emptyset$, β' is an order arc in $C(X)$ from A_1 to $A_1 \cup C$, and $H(C \cup A_1, A_1) < \epsilon$, then $\gamma = \alpha \cup \beta'$ is an order arc from A_0 to $C \cup A_1$ such that $H^2(\alpha,\gamma) < \epsilon$.*

Proof: First we prove that the set β is a continuous image of α. Define $g : \alpha \to \beta$ by $g(A_t) = C \cup A_t$ for each $A_t \in \alpha$. Let $\epsilon > 0$. Let $A_t, A_s \in \alpha$ such that $H(A_t, A_s) < \epsilon$. Then $H(g(A_t), g(A_s)) = H(C \cup A_t, C \cup A_s) \leq H(A_t, A_s) < \epsilon$ by [7, Lemma 1.4]. Hence g is continuous. It is clear that g is onto. Since β is linearly order by the strict set inclusion $\subset$, we use a Whitney map $\mu : \beta \to [\mu(A_0 \cup C), \mu(A_1 \cup C)]$ which is one-to-one and onto. Hence β is an order arc.

Now suppose $H(C, A_0) < \epsilon$. Let $C \cup A_t \in \beta$. Then $H(A_t, C \cup A_t) = H(A_0 \cup A_t, C \cup A_t) \leq H(C, A_t) < \epsilon$ by [7, Lemma 1.4]. Hence $H(g(A_t), A_t) < \epsilon$. Hence $H^2(\alpha, \beta) < \epsilon$.

The proof for the part (b) is similar.

(1.11) Lemma. *Let X be a metric continuum. Let $\alpha = \{A_t\}_{t \in I}$ be an order arc in $C(X)$ from A_0 to A_1 such that A_0 has a point a at which X is connected im kleinen. Then, for each $\epsilon > 0$, there is a $\delta > 0$ such that if B is an element of the δ-neighborhood $\mathcal{V}_\delta$ of A_0 in $C(X)$ then there is an order arc β in $C(X)$ from B to an element of $C(X)$ such that $H^2(\alpha, \beta) < \epsilon$.*

Proof: Let $0 < \tau < \frac{1}{2}\min\{\epsilon, H(A_0, A_1)\}$. Since X is connected im kleinen at a, there is $0 < \delta < \frac{\tau}{2}$ such that the component D of $N(\frac{\tau}{2}, a)$ containing a contains $N(\delta, a)$. Let $\mathcal{V}_\delta$ be the δ-neighborhood of A_0 in $C(X)$ and $B \in \mathcal{V}_\delta$. Then $B \cap \bar{D} \neq \emptyset$ so that $B \cup \bar{D} \cup A_0$ is a subcontinuum of X. Since $\bar{D}$ is contained in the closure of $N(\frac{\tau}{2}, A_0)$ and $B \subset N(\delta, A_0)$, we have $B \cup \bar{D} \cup A_0 \subset N(\tau, A_0)$. This fact together with $A_0 \subset N(\tau, B \cup \bar{D} \cup A_0)$ yields $H(B \cup \bar{D} \cup A_0, A_0) < \tau$. Since $\frac{\tau}{2} < H(A_0, A_1)$ and $A_0 \subset A_1$, we have $A_0 \subset N(\tau, A_1)$ and $A_1 \not\subset N(\tau, A_0)$ so that $A_1 \setminus (B \cup \bar{D} \cup A_0) \neq \emptyset$. Let $\beta^2 = \{B \cup \bar{D} \cup A_t : A_t \in \alpha\}$ to be an order arc in $C(X)$ from $B \cup \bar{D} \cup A_0$ to $B \cup \bar{D} \cup A_1$ such that $H^2(\alpha, \beta^2) < \tau$ by the part (a) of (1.10). Let β^1 be an order arc in $C(X)$ from B to $B \cup \bar{D} \cup A_0$, and let $\beta = \beta^1 \cup \beta^2$. Then β is an order arc from B to $B \cup \bar{D} \cup A_1$. We show that $H^2(\alpha, \beta) < \epsilon$. For each $C \in \beta^1$, we have $B \subset C \subset B \cup \bar{D} \cup A_0$ and $H(A_0, C) \leq H(A_0, B \cup \bar{D} \cup A_0) < \tau$ and $H^2(\{A_0\}, \beta^1) < \tau$.

Thus this fact together with $H^2(\alpha, \beta^2) < \tau$ yields $H^2(\alpha, \beta) < \tau < \epsilon$.

Definition. A continuous mapping f of a topological space X onto a topological space Y is said to be *confluent* if, for each subcontinuum K of Y, each component of the inverse $f^{-1}(K)$ is mapped by f onto K.

A pair $\{\{X_n\}_{n=0}^{\infty}, f\}$ consisting of a sequence $\{X_n\}_{n=0}^{\infty}$ of pairwise disjoint subcontinua of a metric space X and a continuous map $f : \cup_{n=0}^{\infty} X_n \to X_0$ is said to be a *c-pair* if it satisfies the following property: for each n, the restriction $f \mid X_n = f_n : X_n \to X_0$ is a confluent map and f_0 is the identity map on X_0 such that, for each $\epsilon > 0$, there is an N such that $f_n^{-1}(x) \subset N(\epsilon, x)$ for all $x \in X_0$ and for all $n \geq N$.

Let $\{X_n\}_{n=1}^{\infty}$ be a sequence of subsets of a space X. Denote the limit superior of the sequence by LsX_n, the limit inferior of it by LiX_n, and the limit of the sequence by LtX_n.

(1.12) Lemma. *Let $\{\{X_n\}_{n=0}^{\infty}, f\}$ be a c-pair. Then $LtX_n = X_0$ and $LtC(X_n) = C(X_0)$.*

Proof: Let us first prove that $LtX_n = X_0$. It is clear from the definition that $X_0 \subset LiX_n$. Let $x \in LsX_n$, and $\{x_{n_k}\}_{k=0}^{\infty}$, $x_{n_k} \in X_{n_k}$, be a sequence which converges to x. By the continuity of f the sequence $\{f(x_{n_k})\}_{k=0}^{\infty} = \{f_{n_k}(x_{n_k})\}_{k=0}^{\infty}$ converges to a point $y \in X_0$. Let $\epsilon > 0$ be given. There is a positive N_1 such that $f_{n_k}(x_{n_k}) \in N(\frac{\epsilon}{2}, y)$ for all $k > N_1$. By the hypothesis, there is a positive integer N_2 such that $x_{n_k} \in f_{n_k}^{-1} f_{n_k}(x_{n_k}) \subset N(\frac{\epsilon}{2}, f_{n_k}(x_{n_k}))$ for all $k > N_2$ so that $x_{n_k} \in N(\epsilon, y)$ for all $k > \max\{N_1, N_2\}$. Therefore $y = x \in X_0$. Thus $LsX_n \subset X_0$. This proves that $LtX_n = X_0$.

We now prove the second part. Let $\epsilon > 0$. Let N be an interger such that $f_n^{-1}(x) \subset N(\epsilon, x)$ for all $x \in X_0$ and for all $n > N$. Let $K \in C(X_0)$. For $n > N$, let B_n be a component of $f_n^{-1}(K)$. Then $B_n \subset \cup\{f_n^{-1}(y) : y \in K\} \subset N(\epsilon, K)$. On the other hand for each $x \in K$, there is some $y \in B_n$ such that $f_n(y) = x$. So that $d(x,y) < \epsilon$. Hence $x \in N(\epsilon, y)$. Thus $K \subset N(\epsilon, B_n)$. Therefore $H(B_n, K) < \epsilon$. This proves that the ϵ-neighborhood of K in $C(X)$ intersects $C(X_n)$ for all $n > N$. Hence $C(X_0) \subset LiC(X_n)$.

Now suppose $B \in LsC(X_n)$. Since $LtX_n = X_0$, it is obvious that $B \in C(X_0)$. Thus $LsC(X_n) \subset C(X_0)$. This proves $C(X_0) = LtC(X_n)$.

(1.13) Theorem. *Let $\{\{X_n\}_{n=0}^{\infty}, f\}$ be a c-pair. Let $\epsilon > 0$ and let α be an order arc in $C(X_0)$. Then there is a positive interger N such that, for each $n > N$, there is an order arc τ in $C(X_n)$ such that $H^2(\alpha, \tau) < \epsilon$. Futhermore, if A is the initial point of α which contains a point at which X_0 is connected im kleinen, then there are a positive integer N and a δ-neighborhood $\mathcal{V}_\delta$ of A in $C(\cup_{n=0}^{\infty} X_n)$ such that, for $n > N$ and for each element $B \in \mathcal{V}_\delta \cap C(X_n)$ there is an order arc γ in $C(X_n)$ having its initial point B such that $H^2(\alpha, \gamma) < \epsilon$.*

Proof: We prove the first part. Let $\epsilon > 0$ be given. Let α be an order arc in $C(X_0)$ from A_0 to A_1. We parametrize $\alpha = \{A_t\}_{t \in [0,1]}$ so that $A_s \subset A_t$ and $A_s \neq A_t$ whenever $s < t$. Let N be a positive integer such that $f_n^{-1}(x) \subset N(\epsilon, x)$ for all $x \in X_0$ and for all $n > N$. For each fixed $n > N$, let B_0 be a component of $f_n^{-1}(A_0)$. We fix this component. Let β_n be the collection of all B_t, where B_t is the component of $f_n^{-1}(A_t)$, $A_t \in \alpha$, which contains B_0. We claim that there is an an order arc in $C(X_n)$ containing β_n. Let B_1 be the component of $f_n^{-1}(A_1)$ which contains B_0. Let $\mathcal{S}_n = \{B \in C(B_1) : B_0 \subset B\}$. We give an relation $\prec$ on $\mathcal{S}_n$ to be $B_\alpha \prec B_\beta$ if $B_\alpha \subset B_\beta$ and $B_\alpha \neq B_\beta$. Then $\prec$ is a strict partial order. We claim that β_n is a simply ordered subset of $\mathcal{S}_n$. To see it let $B_t, B_s \in \beta_n$. Let $A_t, A_s \in \alpha$ such that B_t and B_s are the components of $f_n^{-1}(A_t)$ and $f_n^{-1}(A_s)$ respectively containing B_0. Since α is also simply ordered set by $\prec$ we may assume that $A_s \subset A_t$ and $A_s \neq A_t$. Then by the confluency of f_n, it is easily seen that $B_s \prec B_t$. The transitivity is obvious. So that β_n is simply ordered set. Then by the maximal principle, there is a maximal simply ordred subset τ of $\mathcal{S}_n$ containing β_n. Then τ is an order arc in $C(X_n)$ from B_0 to B_1.

We show that $H^2(\alpha, \tau) \leq \epsilon$. First we note that (i). $H(A_t, B_t) < \epsilon$ for each $t \in [0,1]$.

We show that (ii). if $B \in \tau \setminus \beta_n$, then there is $A_{s_0} \in \alpha$ such that $B \prec B_{s_0}$, $f_n(B) = A_{s_0}$ and $H(B, A_{s_0}) < \epsilon$.

Let $B \in \tau \setminus \beta_n$ be an arbitray element. Then $B_0 \prec B \prec B_1$. Let $S = \{s : B_s \prec B\}$ and $T = \{t : B \prec B_t\}$. Then S and T are both nonempty. We let $s_0 = \sup S$ and $t_0 = \inf T$. Then $s_0 \leq t_0$.

Let $\{t_m\}_{m=1}^{\infty}$ be a decreasing sequence in T which converges to t_0, and let $\{B_{t_m}\}_{m=1}^{\infty}$ be the corresponding sequence of elements of β_n. Without loss of generality we assume that $\{B_{t_m}\}_{m=1}^{\infty}$ converges to $C \in \tau$ and the sequence $\{A_{t_m}\}_{m=1}^{\infty}$ converges to A_{t_0}. Since $B_{t_0} \prec B_{t_m}$ for all m, $B_{t_0} \subset Lt B_{t_m} = C$. On the other hand by the property of the map f_n, we have $C \subset B_{t_0}$. Hence we have $C = B_{t_0}$. Therefore $B \preceq B_{t_0} = C$.

Now let $\{s_m\}_{m=1}^{\infty}$ be an increasing sequence in S converging to s_0. We may again assume that the corresponding sequences $\{B_{s_m}\}_{m=1}^{\infty}$ and $\{A_{s_m}\}_{m=1}^{\infty}$ converge to $C' \in \tau$ and A_{s_0} respectively. Then $B_{s_m} \prec C' \preceq B_{s_0}$ for all m and $C' \preceq B$. Since $s_0 \leq t_0$, we also have $B_{s_0} \preceq B_{t_0}$. Hence we have either $B \prec B_{s_0}$ or $B_{s_0} \prec B$. If $B_{s_0} \prec B$, then there would exist an s, $s_0 < s < t_0$, such that $B_{s_0} \prec B_s \prec B_{t_0}$, $B_s \in \beta_n$. This contradicts the choice of either s_0 or t_0. Thus we have $C' \preceq B \prec B_{s_0}$.

Since $f_n(B_{s_m}) = A_{s_m} \to f_n(C')$ as $m \to \infty$, and $A_{s_0} = f_n(B_{s_0})$, we have $f_n(B) = A_{s_0}$.

We now show that $H(B, A_{s_0}) < \epsilon$. Since $B \subset B_{s_0}$ and $H(B_{s_0}, A_{s_0}) < \epsilon$, we have $B \subset N(\epsilon, A_{s_0})$. Let $x \in A_{s_0}$. Then there is $y \in B \cap f_n^{-1}(x)$ such that $y \in f_n^{-1}(x) \subset N(\epsilon, x)$. Hence $x \in N(\epsilon, y)$. $A_{s_0} \subset N(\epsilon, B)$. Therefore $H(B, A_{s_0}) < \epsilon$.

Now we combine (i) and (ii), one conclude that $H^2(\alpha, \tau) < \epsilon$.

We prove for the second part. Let N be a positive integer such that $f_n^{-1}(x) \subset N(\frac{\epsilon}{2}, x)$ for all $x \in X_0$ and all $n > N$. Let

$A \in C(X_0)$ contains a point a at which X_0 is connected im kleinen. Let $Z = \cup_{n=0}^{\infty} X_n$. Since X_0 is connected im kleinen at a, there is $0 < \delta_1 < \frac{\epsilon}{16}$ such that the δ_1-neighborhood $N(\delta_1, a)$ in the subspace X_0 of a is contained in the component C of $N(\frac{\epsilon}{16}, a)$. Since the induced map $f^* : C(Z) \to C(X_0)$ by f is continuous at A, there is a $0 < \delta < \delta_1$ such that the δ-neighborhood $\mathcal{V}_\delta$ of A in $C(Z) \setminus \cup_{n=1}^{N} C(X_n)$ is mapped into the δ_1-neighborhood $\mathcal{V}_{\delta_1}$ of A in $C(X_0)$.

Let $B \in \mathcal{V}_\delta \cap C(X_n)$ for some $n > N$. Then $H(A,B) < \delta$ and $H(f_n(B), A) < \delta_1$ so that $H(f_n(B), B) < \frac{\epsilon}{8}$.

Since $f_n(B) \cap C \neq \emptyset$ (becuase $H(f_n(B), A) < \delta_1$, $f(B) = f_n(B)$, and $f(A) = f_0(A) = A$), $A \cup f_n(B) \cup \bar{C}$ is a subcontinuum of X_0. Let τ_1 be an order arc in $C(X_0)$ from A to $A \cup f_n(B) \cup \bar{C}$ and let $\tau_2 = \{A \cup f_n(B) \cup \bar{C} \cup A_t : A_t \in \alpha\}$. Then $\tau = \tau_1 \cup \tau_2$ is an order arc in $C(X_0)$. Since $H(A, A \cup \bar{C}) < \frac{\epsilon}{15}$ $(\bar{C} \subset \overline{N(\delta_1, a)} \subset N(\frac{\epsilon}{15}, A))$ and $H(A, f_n(B)) < \delta_1$, $H(A, A \cup \bar{C} \cup f_n(B)) = H(A \cup A, A \cup \bar{C} \cup f_n(B)) \leq \max\{H(A, A \cup \bar{C}), H(A, f_n(B))\} < \frac{\epsilon}{15}$ by [7, Proposition 1.5], we have $H^2(\alpha, \tau_2) \leq \frac{\epsilon}{15}$ by (1.8).

Now let $A' \in \tau_1$. Then $A' \subset A \cup \bar{C} \cup f_n(B)$ so that $H(A, A') \leq H(A, A \cup \bar{C} \cup f_n(B)) < \frac{\epsilon}{15}$ by [9, (0.63.3), p.34]. This means that $H^2(\{A\}, \tau_1) < \frac{\epsilon}{15}$. Hence $H^2(\alpha, \tau) < \frac{\epsilon}{15}$.

Let D be the component of $f_n^{-1}(A \cup \bar{C} \cup f_n(B))$ which contains B. Let γ_1 be an order arc in $C(X_n)$ from B to D, and let γ_2 be an order arc in $C(X_n)$ whose initial point is D such that $H^2(\tau_2, \gamma_2) < \frac{\epsilon}{2}$ which is provided by the first part for $\frac{\epsilon}{2}$. Let $\gamma = \gamma_1 \cup \gamma_2$. Then γ is an order arc in $C(X_n)$.
In order to show $H^2(\alpha, \gamma) < \epsilon$, we show first that $H^2(\gamma_1, \tau_1) < \frac{19\epsilon}{30}$.
For each $B' \in \gamma_1$, $H(A, B') \leq H(A,B) + H(B, B')$. Since $H(B, B') = H(B, B \cup B') \leq H(B, D)$ by [9, (0.63.4), p.34], we compute $H(B, D) \leq H(A,B) + H(A, A \cup \bar{C} \cup f_n(B)) + H(A \cup \bar{C} \cup f_n(B), D) < \frac{17\epsilon}{30}$. So that $H(A, B') < \frac{19\epsilon}{30}$. On the other hand, for each $A' \in \tau_1$, $H(A', B) \leq H(A, A') + H(A, B) \leq H(A, A \cup \bar{C} \cup f_n(B)) < \frac{2\epsilon}{15}$. Combining these two, we conclude that $H^2(\tau_1, \gamma_1) < \frac{19\epsilon}{30}$.

Since $H^2(\tau_2, \gamma_2) \leq \frac{\epsilon}{2}$, we see that $H^2(\tau, \gamma) < \frac{19\epsilon}{30}$. Hence $H^2(\alpha, \gamma) \leq H^2(\alpha, \tau) + H^2(\tau, \gamma) < \frac{21\epsilon}{30} < \epsilon$. This completes the proof.

2. NONCONTRACTIBLE HYPERSPACE WITHOUT R^i-CONTINUUM.

Charatonik exhibited a dendroid X [1, Example 5, p.209] without R^i-continuum whose hyperspace $C(X)$ is not contractible. We take this space X to prove that $C(X)$ does not contain any R^i-continuum by showing that the adimissible fiber $\mathcal{A}(A)$ at A in $C(X)$ is path-connected.

Let X be the dendroid (Example below). We use the same notations as in the example. We need some additional notations: Let T_n be the triod in Q_n whose vertices are c_n, a_n, and d_n. Let T'_n be the image under the central symmetry map g with respect to the origin b. We denote the unique arc between two points x and y by $[x, y]$. If $x = y$ then $[x, y] = \{x\}$.

We now define several subsets of $C(X)$: For each positive in teger n, let $\mathcal{L}_n$ be the collection of all triods A in $T_n \setminus \{a_n\}$ such that one vertex of A is a point of the half-open interval $[c_n, b_n)$, the other vertex lies in $[d_n, b_n)$, the third vertex of A lies in $[b_n, a_n)$ (here we allow the third vertex can be b_n). Let $\mathcal{L}'_n$ be the set of all images of elements of $\mathcal{L}_n$ under the symmetry map g. Let $\mathcal{K}_1 = \{A \in C(X)$: A contains a point at which X is locally connected $\}$. Let $\mathcal{K}_2$ be the set consisting of only the vertical arc $[c, a]$. Let $\mathcal{W} = \mathcal{K}_1 \cup \mathcal{K}_2 \cup (\cup_{n=1}^{\infty}(\mathcal{L}_n \cup \mathcal{L}'_n))$.

Lemma 2.1. *$C(X)$ is connected im kleinen at each $A \in \mathcal{W}$.*

Proof: If $A \in \mathcal{K}_1$, we apply (1.5). If $A \in \mathcal{K}_2 \cup \mathcal{L}_n \cup \mathcal{L}'_n$, we apply (1.8) or [5, Theorem 2].

For each positive integer n, let $\mathcal{U}_n = C(T_n \setminus (\mathcal{K}_1 \cup \mathcal{L}_n)$ and let $\mathcal{U}'_n$ be the collection of all images of elements of $\mathcal{U}_n$ under the symmetry map g. Let $\mathcal{V} = C([c, a]) \setminus \mathcal{K}_2$ and let

$\mathcal{N} = \mathcal{V} \cup (\cup_{n=1}^{\infty}(\mathcal{U}_n \cup \mathcal{U}'_n))$. Now one can easily show that:

Lemma 2.2. $\mathcal{N} = C(X) \setminus \mathcal{W}$.

Proof: Let $A \in C(X) \setminus \mathcal{W}$. Then $A \subset \cup_{n=1}^{\infty}([T_n \setminus \{a_n\}] \cup [T'_n \setminus \{a'_n\}]) \cup [c, a]$. Since all T_n, T'_n, $[c, a]$ are disjoint, either $A \subset T_n \setminus \{a_n\}$, $A \subset T'_n \setminus \{a'_n\}$ for some n, or A is a proper subset of $[c, a]$. Also $A \notin (\mathcal{L}_n \cup \mathcal{L}'_n)$ for any n. A must be an arc in X such that either $A \in \mathcal{V}$ or $A \in (\mathcal{U}_n \cup \mathcal{U}'_n)$ for some n.

Proposition 2.3. *Let X be the dendroid. Then, for each $A \in C(X)$, there is an order arc in $\mathcal{A}(A)$ from A to $C(X)$.*

Proof: Suppose $A \in \mathcal{W}$. Then $C(X)$ is connected im kleinen at A. So that A is a k-point of $C(X)$ by (1.2). Hence $\mathcal{A}(A) = \mathcal{T}(A)$, where $\mathcal{T}(A)$ is the total fiber at A in $C(X)$. Thus any order arc $\hat{\alpha}$ in $C^2(X)$ from $\{A\}$ to $C(X)$ is contained $\mathcal{T}(A)$ so that $\hat{\alpha} \subset \mathcal{A}(A)$.

Suppose $A \in \mathcal{N}$. Since each element of $\mathcal{N}$ is an arc in X, we write $A = [x, y]$ with the end points x and y.

Let $A \in \mathcal{V}$. Let $c \leq x \leq y < a$. We find an order arc $\hat{\alpha} = \hat{\alpha}_1 \cup \hat{\alpha}_2$ from $\{A\}$ to $C(X)$ in $\mathcal{A}(A)$ as follows:

Let $\alpha = \{[(1-t)x + tc, (1-t)y + ta] : t \in [0, 1]\}$. Then α is an order arc in $C(X)$ from A to $[c, a]$. For convenience, we let $A_t = [(1-t)x + tc, (1-t)y + ta]$. Then $\alpha = \{A_t : t \in [0, 1]\}$ such that $A = A_0$ and $[c, a] = A_1$. For each $t \in I$, let $\alpha_t = \{A_s \in \alpha : 0 \leq s \leq t\}$. Then α_t is an order arc in $C([c, a])$ for each $t \in (0, 1]$ and $\hat{\alpha}_1 = \{\alpha_t : t \in I\}$ is an order arc in $C^2(X)$ from $\{A\} = \alpha_0$ to $\alpha_1 = \alpha$. We note that $[c, a] \in \alpha_1$.

We show that $\alpha_t \in \mathcal{A}(A)$ for each $t \in [0, 1]$.

Suppose $x \neq c$. Then we take the following c-pair $\{\{X_n\}_{n=0}^{\infty}, f\}$: For each n, let $X_{2n} = Q_n$, $X_{2n-1} = Q'_n$, $X_0 = [c, a]$, and $f_n : X_n \to X_0$ the horizontal projection. Then each f_n is a confluent map and $f = \cup_{n=1}^{\infty} f_n : \cup_{n=0}^{\infty} X_n \to X_0$ is continuous and f_0 is the identity on X_0. For $\epsilon > 0$, let N and

$\delta > 0$ be the same as in the proof of (1.13). We choose δ_1, $0 < \delta_1 < \min \frac{1}{2}\{d(x,c), \delta, \epsilon, d(a,y)\}$. Let B be an element of the δ_1-neighborhood of A in $C(X)$. Then B is entirely contained in Q_n (or Q'_n). Hence by (1.13), there is an order arc γ in $C(Q_n)$ (or $C(Q'_n)$) such that $H^2(\alpha_1, \gamma) < \epsilon$ and so that $H^3(\hat{\alpha}_1, \hat{\gamma}) < \epsilon$ by (1.9). Hence there is $\gamma_s \in \hat{\gamma}$ such that $H^2(\alpha_t, \gamma_s) < \epsilon$. If $B \in C([c,a])$, by (1.10) we have an order arc $\hat{\gamma} \in C^2(X)$ such that $H^2(\alpha_1, \gamma) < \epsilon$ so that $H^3(\hat{\gamma}, \hat{\alpha}_1) < \epsilon$. Hence, by (1.9.1) $\alpha_t \in \mathcal{A}(A)$ for each $t \in [0,1]$.

If $x = c$, let N and $0 < \delta$ be the same is in (1.13) for $\frac{\epsilon}{2}$. Let $B \in C(X)$ and $H(B,A) < \delta$. If B in entirely contained in Q_n or in Q'_n, then we get the same γ and $\hat{\gamma}$ as above. Otherwise B must contain more than one ramification points a'_n of Q'_n. Let $a'_m \in B$ such that $d(a'_m, c) \geq d(a'_n, c)$ for all $a'_n \in B$, and let $C = [a'_m, c]$. Then $H(A, C \cup A) < \delta$ and $H(B, C \cup A) < \delta$. Then applying (1.10) twice we get an order arc γ in $C(X)$ with the initial point B such that $H^2(\gamma, \alpha_1) < \epsilon$ so that and $\hat{\gamma}$ such that $H^3(\hat{\alpha}_1, \hat{\gamma}) < \epsilon$. Hence there is an element $\gamma_s \in \hat{\gamma}$ such that $H^2(\alpha_t, \gamma_s) < \epsilon$. This proves that $\alpha_t \in \mathcal{A}(A)$ for each $t \in [0,1]$.

Let $\hat{\alpha}_2 = \{\sigma_t\}_{t \in I}$ be any order arc in $C^2(X)$ from α_1 to $C(X)$. Since $[c,a]$ is a k-point of $C(X)$ and $[c,a] \in \alpha_1 \subset \sigma_t$ for each $t \in I$, $\sigma_t \in \mathcal{A}([c,a])$ for each $t \in I$. Since $\alpha_1 \in \mathcal{A}(A)$, $\sigma_t \in \mathcal{A}([c,a])$, and $[c,a] \in \alpha_1 \cap \sigma_t$, $\alpha_1 \cup \sigma_t = \sigma_t \in \mathcal{A}(A)$ by (1.1). It is clear that $\hat{\alpha}$ is an order arc in $\mathcal{A}(A)$ from $\alpha_0 = \{A\}$ to $C(X)$.

Now suppose $A = [x,y] \in \mathcal{U}_n \cup \mathcal{U}'_n$. Suppose $A \in \mathcal{U}_n$. Then either $A \subset [c_n, a_n]$ or $A \subset [d_n, a_n]$. We prove only for $c_n \leq x \leq y \leq a_n$. Let $\epsilon > 0$ be given. Since $a_n \notin A$, so we must have $d(y, a_n) > 0$. There are two cases to consider:

Case 1. $c_n \leq x < b_n$. Let $0 < \epsilon' < \frac{1}{3}\min\{\epsilon, d(y, a_n), d(b_n, a_n), d(x, [b_n, d_n])\}$.

Let e_m be the unique point of the intersection of $[c_{n,m}, a_n]$ and the horizontal line $y = (1 - \epsilon')$. For each m, let e_m be the unique point of the intersection of $[c_n, a_n]$ and the horizontal line, and let e_0 be the point in the intersection of the line and $[c_n, a_n]$. Then $d(a_n, e_0) = \epsilon'$. Let $\{\{X_m\}_{m=0}, f\}$ be the c-pair defined by, for each m, $X_n = [c_{n,m}, e_m]$ and $X_0 = [c_n, e_0]$ and $f_m : X_m \to X_0$ be the horizontal projection of X_m onto X_0, and f_0 the identity on X_0 and $f = \cup_{m=1}^{\infty} f_m$. Then each f_m is a confluent map and $f : Z = \cup_{n=0}^{\infty} X_m \to X_0$ is continuous. Let α^1 be an order arc in $C(X)$ from A to $[x, e_0]$ and let α^2 be an order arc in $C(X)$ from $[x, e_0]$ to $[x, a_n]$. Then α^1 is an order arc in $C(X_0)$ and $\alpha = \alpha^1 \cup \alpha^2$ is an order arc from A to $[x, a_n]$. We note that, since $H([x, e_0], [x, a_n]) < \epsilon'$, $H^2(\alpha^1 \cup \alpha^2, \alpha^1) < \epsilon'$ by (1.10). Let $\hat{\alpha} = \{\alpha_t\}_{t \in [0,1]}$ denote the induced order arc by α. Now let N and $\delta > 0$ be given by (1.13) for ϵ'. Let $\mathcal{V}_\delta$ be the δ-neighborhood of A in $C(X)$, and $B \in \mathcal{V} \cap C(Z)$. If $B \in C(X_m)$, $m > N$, then there is an order arc β from B such that $H^2(\alpha^1, \beta) < \epsilon'$ by (1.13) and $H^3(\hat{\alpha}^1, \hat{\beta}) < \epsilon'$, where $\hat{\alpha}^1 = \{\alpha_t\}_{t \in [0,1]}$ and $\hat{\beta} = \{\beta_t\}_{t \in [0,1]}$ are the induced order arcs by α^1 and β respectively as in (1.9). If $B \in C(T_n)$ then $B \cap A \neq \emptyset$ and $H(B, A) < \delta_1$ so that the order arc β obtained by (1.10) is such that $H^2(\beta, \alpha^1) < \delta_1$ and $H^3(\hat{\beta}, \hat{\alpha}^1) < \delta_1$. In any case, we have $H^2(\alpha, \beta) \leq H^2(\alpha^1 \cup \alpha^2, \alpha^1) + H^2(\alpha^1, \beta) < 2\epsilon' < \epsilon$ and hence $H^3(\hat{\alpha}, \hat{\beta}) < \epsilon$ by (1.9). Furthermore, one can show that, for each $\alpha_t \in \hat{\alpha}$, there is $\beta_s \in \hat{\beta}$ such that $H^2(\alpha_t, \beta_s) < \epsilon$. Hence $\alpha_t \in \mathcal{A}(A)$ for each α_t. Let $\hat{\alpha} = \{\alpha_t\}_{t \in I}$ and let $\hat{\gamma} = \{\gamma_s\}_{s \in I}$ be any order arc in $C^2(X)$ from α_1 to $C(X)$. Since $[x, a_n] \in \alpha_1 \in \mathcal{A}(A)$ and $[x, a_n]$ is a k-point of $C(X)$ and $\alpha_1 \subset \gamma_s$ implies $[x, a_n] \in \gamma_s$ for each $\gamma_s \in \hat{\gamma}$, so that by (1.1) $\gamma_s \in \mathcal{A}([x, a_n])$ for each $\gamma_s \in \hat{\gamma}$. This prove that $\hat{\alpha} \cup \hat{\gamma}$ is an order arc in $\mathcal{A}(A)$ from $\{A\}$ to $C(X)$.

Case 2. $b_n \leq x$. In this case we let $0 < \epsilon' < \frac{1}{3}\{\epsilon, d(y, a_n), \frac{1}{10}\}$. We need two c-pairs. Let $\{\{X_m\}_{m=0}^{\infty}, f\}$ be the c-pair for this ϵ' as in Case 1. Let $y = (1 - \epsilon')$ be horizontal line. For each positive integer m, let e'_m be the point of the intersection of the line y with $[d_{n,m}, a_n]$. Let e'_0 the point of intersection of the line y with $[d_n, a_n]$. Let

$Y_m = [p'_m, e'_m]$ for each m and $Y_0 = [d_n, e'_0]$. Let $g_m : Y_m \to Y_0$ be the horizontal projections. Let $g = \cup g_n$. Then $\{\{Y_m\}_{m=0}^{\infty}, g\}$ is a c-pair. Let α^1 be an order arc in $C(X)$ from

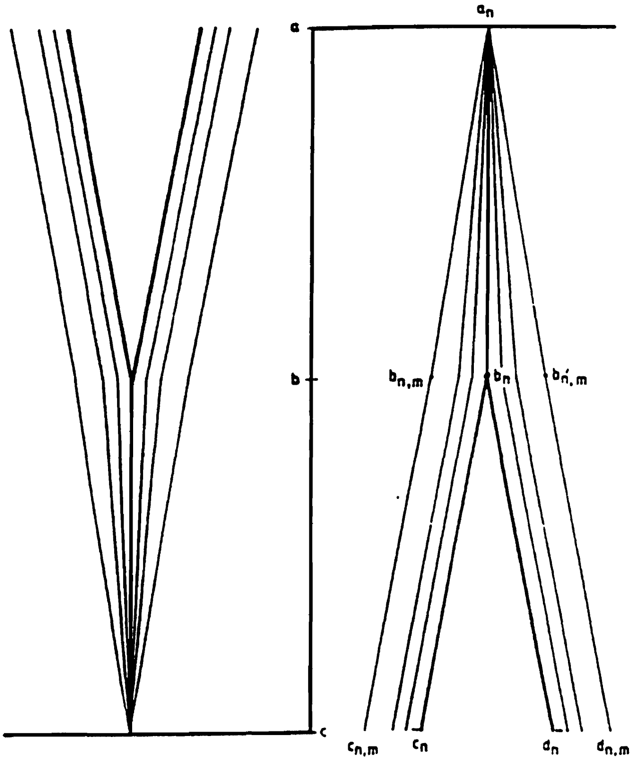
$[x, y]$ to $[x, e_0]$ and let α^2 be an order arc in $C(X)$ from $[x, e_0]$ to $[x, a_n]$. Let $\alpha = \alpha^1 \cup \alpha^2$. Then α is an order arc in $C(X)$ from $[x, y]$ to $[x, a_n]$. Applying the same technique and argument as in Case 1 with each one of the c-pairs, we see that each element α_t of the induced order arc $\hat{\alpha} = \{A_t\}_{[0,1]}$ by α is admissible at $A = [x, y]$ in $C(X)$. This order arc $\hat{\alpha}$ is an order arc in $C^2(X)$ from $\alpha_0 = \{A\}$ to $\alpha_1 = \alpha$. As before, we let $\hat{\gamma} = \{\gamma_t\}_{t \in [0,1]}$ be an order arc in $C^2(X)$ from α_1 to $C(X)$. Since $[x, a_n] \in \alpha_1 \in \mathcal{A}(A)$, and $[x, a_n]$ is a k-point of $C(X)$ and $[x, a_n] \in \gamma_t$ for each $\gamma_t \in \hat{\gamma}$, we have $\gamma_t \in \mathcal{A}(A)$ for each $\gamma_t \in \hat{\gamma}$ by (1.1). Thus $\hat{\alpha} \cup \hat{\gamma} \subset \mathcal{A}(A)$ is an order arc in $C^2(X)$ from $A = \alpha_0$ to $C(X)$.

Corollary 2.4. *$C(X)$ does not contain any R^i-continuum.*

By (2.3) there is an order arc in $\mathcal{A}(A)$ from A to $C(X)$ for each $A \in C(X)$. Hence by (1.3) and (1.4) we conclude that $C(X)$ does not contain any R^i-continuum.

Charatonik's Example [1, Example 5, p.209]. In the Euclidean plane let $a = (0, 1)$, $b = (0, 0)$, $c = (0, -1)$ and for each positive integer n let $a_n = (2^{-3n}, 1)$, $b_n = (2^{-3n}, 0)$, $c_n = (2^{-(3n+1)}, -1)$, and $d_n = (2^{-(3n-1)}, -1)$. For each positive integer m, let $b_{n,m} = (2^{-3n}(1 - 2^{(m+3)}), 0)$, $b'_{n,m} = (2^{-3n}(1 + 2^{-(m+3)}), 0)$, $c_{n,m} = (2^{-(3n+1)}(1 - 2^{-(m+3)}), -1)$,and $d_{n,m} = (2^{-(3n-1)}(1 + 2^{-(m+3)}), -1)$.

For each n let $Q_n = [a_n, b_n] \cup [b_n, c_n] \cup [b_n, d_n] \cup \bigcup_{m=1}^{\infty}([b_{n,m}, a_n] \cup [c_{n,m}, b_{n,m}]) \cup \bigcup_{m=1}^{\infty}([b'_{n,m}, a_n] \cup [d_{n,m}, b'_{n,m}])$. Let $Y = [a, a_1] \cup [a, c] \cup \bigcup_{n=1}^{\infty} Q_n$ and let Q'_n and Y' respectively be the images of Q_n and Y under the symmetry map g with respect to the origin b. Finally we put $X = Y \cup Y'$.



References

1. Charatonik, W. J., *R^i-continua and hyperspaces*, Topology and its Applications, **23** (1986), 207-216.
2. Czuba, S. T., *R^i-continua and contractibility of dendroids*, Proc. International Conference on Geometric Topology, pwn-Polish Sci. Publishers, Warszawa, (1980) 77-79.
3. Eberhart, C., *Continua with locally connected Whitney continua*, Houston J. Math., **4** (1978), 165-173.
4. Goodykoontz, J.T., *Connectedness im kleinen and local connectedness in 2^X and $C(X)$*, Pacific J. Math., **53** (1974), 387-397.
5. ———, *More on connectedness im kleinen and local connectedness in $C(X)$*, Proc.Amer.Math.Soc., **65** (1977), 357-364.
6. Kelley, J. L., *Hyperspaces of a continuum*, Trans.Amer.Math.Soc., **52** (1942), 22-36.

7. Rhee, C. J. and Nishiura, T., *An admissible condition for contractible hyperspaces*, Topology Proc., **8** (1983), 303-314.
8. ———, *Contractible hyperspace of subcontinua*, Kyungpook Math. J., **24** (1984), 143-154.
9. Rhee, C. J., *Obstructing sets for hyperspace contraction*, Topology Proc., **10** (1985), 159-173.

Wayne State University
Detroit, Michigan

Won Kwang University
Iri, Chunbuk, Korea