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PROBLEM SECTION

Problems were contributed by participants in either the 1993 Columbia, South Carolina conference (vol. 18) or the 1994 Auburn, Alabama conference (vol. 19). The number in brackets after each problem identifies which conference it came from.

B. Generalized Metric spaces and Metrization

36. *(Howard Wicke)* Is every bi-sequential space the continuous image of a metrizable space under a map with completely metrizable (or even discrete) fibers? [93]

BB. Metric spaces

1. *(Hattori and Ohta)* A metric space is said to have UMP (resp. WUMP) if for every pair of distinct points x, y there exists exactly (resp. at most) one point p such that $d(x,p) = d(y,p)$. Is a separable metric space having UMP homeomorphic to a subspace of the real line? [94]

2. *(Hattori and Ohta)* Is a peripherally compact (*i.e.,* each point has a neighborhood base consisting of sets with compact boundary) and separable metric space having WUMP homeomorphic to a subspace of the real line? [94]

See also problem E12.

C. Compactness and Generalizations

60. *(Friedler, Girou, Pettey, and Porter)* A regular T_1 [*resp.* Urysohn] space X is *R-closed* [resp. *U-closed*] if X is a closed subspace of every regular T_1 [*resp.* Urysohn] space containing X as a subspace. Is a space in which each closed set is R-closed [*resp.* U-closed] necessarily compact? [93]

61. *(Friedler, Girou, Pettey, and Porter)* A regular T_1 space is *RC-regular* if it can be embedded in an R-closed space. Find an internal characterization of RC-regular spaces. [93]

62. *(Friedler, Girou, Pettey, and Porter)* Is the product of two R-closed spaces necessarily RC-regular? [93]

63. *(Friedler, Girou, Pettey, and Porter)* Is there only one minimal regular topology coarser than an R-closed topology that has a proper regular subtopology? [93]

64. *(Friedler, Girou, Pettey, and Porter)* If the product of spaces X and Y is strongly minimal regular [*resp.* RC-regular] then must each of X and Y be strongly minimal regular [*resp.* RC-regular]? [93]

See also problem O14.

D. Paracompactness and Generalizations

41. *(Kenichi Tamano)* Is the space ω^{ω_1} weakly $\delta\theta$-refinable? [94]

E. Separation and Disconnectedness

12. *(Hattori and Ohta, attributed to Nadler)* Must a totally disconnected separable metric space having UMP be 0-dimensional? [94]

Remark: A positive answer to BB1 also answers this question positively.

F. Continua Theory

33. *(B. E. Wilder)* Which of the known results concerning aposyndetic continua can be extended to the class of C-continua? [93]

34. *(Sergio Macías)* In this and the following two problems, let $\Gamma = \mathcal{S}^1 \times \mathbb{Q}$ or $\Gamma = \mathcal{W} \times \mathbb{Q}$, where $\mathcal{W}$ is the figure eight, and $\mathbb{Q}$ is the Hilbert cube. Let $\sigma : \tilde{\Gamma} \to \Gamma$ be the universal covering space. Let X be a homogeneous continuum essentially embedded in Γ and let $\tilde{X} = \sigma^{-1}(X)$. Two points of $\tilde{X}$ are said to be in the same *continuum component* if there is a continuum

in $\tilde{X}$ containing them. Do the continuum components and components of $\tilde{X}$ coincide if X is homogeneous? [Without homogeneity, the answer is known to be negative.] [93]

35. *(Sergio Macías)* Let $\tilde{K}$ be a component of $\tilde{X}$. If $\sigma(\tilde{K})) \neq X$, is it true that $\sigma(\tilde{K})$ is contained in a composant of X? is it equal to a composant? [93]

36. *(Sergio Macías)* Suppose that $\sigma(\tilde{K}) \neq X$. If $\tilde{K}_C$ is a continuum component of $\tilde{X}$, is it true that $\sigma(K_C)$ is equal to a composant of X? [93]

G. Mappings of Continua and Euclidean Spaces

29. *(Ryszard J. Pawlak)* A function is called a *Darboux* function if it takes connected sets to connected sets. If X is connected and locally connected space, under what additional assumption does there exist a connected Alexandroff compactification X^* such that a theorem analogous to the following theorem holds?

Theorem. Let X be a continuum having an extension X^* with a one-point remainder x_0, such that X^* has an exploding point with respect to x_0. Then there is a closed Darboux function $f : X^* \to [0,1]$ which is discontinuous at x_0.
In general, what kinds of hypotheses on a space X^* (weaker than compactness) allow one to prove a theorem analogous to this one? [94]

30. *(Ryszard J. Pawlak)* Do there exist for a nondegenerate locally connected continuum X and any homeomorphism $h : X \to X$, spaces X_1, X_2 "close to compactness" such that X is a subspace of X_1 and X_2 and there exists a d-extension $h^* : X_1 \to X_2$ of the function h such that h^* is a discontinuous and closed Darboux function? [94]

See also O14.

I. Infinite Dimensional Topology and Shape Theory

9. *(Helma Gladdines)* Let $L(\mathbb{R}^2)$ denote the collection of Peano continua in $\mathbb{R}^2$. Is $L(\mathbb{R}^2)$ homeomorphic to the product of infinitely many circles? [93]

L. Topological Algebra

32. *(Dikranjan and Shakhmatov)* Which infinite groups admit a pseudocompact topology? In other words, what special algebraic properties must pseudocompact groups have? [93]

33. *(Dikranjan and Shakhmatov)* If G is a pseudocompact Abelian group, must either the torsion subgroup $t(G) = \{g \in G : ng = 0 \text{ for some } n \in \mathbb{N} \setminus \{0\}\}$ or $G/t(G)$ admit a pseudocompact group topology? [93]

34. *(Dikranjan and Shakhmatov)* If an Abelian group G admits a pseudocompact group topology, must the group $G/t(G)$ admit a pseudocompact group topology? *Remarks.* The answer to this and the preceding question is affirmative for torsion and torsion-free groups (both trivially), for divisible groups, for groups with $|G| = r(G)$, where $r(G)$ is the free rank of G, and when $t(G)$ admits a pseudocompact topology or is bounded, *i.e.,* there is some $n \in \mathbb{N} \setminus \{0\}$ such that $ng = 0$ for all $g \in G$. [93]

35. *(Dikranjan and Shakhmatov)* Suppose that G is an Abelian group, $n \in \mathbb{N}\{0\}$ and both $nG = \{ng : g \in G\}$ and G/nG admit pseudocompact group topologies. Must then G also admit a pseudocompact topology? [93]

36. *(Dikranjan and Shakhmatov)* Let $D(G)$ denote the maximal divisible subgroup of an Abelian group G. If G is pseudocompact, must either $D(G)$ or $G/D(G)$ admit a pseudocompact topology? [93]

37. *(Dikranjan and Shakhmatov)* Let G be an Abelian group with $D(G) = \{0\}$, i.e. a reduced Abelian group. If G admits a pseudocompact group topology, must G admit also a zero-dimensional pseudocompact group topology? [93]

38. *(Dikranjan and Shakhmatov)* Let G be a non-torsion pseudocompact Abelian group. Do there exist a cardinal σ and a subset of cardinality $r(G)$ of $\{0,1\}^\sigma$ whose projection on every countable subproduct is a surjection? [93]

39. *(Dikranjan and Shakhmatov)* Characterize (Abelian) groups which admit a group topology which has one of the

following properties:

(i) countably compact,

(ii) σ-compact, or

(iii) Lindelöf.

40. *(Dikranjan and Shakhmatov)* For which cardinals τ does the free Abelian group with τ generators admit a countably compact group topology? [93]

41. *(H. Teng)* Let X be fortissimo space and p the particular point of X. Let $Y = X \setminus \{p\}$. Is $C_p(Y|X)$ normal? [93]

42. *(H. Teng)* With X and Y as in the previous problem, is $C_p(Y|X)$ homeomorphic to the Σ-product of $|X|$-many real lines? [93]

O. Theory of Retracts; Extension of Continuous Functions

14. *(Friedler, Girou, Pettey, and Porter)* Let Y be an R-closed [*resp.* U-closed] extension of a space X and f a continuous function from X to an R-closed [*resp.* U-closed] space Z. Find necessary and sufficient conditions that f can be extended to a continuous function from Y to Z. [93]

15. *(Ryszard J. Pawlak)* Characterize those spaces which possess Borel Darboux retracts. [93]

See also G29 and G30.

P. Products, Hyperspaces, Remainders, and Similar Constructions

33. *(Tim LaBerge)* Is there a countable collection $\{A_n : n \in \omega\}$ of non-Lindelöf ACRIN spaces whose topological sum is ACRIN? [93]

34. *(Tim LaBerge)* Is there an ACRIN space X such that $X + X$ is not ACRIN? [93]

35. *(Tim LaBerge)* Are there Lindelöf spaces X and Y such that $X \times Y$ is ACRIN but not Lindelöf? [93]

R. Dimension Theory

7. *(Takashi Kimura)* Does there exist a normal (or metrizable) space X having *trind* such that every compactification of X fails to have *trind*? [93]

T. Algebraic and Geometric Topology

14. *(Stasheff)* The structure of a (based) loop space ΩX allows the reconstruction of a space BY of the homotopy type of X. The parametrization of higher homotopies by the associahedra plays a crucial role. Does the joining of closed strings (= free loops) described in my talk lead in an analogous way to constructing from a free loop space $Z = \mathcal{L}X$ a space of the homotopy type of X? perhaps with the moduli space described in the article or some variant playing the role of the associahedra? [93]

Y. Topological Games

6. *(M. Scheepers)* Let λ be an uncountable cardinal of uncountable cofinality. Let κ be a cardinal such that $\lambda^{<\lambda} < cof([\kappa]^\lambda, \subset) \leq 2^\lambda$. Does TWO have a winning remainder strategy in any of $WMEG([\kappa]^\lambda)$, $WMG([\kappa]^\lambda)$ or $VSG[\kappa]^\lambda$? [93]