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PROBLEM SECTION

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Editor's Note: After twenty years as your Problems Editor, I am turning the Problem Section over to John Mayer. It has been a most rewarding experience for me, and I hope John will find it one too. I will continue to contribute problems and information about results on earlier problems to this journal through him and encourage everyone reading this to do the same. Next year and the following year, I plan to do a retrospective, "The Classical Problems–Twenty Years Later" for the Problems Section and encourage anyone with information on these problems (which appeared in Volumes 1 and 2) to send them to:

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CONTRIBUTED PROBLEMS

Many problems in this subsection are related to a paper by the contributor in this volume, where additional information about the problems can be found.

B. Generalized Metric Spaces and Metrization

37. (*Shou Lin, "On spaces wsith a k-network consisting of compact subsets"*) Suppose X is a space with a point-countable closed k-network. Does X have a point countable compact k-network if every first countable closed subspace of X is locally compact?

38. (*ibid.*) Suppose X is a quotient s-image of a metric space. Does X have a point-countable closed k-network if every first countable closed subspace of X is locally compact?

39. (*ibid.*) Supposse X has a σ-closure-preserving compact k-network. Is X a k-space if X is a k_R-space?

E. Separation and Disconnectedness

13. (*A. Arhangel'skiĭ, "Topological Homogeneity"*) Is there in ZFC a non-discrete extremally disconnected topological group?

G. Mappings of Continua and Euclidean Spaces

31. (*Donna Saritz*) If a homogeneous compact metric space is locally n-connected for all n, is the space necessarily 2-homogeneous?

See also Problem M13 and R11.

H. Homogeneity and Mappings of General Spaces

31. (*A. Arhangel'skiĭ, "Topological Homogeneity"*) Let X be an infinite homogeneous compactum. Is there a non-trivial convergent sequence in X? What if we assume X to be 2-homogeneous? countable dense homogeneous?

32. (*ibid.*) Is there a homogeneous compactum of cellularity greater than 2^ω? One that is 2-homogeneous? Negative answers would imply negative ones to the respective parts of the following problem.

33. (*ibid.*) Can every compactum be represented as a continuous image of a homogeneous compactum? Of a 2-homogeneous compactum?

34. (*ibid.*) Is every first countable compactum the continuous image of a first countable homogeneous compactum? [Yes if CH.]

35. (*ibid.*) Is every separable space [*resp.* separable compactum] the continuous image of a countable dense homogeneous space [*resp.* compactum]?

36. (*ibid.*) If Y is a zero-dimensional compactum, is there a compactum X such that $X \times Y$ is homogeneous? 2-homogeneous?

37. (*ibid.*) If Y is a Tychonoff space, is there a Tychonoff space X such that $X \times Y$ is 2-homogeneous?

38. (*ibid.*) Let Y be a compactum. Is there a homogeneous compactum X which contains an l-embedded topological copy of Y? A t-embedded topological copy?

39. (*Dennis Garity, "On Finite Products of Menger Spaces and 2-Homogeneity"*) Is there a compact metric space of dimension less than $(n + 2)$ that is homogeneous, locally n-connected, and not 2-homogeneous?

40. (*ibid*) If a homogeneous compact metric space is locally n-connected for all n, is the space necessarily 2-homogeneous?

See also E13, G31, and R11.

M. Manifolds and Cell Complexes

11. (*Chris Good, "Dowker Spaces, Anti-Dowker Spaces, etc."*) Is there a hereditarily normal Dowker manifold?

12. (*Beverly Brechner and Joo S. Lee, "A Three Dimensional Prime End Theory"*) Characterize those bounded domains U in E^3 which admit a prime end structure.

13. (*ibid.*) Characterize those bounded domains U in E^3 which admit a C-transformation onto the interior of some compact 3-manifold.

See also P36 and P37.

P. Products, Hyperspaces, Remainders and Similar Constructions

36. (*Chris Good, "Dowker Spaces, Anti-Dowker Spaces, etc."*) Can the square of a perfectly manifold be a Dowker space? [No if $MA + \neg CH$ because it implies perfectly normal manifolds are metrizable.]

37. (*ibid.*) Does $MA + \neg CH$ imply the existence of a Dowker manifold, or even a locally compact Dowker space?

38. (*ibid.*) If X is a normal, countably paracompact space and X^2 is normal, does $MA + \neg CH$ imply X^2 is countably paracompact? What if X is also perfectly normal?

39. (*ibid.*) Is there a Dowker space X such that X^2 is Dowker? Such that X^n is Dowker for all finite n?

40. (*ibid.*) Can the square of a monotonically normal space or of a Lindelöf space be Dowker?

R. Dimension Theory

9. (*V. A. Chatyrko, "On the transfinite dimension dim and essential mappings"*) If C is the Cantor set, is $trdim X = trdim(X \times C)$? [Yes if $trdim(X \times C) \geq \omega^2$: see the article.]

10. (*ibid.*) Is it true that if X is a space and α is a countable ordinal number $\geq \omega^2$, then $trdim X \geq \alpha$ iff X admits an essential map onto Henderson's cube H^α? [Yes for limit ordinals, see article.]

11. (*Donna Saritz*) Is there a homogeneous compact metric space of dimension less than $n+2$ that is locally n-connected but not 2-homogeneous?

12. (*Takashi Kimura "A note on compactification theorem for trdim"*) Does there exists a S-w. i. d. space X such that trdim $X \geq w(X)^+$?

U. Uniform Spaces and Generalizations

4. (*Hans-Peter Kunzi, " The Bourbaki Quasi-Uniformity"*) Try to characterize those properties P of quasi-uniform spaces $(X,\mathcal{U})$ that fulfill the following condition: $(X,\mathcal{U})$ has Property P whenever $(\mathcal{P}_0(X),\mathcal{U}_*)$ has Property P.

PROBLEMS FROM OTHER SOURCES

At the AMS Regional Meeting in Chattanooga, Tennessee, October 11–12, 1996, during the Special Session in Set-theoretic Topology, there was a problem session at which the following problems were posed.

1. (*W. W. Comfort, attributed to N. Noble*) Can there be an uncountable family of noncompact Tychonoff spaces whose

product is a k-space? [Transmitted by P. Nyikos. *Remarks.* N. Noble showed in his Ph.D. thesis that a co-countable subfamily must have pseudocompact product, hence all but countably many factors must be countably compact.]

2. (*F. D. Tall, attributed to W. Fleissner*) Is there a normal k-space which is not collectionwise normal? *Remarks.* Peg Daniels has shown the consistency of every normal k'-space being collectionwise normal, assuming large cardinal axioms.

3. (*D. J. Lutzer*) Can every perfectly normal suborderable space be embedded in a perfectly normal LOTS?

4. (*D. J. Lutzer*) Let X be a suborderable space with a σ-discrete dense subspace. Can X be embedded in a perfectly normal LOTS? a perfectly normal LOTS with a σ-discrete dense subspace?

5. (*Chunliang Pan*) Dowker showed that a space X is normal and countably paracompact if, and only if, it is possible to choose, for each USC real-valued function g and each LSC real-valued function h such that $g(x) < h(x)$ for all x, a continuous function $\Phi(g,h)$ such that $g < \Phi(g,h) < h$ everywhere. Can we characterize internally those spaces X for which this choice can be done monotonically, i.e. if $g < g'$ and $h < h'$ then $\Phi(g,h) < \Phi(g',h')$ everywhere? *Remarks.* If $\leq$ is substituted for $<$ everywhere, then we get a condition equivalent to perfect normality.

6. (*G. Gruenhage, attributed to R. McCoy*) Find a property P such that X has P iff $C(X)$ with the compact-open topology is a Baire space. Does the Moving-Off Property (MOP) provide such a characterization? [*Remarks.* If $C(X)$ is Baire in the compact-open topology, then X has the MOP, which is the property that eveery collection $\mathcal{L}$ of compact sets that moves off the compact sets contains an infinite subcollection with a discrete open expansion. A family $\mathcal{L}$ is said to *move off the compact sets* if for each compact subset K of X there is a member of $\mathcal{L}$ that is disjoint from it.]

7. (*S. Purisch*) Can we characterize the compact spaces of

diversity 2, i.e. those compact spaces with exactly two open subspaces up to homeomorphism?

8. (*P. J. Nyikos, attributed to A. Dow and K.-P. Hart*) If a continuum is the continuous image of the Stone-Čech remainder ω^*, is it the continuous image of the Stone-Čech remainder H^* of the closed half-line?

INFORMATION ON EARLIER PROBLEMS

Classic Problem III, *vol. 1, p. 363* Is every screenable normal space paracompact? *Solution.* No (*Z. Balogh*). Earlier, M. E. Rudin had shown a negative answer to be consistent.

Problem 8 of "Basic Problems in General Topology", by S. Watson, *vol. 15, p. 218* Are locally compact normal metacompact spaces paracompact? *Solution.* This is ZFC-independent. (*G. Gruenhage and P. Koszmider*) Yes if $\mathfrak{p} > \omega_1$, no in a forcing model constructed by Gruenhage and Koszmider.