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Making Some Ideals Meager On Sets Of Size Of The Continuum

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Abstract

We obtain results connected with the recent studies of Ciesielski and Jasiński [CJ]. An ideal I on a set X is called τ-meager if the τ-meager sets are exactly the sets in I. Using a modified method of the classical Sierpiński duality theorem [S], we show that (under some set-theoretical assumptions) several ideals on X with $|X| = \mathfrak{c}$ are τ-meager where τ is a topology on X homeomorphic to the natural topology on $\mathbb{R}$, or to the density topology on $\mathbb{R}$. We also prove that, if I is a Σ_2^0-supported ccc σ-ideal containing all singletons in a Polish space X, then there exists a Polish topology τ on X which makes I meager and has the same Borel sets as the original topology. That improves the earlier result of the second author [R].

1 Introduction

We use the standard set-theoretical notation (see [K]). By $\mathfrak{c}$ we denote the cardinality of the continuum. For an ideal I of subsets of a fixed nonempty set X, we say that a topology τ on X *makes I meager* (*nowhere dense*) if I is exactly the family of all τ-meager (τ-nowhere dense) sets. In [CJ] the authors considered the question how to find the "best" possible topology τ with the above properties when I is a fixed given ideal. The answers are obtained in several general and more particular cases. In the present paper we show some new results of that type. We only study the situation when $|X| = \mathfrak{c}$ and I contains all singletons. Of course, I is assumed proper, i.e. I is different from $\mathcal{P}(X)$ (the power set of X).

Recall some observations and results of Ciesielski and Jasiński [CJ]:

(1) For each σ-ideal I on X, the family $\tau = \{X \setminus A : A \in I\} \cup \{\emptyset\}$ is a topology making I meager and nowhere dense; then τ is T_1 but not T_2. See [CJ, Facts 1.1 and 1.6]. It is a simple exercise.

(2) Under CH, for each σ-ideal I on a set X of cardinality $\mathfrak{c}$, there exists a Hausdorff topology τ on X making I meager [CJ, Th.3.11]. The proof is based on the existence of a Luzin space (under CH).

(3) $MA + \neg CH$ implies that there is no uncountable Hausdorff space X whose topology makes the ideal of all countable sets in X meager [CJ, Fact 3.7]. It is due to Kunen [K1].

(4) Under CH, for each σ-ideal I on $\mathbb{R}$ having cofinality ω_1, there exists a zero-dimensional Hausdorff topology (thus $T_{3.5}$) on $\mathbb{R}$ making I meager and nowhere dense [CJ, Corollary 4.2]. This follows from a deep result of Ciesielski and Jasiński [CJ, Th.3.12] proved by the technique of forcing.

2 Application of the Sierpiński duality theorem

In this section, we fix a set X with $|X| = \mathfrak{c}$. For an ideal I on X, we define

$$add(I) = \min\{|\mathcal{F}| : \mathcal{F} \subseteq I \ \& \ \bigcup \mathcal{F} \notin I\},$$

$$cof(I) = \min\{|\mathcal{F}| : \mathcal{F} \subseteq I \;\&\; (\forall A \in I)\,(\exists B \in \mathcal{F})\,(A \subseteq B)\}.$$

(See e.g. [F] or [V].) We say that a family $\mathcal{F} \subseteq I$ forms a *base* of I if each $A \in I$ is contained in a $B \in \mathcal{F}$ such that $A \subseteq B$. So, $cof(I)$ is the minimal cardinality of a base of I. We say that two ideals I and J on X are *isomorphic* if there is a bijection f from X onto X such that $E \in I$ iff $f[E] \in J$, for each $E \subseteq X$. We shall say that an ideal I on X *admits* a $\mathfrak{c}$-*tower* if there is a family $\mathcal{F} = \{B_\alpha : \alpha < \mathfrak{c}\} \subseteq I$ (called a $\mathfrak{c}$-*tower* for I) such that:

(i) $\bigcup \mathcal{F} = X$,

(ii) $B_\gamma \subseteq B_\alpha$ for any $\gamma < \alpha < \mathfrak{c}$,

(iii) $|B_0| = |B_{\alpha+1} \setminus B_\alpha| = \mathfrak{c}$ for each α, $0 < \alpha < \mathfrak{c}$,

(iv) $B_\alpha = \bigcup_{\gamma<\alpha} B_\gamma$ for each limit ordinal α, $0 < \alpha < \mathfrak{c}$,

(v) $\mathcal{F}$ is a base of I.

Note that, if an ideal I admits a $\mathfrak{c}$-tower, then it forms a σ-ideal. Our notion of $\mathfrak{c}$-tower is different from the usual notion of a tower used in combinatorics on ω (cf. [V]).

The origins of ideas presented in Propositions 2.1 and 2.2 (given below) come from the classical theorem of Sierpiński [S] about duality between small sets in the sense of measure and category, when CH is assumed. (See also [O] where the Sierpiński-Erdös theorem is shown.) Some years ago, the first author learnt from L. Bukovský (Košice) about assumptions weaker than CH in theorems of that type. Some modifications and extensions of the Sierpiński-Erdös duality theorem can be found in [BJ], [CKW], [M], [B]. Our version contained in Proposition 2.2 uses the notion of $\mathfrak{c}$-tower. For recent applications of $\mathfrak{c}$-towers, see [BR].

Proposition 2.1 *If I is an ideal on X such that:*

(a) $\bigcup I = X$,

(b) $add(I) = cof(I) = \mathfrak{c}$,

(c) $(\forall A \in I)(\exists D \in I)(D \cap A = \emptyset \ \&\ |D| = \mathfrak{c})$,

then I admits a $\mathfrak{c}$-tower. If $\mathfrak{c}$ is regular, the converse is also true.

Proof: Since (by (b)) $cof(I) = \mathfrak{c}$, we can pick a base $\{A_\alpha : \alpha < \mathfrak{c}\}$ of I. We define sets B_α, $\alpha < \mathfrak{c}$, as follows. Let $B_0 = A_0 \cup D_0$ where D_0 is the respective set D choosen in (c) if $A = A_0$. Assume that $0 < \alpha < \mathfrak{c}$ and that the sets B_γ, $\gamma < \alpha$, are defined. If α is a limit ordinal, we put $B_\alpha = \bigcup_{\gamma<\alpha} B_\gamma$. Otherwise, let $B_\alpha = B_{\alpha-1} \cup A_{\alpha-1} \cup A_\alpha \cup D_\alpha$ where D_α is the respective set D chosen in (c) if $A = B_{\alpha-1} \cup A_{\alpha-1} \cup A_\alpha$. In any case, $B_\alpha \in I$ since $add(I) = \mathfrak{c}$. It is easy to check that $\{B_\alpha : \alpha < \mathfrak{c}\}$ forms a $\mathfrak{c}$-tower for I.

To show the converse, observe that (a) follows from (i), and (c) follows from (iii). Also, $add(I) \le cof(I) \le \mathfrak{c}$ is clear. From (ii) and the regularity of $\mathfrak{c}$ it is not hard to infer that $add(I) = \mathfrak{c}$. $\square$

Remark 2.1 *Assume that X is an uncountable Polish space and I is a σ-ideal on X such that I has a base consisting of coanalytic sets, and each perfect set in X contains a perfect set from I. Then I fulfils (c) from Proposition 2.1. Indeed, if $A \in I$, there is a coanalytic set $B \in I$ in a base of I such that $A \subseteq B$. Then $X \setminus B$ is an uncountable analytic set, and thus, by the Souslin theorem [Ku, §39 I], it contains a perfect set P. So, it suffices to pick a perfect set $D \subseteq P$ belonging to I. Since $\omega_1 \le add(I) \le cof(I) \le \mathfrak{c}$ therefore condition (b) of Proposition 2.1 holds, if one assumes CH.*

Proposition 2.2 *If two ideals I and J on X admit $\mathfrak{c}$-towers, then they are isomorphic.*

Proof: Let $\mathcal{F}(I) = \{B^I_\alpha;\ \alpha < \mathfrak{c}\}$ and $\mathcal{F}(J) = \{B^J_\alpha;\ \alpha < \mathfrak{c}\}$ be $\mathfrak{c}$-towers for I and J. Fix bijections $f_{-1} : B^J_0 \to B^J_0$ and $f_\alpha : (B^J_{\alpha+1} \setminus B^J_\alpha) \to (B^J_{\alpha+1} \setminus B^J_\alpha)$ for each $\alpha < \mathfrak{c}$. Then $f = f_{-1} \cup \bigcup_{\alpha<\mathfrak{c}} f_\alpha$ shows that I and J are isomorphic. □

Let $\mathbb{K}$ and $\mathbb{L}$ denote the ideals of meager and of Lebesgue null sets in $\mathbb{R}$, respectively. For information about the density topology we refer the reader to [CLO].

Theorem 2.1 *Let I be an ideal on X which admits a $\mathfrak{c}$-tower.*

(I) If $add(\mathbb{K}) = cof(\mathbb{K}) = \mathfrak{c}$, there exists a topology on X, homeomorphic to the natural topology on $\mathbb{R}$, and making I meager.

(II) If $add(\mathbb{L}) = cof(\mathbb{L}) = \mathfrak{c}$, there exists a topology on X homeomorphic to the density topology on $\mathbb{R}$ (thus $T_{3.5}$), and making I meager and nowhere dense.

Proof: We may assume that $X = \mathbb{R}$. Note that $\mathbb{K}$ and $\mathbb{L}$ admit $\mathfrak{c}$-towers, provided that $add(\mathbb{K}) = cof(\mathbb{K}) = \mathfrak{c}$ and $add(\mathbb{L}) = cof(\mathbb{L}) = \mathfrak{c}$, since condition (a) in Proposition 2.1 is evident, and (c) follows from Remark 2.1. Thus, by Proposition 2.2, there are bijections $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ witnessing that the ideals in the pairs $\mathbb{K}$, I and $\mathbb{L}$, I are isomorphic. Let τ_n and τ_d denote the natural and the density topologies on $\mathbb{R}$. Note that τ_d makes $\mathbb{L}$ meager and nowhere dense, and τ_d is $T_{3.5}$ (see [CLO, Th.1.2.3]). Now, it is clear that $\{f[U] : U \in \tau_n\}$ and $\{g[U] : U \in \tau_d\}$ are topologies good for (I) and (II), respectively (the functions f and g form the corresponding homeomorphisms). □

Remark 2.2 *Both assumptions $add(\mathbb{K}) = cof(\mathbb{K}) = \mathfrak{c}$ and $add(\mathbb{L}) = cof(\mathbb{L}) = \mathfrak{c}$ are somewhat weaker than CH or MA* (see [F] and [BJ]). *Thus, comparing Theorem 2.1(II) with the fact (4) quoted in Section 1, we see that our set-theoretical assumptions are different and our topology τ is not zero-dimensional (since the density topology is not zero-dimensional).*

Note that it is impossible to prove in ZFC the existence of an ideal I on X with $|X| = \mathfrak{c}$, fulfilling $\bigcup I = X$ and add(I) = cof(I) = $\mathfrak{c}$. Indeed, it is enough to consider a model of ZFC in which $\mathfrak{c}$ is singular. Since add(I) is always regular (see e.g. [BJ]), in that model we have add(I) $< \mathfrak{c}$.

If CH holds, we derive from Remark 2.1 a simple application of Theorem 2.1.

Corollary 2.1 *Assume CH. Let I be a σ-ideal on an uncountable Polish space X, containing all singletons, possessing a base of coanalytic sets, and such that each perfect subset of X contains a perfect set in I. Then there are topologies τ_1 and τ_2 on X homeomorphic (respectively) to the natural topology on $\mathbb{R}$ and to the density topology on $\mathbb{R}$, and making I (respectively) meager, and meager nowhere dense.*

Example 2.1 Let I be the σ-ideal of all σ-porous sets in $\mathbb{R}$. Recall (cf. [Z]) that a subset of $\mathbb{R}$ is called *σ-porous* if it is a countable union of porous sets. A set $E \subseteq \mathbb{R}$ is called *porous* if $\limsup_{r\to 0^+}(\gamma(E,x,r)/r) > 0$ for each $x \in E$ where $\gamma(E,x,r)$ is the length of the longest interval $(a,b) \subseteq (x-r,x+r)\setminus E$ (or, $\gamma(E,x,r) = 0$ if there is no such interval). Note that I has a base consisting of Borel sets [FH]. It is easy to check that each perfect subset of $\mathbb{R}$ contains a perfect porous set. (See also [BW] where a stronger result is shown.) Thus, if CH holds, I fulfils the assertion of Corollary 2.1. Observe that here CH is really needed to get $add(I) = cof(I)$ since Brendle [Br] proved (in ZFC) that $add(I) = \omega_1$ and $cof(I) = \mathfrak{c}$.

An example of a σ-ideal I where an application of Theorem 2.1 deos not need CH will be given at the end of Section 3.

3 Σ^0_2-supported ccc σ-ideals

In this section we assume that X is an uncountable Polish space (thus $|X| = \mathfrak{c}$). For the basic facts and notation concerning descriptive set theory we refer the reader to [Mo] or [Ku]. An ideal I on X is called Σ_2^0-*supported* if I has a base consisting of F_σ sets (i.e. of sets from the class Σ_2^0, according to the notation from [Mo]). In [KS], the authors obtained a deep theorem classifying Σ_2^0-supported σ-ideals containing all singletons on X. Here we will work only with Σ_2^0-supported ccc σ-ideals. Recall that I fulfils the *countable chain condition* (or is a ccc ideal) if each disjoint family of Borel sets in X that are not in I is countable. For a nonempty family $\mathcal{F} \subseteq \mathcal{P}(X)$ let $MGR(\mathcal{F})$ consist of all sets $E \subseteq X$ such that $E \cap F$ is meager in F for each $F \in \mathcal{F}$. The following proposition can be derived immediately from the above-mentioned theorem of Kechris and Solecki [KS, Th.2].

Proposition 3.1 *A σ-ideal I containing all singletons on X forms a Σ_2^0-supported ccc σ-ideal if and only if $I = MGR(\mathcal{F})$ for a countable family $\mathcal{F} = \{F_\gamma : \gamma < \alpha\}$, $\alpha < \omega_1$, of nonempty closed sets in X such that $F_\gamma \subseteq F_\beta$ whenever $\beta < \gamma < \alpha$, and $F_{\gamma+1}$ is nowhere dense in F_γ for $\gamma + 1 < \alpha$.*

Corollary 3.1 *If I is a ccc Σ_2^0-supported σ-ideal containing all singletons in X, then $I = MGR(\mathcal{F}^*)$ where $\mathcal{F}^* = \{F^*_{\gamma+1} : \gamma < \alpha\}$, $\alpha < \mathfrak{c}$, is a countable family of dense-in-itself uncountable sets of type F_σ and G_δ, pairwise disjoint, contained in X and such that $X \setminus \bigcup \mathcal{F}^*$ is of type F_σ and G_δ.*

Proof: We may assume that $I = MGR(\mathcal{F})$ where $\mathcal{F} = \{F_\gamma : \gamma < \alpha\}$ satisfies all requirements given in Proposition 3.1. We will modify the family $\mathcal{F}$ as follows. Let additionally $F_\alpha = \emptyset$. Define $D_0 = X$; $D_{\gamma+1} = F_\gamma$ if $\gamma \leq \alpha$, and $D_\lambda = \bigcap_{\gamma<\lambda} F_\gamma$ if $\lambda \leq \alpha$ is a limit ordinal. Note that $D_\gamma \subseteq D_\beta$ whenever $\beta < \gamma \leq \alpha + 1$, and all sets D_γ are closed (which follows from the properties of the sets F_γ). Put $F^*_\gamma = D_\gamma \setminus D_{\gamma+1}$ for all $\gamma \leq \alpha$. Then the sets F^*_γ are of type F_σ and G_δ, pairwise disjoint, and $\bigcup_{\gamma\leq\alpha} F^*_\gamma = X$. Let $\mathcal{F}^* = \{F^*_{\gamma+1} : \gamma < \alpha\}$. Fix

$\gamma < \alpha$. Since $F_\gamma \neq \emptyset$ and $F_{\gamma+1}$ is nowhere dense in F_γ, we have $F^*_{\gamma+1} = D_{\gamma+1} \setminus D_{\gamma+2} = F_\gamma \setminus F_{\gamma+1} \neq \emptyset$. Next, observe that F_γ is perfect. Indeed, if x is an isolated point of F_γ, then $\{x\} \notin I$ since $\{x\}$ is nonmeager in F_γ by the Baire category theorem. It yields a contradiction since we have assumed that I contains all singletons. As F_γ is perfect, $F_\gamma \setminus F_{\gamma+1}$ is dense in itself of size $\mathfrak{c}$. Further, note that $X \setminus \bigcup \mathcal{F}^* = \bigcup\{F^*_\gamma : \gamma \leq \alpha, \gamma \text{ is a limit ordinal }\}$ is of type F_σ (as a countable union of F_σ sets) and of type G_δ (as the complement of the set $\bigcup \mathcal{F}^*$ of type F_σ). Since $F^*_{\gamma+1} = F_\gamma \setminus F_{\gamma+1}$ and $F_{\gamma+1}$ is nowhere dense in F_γ (for each $\gamma < \alpha$), we have $A \cap F_\gamma$ is meager in F_γ iff $A \cap F^*_{\gamma+1}$ is meager in $F^*_{\gamma+1}$, for any $A \subseteq X$ and $\gamma < \alpha$. Consequently, $MGR(\mathcal{F}^*) = MGR(\mathcal{F}) = I$. □

Remark 3.1 *The converse of the implication given in Corollary 3.1 is also true. Indeed, let $I = MGR(\mathcal{F}^*)$. Since $\mathcal{F}^* \cup \{X \setminus \mathcal{F}^*\}$ is a countable partition of X consisting of F_σ sets, and each meager set is countained in an F_σ meager set, therefore I forms a Σ^0_2- supported σ-ideal (note that $X \setminus \bigcup \mathcal{F}^* \in I$). Also I is a ccc ideal because $\mathcal{F}^*$ is a countable family of G_δ sets (thus Polish spaces) and the ideal of meager sets in a Polish space is ccc.*

Theorem 3.1 *For each Σ^0_2-supported ccc σ-ideal I containing all singletons in an uncountable Polish space X, there exists a Polish topology τ on X making I meager and having the same Borel sets as the original topology on X.*

Proof: We may assume that $I = MGR(\mathcal{F}^*)$ where $\mathcal{F}^* = \{F^*_{\gamma+1} : \gamma < \alpha\}$, $\alpha < \mathfrak{c}$, satisfies all conditions given in Corollary 3.1. Let $E = X \setminus \bigcup \mathcal{F}^*$. Since F^*_1 is an uncountable Borel set (by Corollary 3.1), we can pick a closed set $D \subseteq F^*_1$ homeomorphic to the Cantor set and nowhere dense in F^*_1 (see [Ku, §37, Th.3]). Consider a Borel isomorphism g from $E \cup D$ onto

D [Ku, §37, II]. Then define $f : E \cup F_1^* \to F_1^*$ by

$$f(x) = \begin{cases} g(x), & \text{if } x \in E \cup D \\ x, & \text{otherwise.} \end{cases}$$

It is clear that f is a bijection and $f[E \cup D] = D$. Additionally f forms a Borel isomorphism which maps $E \cup F_1^*$ onto F_1^*.

Let ρ denote the original (complete and separable) metric on X. Since each set $F_{\gamma+1}^*$, $\gamma < \alpha$, is of type G_δ in $\langle X, \rho \rangle$, we can change the original metric on $F_{\gamma+1}^*$ to an equivalent metric $\rho_{\gamma+1}$ making $F_{\gamma+1}^*$ a complete space (by the Alexandrov theorem [Ku, §33, VI]). Plainly, we may assume that $\rho_{\gamma+1} \leq 1$ for each $\gamma < \alpha$. Finally, we define a new metric ρ^* on X by

$$\rho^*(x, y) = \begin{cases} \rho_1(f(x), f(y)), & \text{if } x, y \in E \cup F_1^*; \\ \rho_{\gamma+1}(x, y), & \text{if } x, y \in F_{\gamma+1}^*, \ 0 < \gamma < \alpha; \\ 1 & \text{otherwise.} \end{cases}$$

Then $\langle X, \rho^* \rangle$ is a direct sum [E, Th.4.2.1] of sets $E \cup F_1^*$ and $F_{\gamma+1}^*$ (for $0 < \gamma < \alpha$) which with the respective metrics form complete and separable spaces. Thus $\langle X, \rho^* \rangle$ is complete and separable, and the topology τ generated by ρ^* is Polish.

If $A \subseteq X$ then $A = (A \cap E) \cup \bigcup_{\gamma<\alpha} A_{\gamma+1}$ where $A_{\gamma+1} = A \cap F_{\gamma+1}^*$ for $\gamma < \alpha$. From the choice of ρ^* it follows that $A_{\gamma+1}$ (for $\gamma < \alpha$) is meager in $\langle X, \rho^* \rangle$ iff $A_{\gamma+1} \cap F_{\gamma+1}^*$ is meager in $F_{\gamma+1}^*$ with the metric ρ. This and the choice of D show that $I = MGR(\mathcal{F}^*)$ equals the family of all meager sets in $\langle X, \rho^* \rangle$. Since the partition $\{E \cup F_1^*\} \cup \{F_{\gamma+1}^* : 0 < \gamma < \alpha\}$ of X consists of sets that are Borel in both spaces $\langle X, \rho \rangle$ and $\langle X, \rho^* \rangle$, and f is a Borel isomorphism, one can easily check that the above two spaces have the same Borel sets. $\square$

Remark 3.2 *(a) Theorem 3.1 improves the former result of the second author* [R] *where the completeness of a new topology on X is not obtained, and Borel sets in both topologies were not compared. The present proof is different.*

(b) Facts (2) and (3) quoted in Section 1 show that the existence of a Hausdorff topology making the σ-ideal (Σ_2^0-supported, not ccc) of all countable sets on $\mathbb{R}$ meager is independent of ZFC.

Example 3.1 Let I denote the σ-ideal of all sets $E \subseteq \mathbb{R}$ that can be included in F_σ Lebesgue null sets. Then I is Σ_2^0-supported but it does not fulfil ccc. It was shown in [BS] that $add(I) = add(\mathbb{K})$ and $cof(I) = cof(\mathbb{K})$. Thus, by Theorem 2.1(I) and Remark 2.1, condition $add(\mathbb{K}) = cof(\mathbb{K}) = \mathfrak{c}$ (weaker than CH) implies that there is a topology on $\mathbb{R}$, homeomorphic to the natural one, making I meager. From Cichoń's diagram (see [F],[BJ] or [CKW]) it follows that $add(\mathbb{L}) = cof(\mathbb{L})$ implies $add(\mathbb{K}) = cof(\mathbb{K})$ (thus also $add(I) = cof(I)$). Hence, if $add(\mathbb{L}) = cof(\mathbb{L}) = \mathfrak{c}$ holds (which is weaker than CH), then, by Theorem 2.1(II) and Remark 2.1, there is a topology on $\mathbb{R}$, homeomorphic to the density topology, making I meager and nowhere dense.

Problem Let I be as in Example 3.1. Does there exist (within ZFC) a Hausdorff (T_3, $T_{3.5}$,..., metric) topology on $\mathbb{R}$ making I meager?

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