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PROBLEM SECTION

Editor's Note: The Problems Section has been guided by the able hand of Peter Nykios for 20 years. Your new Problems Editor can only hope to do as well. Thanks, Peter.

Contributed Problems

The Problems Editor invites anyone who has published a paper in *Topology Proceedings* or has attended a Spring Topology Conference to submit problems to this section. They need not be related to any articles which have appeared in *Topology Proceedings* or elsewhere, but if they are, please provide references. Please define any terms not in a general topology text nor in referenced articles.

The system for indexing topics and questions of the previous Problems Editor has been continued for this volume. Problems which are stated in, or relevant to, a paper in this volume are accompanied by the title of the paper where further information about the problems may be found. Comments of the proposer or submitter of the question are so noted; comments of the Problems Editor are not specially noted. Information on the status of previously posed questions is always welcome. Submission of questions and comments by email in TEX form is strongly encouraged, either to topolog@mail.auburn.edu or directly to the Problems Editor at mayer@math.uab.edu.

B. Generalized Metric Spaces and Metrization

40. (H. H. Hung) Is there a metrization theorem in terms of weak, non-uniform factors?

Comments of the proposer. This paper [*A Note on a Recent Metrization Theorem*] underlines once again the desirability of a non-uniform metrization theorem [Szekszárd, *Topology and Its Applications* (1993), 259–265], 1.1 being uniform following immediately from Corollary 2.3 of [*Canad. J. Math.* **29** (1977), 1145–1151], and 0.2 non-uniform.

C. Compactness and Generalizations

65. (Tzannes Vasilis, *A Hausdorff countably compact space on which every continuous real-valued function is constant*) Does there exist a regular (first countable, separable) countably compact space on which every continuous real-valued function is constant?

66. (Tzannes Vasilis, *ibid.*) Does there exist for every Hausdorff space R, a regular (first countable, separable) countably compact space on which every continuous function into R, is constant?

67. (Maddalena Bonanzinga, *More on the Property of a Space Being Lindelöf in Another*) Characterize Hausdorff (regular, normal) spaces which can be represented as closed subspaces of Hausdorff (regular, normal) star-Lindelöf spaces.

68. (Maddalena Bonanzinga, *ibid.*) How big can be the extent of a Hausdorff (regular, normal) star-Lindelöf space?

67–68. Comments of the proposer. We say that a space is *star-Lindelöf* if for every open cover $\mathcal{U}$ of X there exists a countable subset $F \subset X$ such that $St^1(F,\mathcal{U}) = X$. Star-Lindelöfness is a joint generalization of Lindelöfness, countable compactness and separability. Partial answers to Questions 67 and 68 were obtained in M. Bonanzinga, Star-Lindelöf and absolutely star-Lindelöf spaces, to appear in *Questions and Answers in General Topology.*

See also Question P. 41, this volume.

D. Paracompactness and Generalizations

42. (Peter Nykios) Does V=L imply that first countable, countably paracompact spaces are strongly collectionwise Hausdorff?

43. (Paul J. Szeptycki) Are first countable, countably paracompact, collectionwise Hausdorff spaces strongly collectionwise Hausdorff?

42–43. Comments of the proposer (Szeptycki). A space is *strongly collectionwise Hausdorff* if closed discrete sets can be separated by a discrete family of open sets. The structure of closed discrete sets in first countable spaces has a long and interesting history beginning with the Normal Moore Space Conjecture. The question whether normal, first countable spaces are collectionwise Hausdorff and whether countably paracompact, first countable spaces are collectionwise Hausdorff is particularly interesting. A series of results by D. K. Burke, W. G. Fleissner, P. Nykios, F. D. Tall, and S. Watson address these questions under V=L, PMEA, and other assumptions. Question 42 of Nykios appears to be one of the last important questions concerning the effect of V=L on the separation of closed discrete sets in first countable spaces.

While Burke has shown that PMEA provides a consistent positive answer (even without the assumption of collectionwise Hausdorff), a positive answer to Question 43 assuming V=L would yield a positive answer to Nykios's question, However, any consistent counterexample would go a long way toward clarifying the distinction between normality and countable paracompactness. Note that the assumption of first countability is essential as a ZFC example with uncountable character has been constructed [S. Watson, Comments on separation, this journal, **14** (1989), 315–372]. Also, if we weaken countable paraompactness to paranormality in Question 43, we get a consistent negative answer [Szeptycki, Paranormal spaces in the constructible universe, this journal, **21** (1996)].

P. Products, Hyperspaces, Remainders, and Similar Constructions

41. (Maddalena Bonanzinga, *ibid.*) Does there exist a ZFC example of two star-Lindelöf topological groups G and H such that the product $G \times H$ is not star-Lindelöf?

See definition following Question C. 68, this volume.

42. (Don A. Mattson, *Rimcompact spaces as remainders of compactifications*) Can a nowhere rimcompact space have a compactification with zero dimensional remainder?

Z. Topological Dynamics, Fractals, and Hausdorff Dimension

4. (Jacek Graczyk and Grzegorz Swiatek) Is there a complex bounds theorem for all real polynomials. including the polymodal ones?

Comments of the proposers. In this case, does it help to assume that all critical values are real? Note that in the polymodal case. it is not immediately clear what the statement of the theorem should be.