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MILYUTIN MAPPINGS AND THEIR APPLICATIONS

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Abstract

This is a survey on Milyutin mappings and their applications in the geometry of Banach spaces, the theory of continuous selections of multivalued mappings and the problem of local triviality of fibrations.

1. Introduction

A well-known classical theorem asserts that every compact metric space X can be expressed as a continuous image of the Cantor set K , $\varphi : K \rightarrow X$. Consider the corresponding Banach spaces $C(K)$ and $C(X)$ of all real-valued continuous functions, equipped with the usual sup-norm. Then φ induces the natural composition map $\varphi^* : C(X) \rightarrow C(K)$, defined by $\varphi^*(g) = g \circ \varphi$, for every $g \in C(X)$.

It is easy to see that φ^* is an injective continuous linear operator and moreover, it is an isometry. Hence, $\varphi^*(C(X))$ is a linear subspace of $C(K)$. However, in general one cannot say anything more about the additional properties of this inclusion.

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More generally, one can ask the following question: Given a continuous surjection $\varphi : X \rightarrow Y$ between arbitrary completely regular spaces X and Y , what properties does the inclusion $\varphi^* : C(Y) \rightarrow C(X)$ between the corresponding Banach spaces (of bounded real-valued continuous functions) have?

In the case when $\varphi : X \rightarrow Y$ is a *Milyutin mapping*, much more can be established about such inclusions. Namely, we shall see that in such a case the embedded subspace $\varphi^*(C(Y))$ is closed and *complementable* in $C(X)$, i.e. there exists a projector (linear retraction) $P : C(X) \rightarrow C(X)$ of $C(X)$ onto $\varphi^*(C(Y))$, hence

$$C(X) \cong \varphi^*(C(Y)) \oplus \text{Ker} P,$$

where \cong denotes an isomorphism.

Moreover, the norm of this projector P is the smallest possible, $\|P\| = 1$. Observe that for projectors it is always true that

$$\|P\| = \|P \circ P\| \leq \|P\|^2 \implies \|P\| \geq 1.$$

So, P looks very much like an orthogonal projector in a Hilbert space, since $\|P(f)\| \leq \|f\|$, for every $f \in C(X)$.

The plan of this survey is first, to introduce Milyutin mappings, then show that there exist many interesting examples, and finally, describe their applications to:

- (i) The geometry of Banach spaces;
- (ii) The theory of continuous selections of multivalued mappings; and
- (iii) The problem of local triviality of fibrations.

2. Milyutin Mappings

First, let us illustrate the important role of complementable spaces in geometry of Banach spaces by considering the problem when are two Banach spaces A and B isomorphic, $A \cong B$.

An answer was provided in the mid 1960's by Bessaga and Pełczyński. Their *decomposition principle* [1] says that

$$(A \cong B \oplus C, B \cong A \oplus D, A \cong A \oplus A, B \cong B \oplus B) \implies A \cong B.$$

In fact,

$$\begin{aligned} A \cong B \oplus C &\cong (B \oplus B) \oplus C \cong B \oplus (B \oplus C) \cong B \oplus A \cong A \oplus (A \oplus D) \\ &\cong (A \oplus A) \oplus D \cong A \oplus D \cong B. \end{aligned}$$

In reality, A and B do not appear in a symmetric form, i.e. only the properties of either A or B are well-known. In such cases one can substitute the last two conditions $A \cong A \oplus A$ and $B \cong B \oplus B$ by the requirement that A be *infinitely divisible*. We shall use the fact that infinite divisibility of A is a consequence of the isomorphism:

$$A \cong c_0(A) = \{(a_i)_{i=1}^\infty \mid a_i \in A, \lim_{i \rightarrow \infty} \|a_i\| = 0\}.$$

In this situation, a different version of the Bessaga-Pełczyński decomposition principle implies that $A \cong B$ (cf. [12]).

Definition 2.1. Let X and Y be completely regular spaces. Then a continuous surjection $\varphi : X \rightarrow Y$ is called a *Milyutin mapping* if there exists a continuous map $\nu : Y \rightarrow P_\beta(X)$, such that

$$\text{supp}(\nu_y) \subset \varphi^{-1}(y), \quad y \in Y.$$

Here $P_\beta(X)$ is the set of all *probabilistic* (i.e. countably additive, nonnegative, normed, and regular) measures on the Stone-Čech compactification βX , whose supports lie inside X , $\text{supp}(\mu) \subset X \subset \beta X$. The space $P_\beta(X)$ is endowed with the weak-star topology, induced by the natural inclusion $P_\beta(X) \subset C(\beta X)^*$ into the first conjugate Banach space. More precisely, the basis of neighborhoods of a point $\mu_0 \in P_\beta(X)$ consists of the sets

$$G_\varepsilon(\mu_0; g_1, g_2, \dots, g_n) = \{\mu \mid \left| \int_{\beta X} g_i d\mu - \int_{\beta X} g_i d\mu_0 \right| < \varepsilon\}.$$

The inclusions $\text{supp}(\nu_y) \subset \varphi^{-1}(y)$, for every $y \in Y$, in Definition 2.1 mean that for each $y \in Y$, the measure ν_y is concentrated precisely in the preimage $\varphi^{-1}(y)$, i.e. $\nu_y(B) = 0$, for every Borel subset B of $X \setminus \varphi^{-1}(y)$.

Proposition 2.2. *Let $\varphi : X \rightarrow Y$ be a Milyutin mapping. Then there exists a projector $P : C(X) \rightarrow C(X)$ such that $\text{Im}P = \varphi^*(C(Y))$. In other words, $C(Y)$ is complementable in $C(X)$.*

Proof. (1) Define P as follows :

$$[P(f)](x) = \int_{\varphi^{-1}(\varphi(x))} f d\nu_{\varphi(x)}, \quad f \in C(X).$$

(2) Let us verify that $\text{Im}P \supset \varphi^*(C(Y))$. Let $f \in \varphi^*(C(Y))$. Then $f = g \circ \varphi$, for some $g \in C(Y)$. Hence

$$\begin{aligned} [P(f)](x) &= [P(g \circ \varphi)](x) \\ &= \int_{\varphi^{-1}(\varphi(x))} (g \circ \varphi) d\nu_{\varphi(x)} \\ &= (g \circ \varphi)(x) \int_{\varphi^{-1}(\varphi(x))} 1 \cdot d\nu_{\varphi(x)} \\ &= (g \circ \varphi)(x) = f(x). \end{aligned}$$

Thus $P(f) = f$.

(3) On the other hand, let $f \in C(X)$. We want to find $g \in C(Y)$ such that $P(f) = g \circ \varphi$. It is easy to see that the following is a solution :

$$g(y) = \int_{\varphi^{-1}(y)} f d\nu_y.$$

Hence $\text{Im}P = \varphi^*(C(Y))$.

It remains to verify that $P \circ P = P$. But for every $f \in C(X)$, we have $(P \circ P)(f) = P(Pf) = Pf$ since $Pf \in \varphi^*(C(Y))$. \square

The construction in (3) above yields a correspondence

$$A : C(X) \rightarrow C(Y), \quad (Af)(y) = \int_{\varphi^{-1}(y)} f d\nu_y.$$

which is usually called a *regular averaging operator*. Having already defined

$$\varphi^* : C(Y) \rightarrow C(X), \quad \varphi^*(g) = g \circ \varphi$$

we observe that

$$A \circ \varphi^* = id_{C(Y)}, \quad \varphi^* \circ A = P.$$

Theorem 2.3. (Existence of Milyutin mappings) *For every paracompact space X there exist a 0-dimensional (in dim sense) paracompact space X_0 and a Milyutin mapping $\varphi : X_0 \rightarrow X$ of X_0 onto X such that φ is perfect (i.e. closed and with compact point-inverses) and φ is inductively open (i.e. φ^{-1} admits a lower semicontinuous selection). Moreover, when X is metrizable (resp., separable metric, Polish, compact, uncountable metric compact) one can assume that X_0 is also metric (resp., separable metric, Polish, compact, a Cantor set).*

For X the unit interval $[0, 1]$, this theorem was proved by Milyutin [11], via a surprising construction of a suitable mapping of the Cantor set onto the unit interval. For compact metric spaces and for compact topological groups the result is due to Pełczyński [13], who in fact proved that the product of Milyutin mappings is again a Milyutin mapping. For (nonmetrizable) compacta Theorem 2.3 was proved by Ditor [4]. For Polish spaces and A not necessarily regular, this is a result of Etcheberry [6], and for Polish spaces and A regular, it was proved by Choban [3]. Valov [20] proved this theorem for the class of all products of metrizable spaces and for the class of p -paracompact spaces. In the general case, this result was proved by Repovš, Semenov and Ščepin [15].

The name "Milyutin mappings" was introduced by Ščepin [18]. Pełczyński used the term "Milyutin space" for a space which admits (in our terminology) a Milyutin surjection from a Cartesian power of the two-points set $D = \{0, 1\}$.

Question 2.4. *Does there exist a version of Theorem (2.3) for normal (normal and countably paracompact) spaces?*

3. An Application to the Geometry of Banach Spaces

Recall that the original Banach problem was asking for a proof of the existence of an isomorphism between the Banach spaces $C([0, 1])$ and $C([0, 1] \times [0, 1])$. Milyutin proved in his 1952 dissertation (and published in 1966 [11]) a considerably more general answer. For the history of this question see [13].

Theorem 3.1. *For every uncountable metric compactum X , the Banach space $C(X)$ is isomorphic to the Banach space $C(K)$, where K is the Cantor set.*

Proof. Let X be an uncountable metric compactum (hence a continual compactum). Then X contains a homeomorphic copy K' of the Cantor set K . Let $A = C(K)$ and $B = C(X)$. Clearly, $A \cong c_0(A)$ and hence A is infinitely divisible. Moreover, due to the Dugundji simultaneous extension theorem we have that

$$B \cong C(K') \oplus \text{Ker}Q \cong C(K) \oplus \text{Ker}Q \cong A \oplus \text{Ker}Q,$$

for some projector $Q : B \rightarrow B$.

It therefore suffices, by the Pełczyński decomposition principle, to verify that $C(X)$ is complementable in $C(K)$. But this follows due to the existence of a Milyutin mapping $\varphi : K \rightarrow X$ from the Cantor set K onto X (cf. Theorem (2.3) and Proposition (2.2)). \square

We remark that such methods work in a more general setting and not only for compact spaces X (cf. [2, 6, 20]).

Theorem 3.2. [6] *Suppose that X is a Polish space (i.e. separable, completely metrizable space). Then:*

- (i) *$BC(X)$ is isomorphic to $BC(\mathbb{N}^\infty)$, provided that X contains an uncountable closed subset which is not locally compact at any point; and*
- (ii) *$BC(X)$ is isomorphic to $BC(K \times \mathbb{N})$, provided that X is locally compact and contains a closed noncompact subset in which every nonempty (relatively) open set contains a two-points subset.*

In this theorem BC denotes the Banach space of all bounded continuous functions and \mathbb{N}^∞ denotes the countable Cartesian power of the set of natural numbers \mathbb{N} or, equivalently, the space of all irrational numbers or, the Baire space $B(\infty)$.

Theorem 3.3. [20] *Suppose that $X = \prod\{X_\alpha \mid \alpha \in A\}$, where $\text{card}A = \lambda$ is infinite and each X_α is a complete metric space of weight τ . Then $C_k(X)$ is isomorphic to $C_k(B(\tau)^\lambda)$.*

Theorem 3.4. [20] *Suppose that $X = M \times \prod\{X_\alpha \mid \alpha \in A\}$, where $\text{card}A = \lambda$, M is a complete metric space of weight τ and each X_α is a compact metric space. Then*

- (i) *$C_k(X)$ is isomorphic to $C_k(B(\tau) \times D^\lambda)$, provided that M is nowhere locally compact and the weight of each open subset of M is τ ; and*
- (ii) *$C_k(X)$ is isomorphic to $C_k(T_\tau \times D^\lambda)$, provided that M is locally compact.*

Here, $B(\tau)$ is the Baire space, T_τ is the discrete set of cardinality τ and C_k stands for the topological vector space of all

continuous functions endowed with the compact-open topology.

4. An Application to the Theory of Continuous Selections of Multi-valued Mappings

Hereafter, 2^Y shall denote the family of all nonempty closed subsets of a topological space Y and $F : X \rightarrow 2^Y$ a lower semicontinuous mapping. A mapping $G : X \rightarrow 2^Y$ is called a *selection* of F , provided that $G(x) \subset F(x)$, for every $x \in X$. As a rule, we shall consider *singlevalued continuous* selections $f : X \rightarrow Y$, $f(x) \in F(x)$. A selection of a lower semicontinuous mapping F exists under some strong restrictions on spaces X and Y , and the family of all values of F . There are four classical selection theorems for a paracompact domain X - all due to Michael [7, 8]:

- (1) *Zero-dimensional theorem*: If $\dim X = 0$ and Y is a complete metric space then there exists a continuous singlevalued selection of F ;
- (2) *Convex-valued theorem*: If all values of F are convex subsets of a Banach space Y , then there exists a continuous singlevalued selection of F ;
- (3) *Compact-valued theorem*: If Y is a complete metric space, then F admits an upper semicontinuous compact-valued selection H , which in turn, admits a lower semicontinuous compact-valued selection G ;
- (4) *Finite-dimensional theorem*: If $\dim X = n + 1$ and Y is a complete metric space, each of values of F is an n -connected subset of Y , and the family of all values of F is equi-locally n -connected, then there exists a continuous singlevalued selection of F .

The zero-dimensional selection theorem is the simplest. It turns out that one can derive two other selection theorems

from the zero-dimensional one. This can be proved by using the theory of Milyutin mappings.

Theorem 4.1. [15] *The convex-valued and compact-valued selection theorems are corollaries of the zero-dimensional theorem.*

Sketch of Proof. Let $F : X \rightarrow 2^Y$ be a lower semicontinuous convex-valued mapping from a paracompact space X to any Banach space Y . Apply Theorem (2.3) to obtain a zero-dimensional paracompact space X_0 and a Milyutin mapping $\varphi : X_0 \rightarrow X$ with compact point-inverses.

The composition $G = F \circ \varphi$ is a lower semicontinuous mapping on X_0 with closed (and convex) values $G(x) \subset Y$. We apply the zero-dimensional Michael selection theorem to the mapping $G : X_0 \rightarrow 2^Y$, to get a singlevalued continuous selection $g : X_0 \rightarrow Y$, $g(z) \in G(z)$, $z \in X_0$. Let $\nu : X \rightarrow P_\beta(X_0)$ be a mapping associated with the Milyutin mapping φ .

In order to get a selection $f : X \rightarrow Y$ of the given mapping F we define, for any $x \in X$:

$$f(x) = \int_{\varphi^{-1}(x)} g d\nu_x \in Cl(\text{conv}(g(\varphi^{-1}(x)))) \subset F(x).$$

In other words, $f(x)$ is the barycenter of the compactum $g(\varphi^{-1}(x)) \subset F(x)$, with respect to the probabilistic measure ν_x . This barycenter lies in $F(x)$, due to the convexity and closedness of the values of the mapping F .

Moreover, the mapping $H = g \circ \varphi^{-1}$ is an upper semicontinuous compact-valued selection of F and the composition of the lower semicontinuous selection of $\varphi^{-1} : X \rightarrow 2^{X_0}$ with $g : X_0 \rightarrow Y$ yields a lower semicontinuous compact-valued selection of H . \square

In this theorem some problems arise with verification of the continuity of f , since measure ν_x continuously depends on x ,

whereas the domain of integration $\varphi^{-1}(x)$ depends on x only upper semicontinuously. For the proof one must use the *construction* of the Milyutin mapping $\varphi : X_0 \rightarrow X$ from Theorem (2.3). Note, that for a *compact* X this can be done by using the coincidence of the closures of a convex subset of Y in the weak topology and in the original topology of Y (cf. [19]).

The universality of the zero-dimensional selection theorem works in more general situations. In fact, one can construct an integration theory in a complete metric space with a suitable axiomatic "convex structure" (cf. [9]), and then prove the convex-valued selection theorem exactly as above.

Observe, that in the original proof [9], a desired singlevalued selection was obtained as a uniform limit of a sequence of δ -continuous singlevalued selections. As a simple variation of such a generalization we have the following Toruńczyk's version of a Bartle-Graves type theorem [1; Proposition II.7.1].

Theorem 4.2. *Let X and Y be complete linear metric (in general, nonlocally convex) spaces. Let $u : Y \rightarrow X$ be a surjective linear mapping with the kernel a locally convex space. Then there exists a continuous mapping $f : X \rightarrow Y$ such that $u \circ f = id_X$.*

Proof. It suffices to apply the construction from the proof of Theorem (4.1) to the case $F = u^{-1}$ and observe that the barycenter of a subcompactum of a point-inverse $u^{-1}(x)$ (with respect to the probabilistic measure) lies in $u^{-1}(x)$. \square

In the same manner one can derive the following Michael's selection theorem.

Theorem 4.3. *Let $F : X \rightarrow E$ be a lower semicontinuous mapping from a paracompact space X into a complete locally convex topological vector space E and let the union M of all values of F admit a compatible metric such that each*

value $F(x)$ is a complete subset of M , $x \in X$. Then there exists a continuous singlevalued mapping $f : X \rightarrow E$ such that $f(x) \in Cl(\text{conv}(F(x)))$, for all $x \in X$.

Here, completeness of E means that the closed convex hull of a subcompactum is also compact. The original proof of Theorem (4.3) [10] appeared as a final result of a series of papers concerning improvements of the Arens-Eells theorem on suitable embeddings of a metric space into a Banach space. Our approach shows that one can weaken the hypotheses of Theorem (4.3), by assuming only the completeness of values $F(x)$, and that the first conjugate E^* separates points of E . The completeness gives existence of an integral above, and the uniqueness of this integral follows from the last separation assumption.

Question 4.4. *Does the zero-dimensional selection theorem also imply the finite-dimensional selection theorem?*

5. An Application to the Theory of Local Triviality of Fibrations

It is a standard question in topology when is a surjective mapping a locally trivial fibration. Clearly, a necessary condition is that all point-inverses are homeomorphic and (in the metric compact case) that point-inverses of close points are homeomorphic under small transformations. Such mappings are called *completely regular*.

Definition 5.1. A mapping $f : X \rightarrow Y$ between metric spaces (X, d) and (Y, ρ) is said to be *completely regular* if for each $y_0 \in Y$ and for each $\varepsilon > 0$, there exists $\delta > 0$ such that the inequality $\rho(y, y_0) < \delta$ implies existence of a homeomorphism $h : f^{-1}(y) \rightarrow f^{-1}(y_0)$ such that $d(x, h(x)) < \varepsilon$, for all $x \in f^{-1}(y)$.

Question 5.2. *Under what conditions is a completely regular mapping a locally trivial fibration?*

A classical answer was given by Dyer and Hamström for finite-dimensional Y . It turns out that the answer is positive whenever Y is a complete metric space with $\dim Y \leq n + 1$, the preimages $f^{-1}(y)$ are compacta, and the homeomorphisms group $H(f^{-1}(y))$ is locally n -connected (cf. [5]). For infinite-dimensional Y or for $\dim(f^{-1}(y)) \geq 4$, the answer is in general negative ([16]). For one-dimensional compact polyhedral fibers the answer is positive without any restrictions on $\dim Y$.

Theorem 5.3. [16] *Let $f : X \rightarrow Y$ be a completely regular mapping between compact metric spaces such that point-inverses are homeomorphic to a fixed one-dimensional polyhedron. Then f is a locally trivial fibration.*

However, in the proof of the Theorem (5.3), compactness is essential because it uses separability of the Banach space of continuous functions on metric compacta and the Michael selection theorem for lower semicontinuous mappings with convex nonclosed values which are subsets of a *separable* Banach space.

Earlier, Pixley [14] used this approach for the case when fibers are homeomorphic to the unit interval. So, for the non-compact case such a technique does not work. The following two theorems give many examples of Milyutin mappings with some additional properties. On the other hand, as a corollary we obtain a positive answer to Question (5.2) when point-inverses are homeomorphic to the line.

Theorem 5.4. [17] *Every open surjection $\varphi : X \rightarrow Y$ between Polish spaces is a Milyutin mapping with the associated mapping $\nu : Y \rightarrow P(X)$ such that*

$$\text{supp}(\nu_y) = \varphi^{-1}(y), y \in Y.$$

The equality in the assertion of Theorem (5.4) means that the value of measure ν_y is positive at every nonempty open subset of the preimage $\varphi^{-1}(y)$. Such mappings φ are called *exact Milyutin mappings*. Note that in a contrast with Theorem (2.3), the point-inverses of an exact Milyutin mapping are in general, noncompact subsets of the domain. So, here we work in general with $P(X)$, not with $P_\beta(X)$.

Note, that Polish spaces are precisely the completely metrizable separable spaces and we can assume that X is a subset of the Hilbert cube. However, in the following corollary the values of the associated mapping automatically have compact supports.

Corollary 5.5. *Let M be a Polish space and $\text{exp}(M)$ the set of all subcompacta of M , endowed with the Vietoris topology. Then there exists a continuous mapping*

$$\mu : \text{exp}(M) \rightarrow P_\beta(M)$$

such that $\text{supp}(\mu_K) = K$, for every subcompactum $K \subset M$.

Proof. The projection $p : M \times \text{exp}(M) \rightarrow \text{exp}(M)$ onto the second factor is an open surjection between Polish spaces. To complete the proof it suffices to apply Theorem (5.4) with $Y = \text{exp}(M)$,

$$X = \{(m, K) \mid K \in \text{exp}(M), m \in K\} \subset M \times \text{exp}(M)$$

and φ the restriction of p onto X . □

Theorem 5.6. [17] *Every completely regular mapping $\varphi : X \rightarrow Y$ between Polish spaces with point-inverses without isolated points is an exact Milyutin mapping, with the associated mapping $\nu : Y \rightarrow P(X)$ such that*

$$\nu_y(\{x\}) = 0,$$

for all $y \in Y$ and $x \in \varphi^{-1}(y)$.

An exact Milyutin mapping φ with the property that $\nu_y(\{x\}) = 0$ is called *atomless*. Observe that the proofs of Theorems (5.4) and (5.6) essentially use the zero-dimensional Michael selection theorem.

Corollary 5.7. *A completely regular mapping $\varphi : X \rightarrow Y$ between Polish spaces is a locally trivial fibration, provided that its point-inverses are homeomorphic to the real line.*

Proof. For each $y \in Y$, there exists a unique point $m_y \in \varphi^{-1}(y)$ such that $\varphi^{-1}(y) \setminus \{m_y\}$ consists of two open rays with measures equal to $1/2$ (with respect to ν_y). The existence of such an intermediate point follows from the atomlessness of measure ν_y whereas its uniqueness follows from the exactness of ν_y . Moreover, the regularity of φ implies that $m : Y \rightarrow X$ is a continuous selection of φ^{-1} .

For a fixed $y \in Y$, we pick one of the components of $\varphi^{-1}(y) \setminus \{m_y\}$ and a point, say n_y , from this component such that

$$\nu_y([m_y, n_y]) = 1/4.$$

We can find disjoint ε -neighborhoods of the points m_y and n_y , and the regularity of φ at the point y gives us a neighborhood $U(y)$ such that for each $z \in U(y)$, there exists a unique component of $\varphi^{-1}(z) \setminus \{m_z\}$ with the unique point, say n_z , in this component such that

$$\nu_z([m_z, n_z]) = 1/4.$$

Moreover, $n : U(y) \rightarrow X$ is also a continuous selection of φ^{-1} . Let us show that $\varphi^{-1}(U(y))$ is homeomorphic to the Cartesian product $U \times (-1/2, 1/2)$. Define the homeomorphism h by

$$h(z) = (\varphi(z), \nu_{\varphi(z)}([m_{\varphi(z)}, z])) \in U \times [0, 1/2)$$

if z and $n_{\varphi(z)}$ are in the same component of $\varphi^{-1}(\varphi(z)) \setminus \{m_{\varphi(z)}\}$, whereas in the case when $m_{\varphi(z)}$ separates the points z and $n_{\varphi(z)}$, define h by

$$h(z) = (\varphi(z), -\nu_{\varphi(z)}([z, m_{\varphi(z)}])) \in U \times (-1/2, 0].$$

This completes the proof. □

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