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IN MEMORY OF JOHN HENDERSON ROBERTS (1906-1997)

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ABSTRACT. This paper is written in memory of J. H. Roberts, who died on October 8, 1997. The paper has six parts: biographical sketch, mathematical career and areas of research, work in dimension theory, teaching, list of Ph.D. students, and list of publications.

1. BIOGRAPHICAL SKETCH

John Henderson Roberts, the seventh Ph.D. student of R.L. Moore, died at the Carolina Meadows Health Center in Chapel Hill, N.C. on October 8, 1997. He had been confined to a wheelchair for several years due to the ill effects of several strokes. He was 91 years old when he died. He was survived by his son, John Edward, grandson George, and great-granddaughters Monica (age 5) and Susan (age 2).

Roberts was born in Raywood, Texas (about 40 miles east of Houston) on September 2, 1906. He received the A.B. degree in mathematics from the University of Texas in 1927 at the age of 21. Two years later in 1929 he received the Ph.D. degree. Roberts related to us that he was able to earn the Ph.D. degree so quickly because he made such a favorable impression in Moore's introductory topology course, that Moore immediately moved him into the advanced class. Roberts said that this event was "indelibly etched in my mind." Moore's confidence was quickly justified: In his first four papers, Roberts answered questions raised by C. Kuratowski, G.T. Whyburn, K. Menger, and R.L. Moore, and by 1933, just four years after receiving his degree, he had published 14 papers.

Gian-Carlo Rota, in his recent book ***Indiscrete Thoughts***, states that mathematicians can be subdivided into two types: problem solvers and theorizers (see p. 45). Roberts falls squarely into the problem solver division. Indeed, he was known to say: "I am a sucker for a good problem."

Roberts visited the University of Pennsylvania during 1929-30, where he worked with J.R. Kline, Moore's first Ph.D. student. During 1930-31 he was adjunct professor at the University of Texas. Due to cutbacks caused by the Great Depression, Roberts was not able to stay at the University of Texas. In 1931 he moved to Duke University, where he remained a member of the faculty until he retired in 1971. During World War II, he served as a lieutenant commander in the Navy.

Roberts met his future wife, Doretta von Boeckman (1904-1988), a native of Austin, Texas, while he was attending the University of Texas. They were married on August 27, 1928. Roberts was a boarder at the von Boeckman's house while a student at Texas, and he was projectionist at a theater in Austin where Doretta played the piano for silent movies.

Professor and Mrs. Roberts were always very kind to new faculty members and graduate students at Duke. They gave frequent Saturday night parties which were greatly appreciated by all. Mary Ellen and Walter Rudin remember going to many parties at the Roberts' house. Walter had been going to the parties as a graduate student at Duke. He received the Ph.D. degree from Duke in June, 1949, and was then hired as an instructor. The following September Mary Ellen Estill was hired as an instructor at Duke, having just received the Ph.D. degree from the University of Texas. They recall that their first "dates" were probably at the Roberts' parties. These parties were unique at Duke, the only departmental social occasions where all of the graduate students and the faculty could meet. Mary Ellen said "John Roberts was very much a mentor for me at Duke and for Walter and other graduate students, and Doretta cheered us all on too. They were also enthusiastic over a possible budding romance between Walter and me but were certainly not pushy about it."

At the parties we attended in the early 1960's, Roberts showed us numerous electronic devices that he had built from kits. This included amateur radio equipment, a color television set, and an appliance timer, which Roberts gave to Vaughan who is still using it. To Mrs. Roberts' delight, Professor Roberts added a switch on a long cord that could, from across the room, turn the sound from the television off and on while leaving the picture visible. She called this device her "blab-off."

At the time of his retirement, Roberts' Ph.D. students gave him a surprise dinner party, and 11 of his 24 students were able to attend, many coming from a considerable distance. As a gift, he was presented with a scrapbook with a list of his students and publications. In addition, each student was allotted two pages in the scrapbook; one a personal letter to Roberts and the other a curriculum vitae. In later years Roberts would refer to this as his "brag book."

Roberts will long be remembered by his colleagues as a remarkable and distinguished mathematician, and by his Ph.D. students as an inspiring teacher and an unselfish thesis advisor.

2. Mathematical career and research

The phrase "he had a long and distinguished career," certainly applies to Roberts. During his 40 years at Duke, he had 24 Ph.D. students, was director of graduate studies (1948-1960), and was chair of the department (1966-68). He served as a managing editor of the ***Duke Mathematical Journal*** (1951-1960) and was Secretary of the American Mathematical Society in 1954. He spent the academic year 1937-38 at Princeton and wrote a joint paper with N.E. Steenrod while there.

Early in his career Roberts worked mainly in continuum theory (especially in the plane), but beginning in the 1940's, his interests shifted, and he began working in dimension theory. In the early 1950's, however, he wrote two papers on integral equations which resulted from questions posed to him by W.R. Mann at the University of North Carolina at Chapel Hill.

Let us give two examples of Roberts' characterization of himself as a sucker for a good problem. In 1959-1960, Hans Debrunner, a Swiss mathematician and an expert in combinatorial geometry, visited Duke for the academic year, having spent the previous year at Princeton working with R.H. Fox. One day he mentioned a geometry problem to Roberts which, according to Debrunner, had been unsolved for several years. When Roberts saw the problem, he said: "I can't believe that problem hasn't been solved." He promptly wrote up a solution and sent it to ***Elemente der Mathematik***, the journal in which the problem first appeared. Roberts proved that if a triangle DEF is inscribed in a triangle ABC with D on BC, E on CA, and F on AB, then the minimum of the perimeters of the four smaller triangles is always assumed by a corner triangle. The only case in which the triangle DEF assumes this minimum is when D, E, and F are respectively the midpoints of BC, CA, and AB, and the four smaller triangles are congruent.

In 1959, Stallings constructed a compact zero-dimensional set $K \subseteq \mathbf{I}^2$ such that the graph of every continuous function $f : \mathbf{I} \to \mathbf{I}$ (where $\mathbf{I} = [0,1]$) intersects K, and he asked whether every such set must also intersect the graph of every connectivity function ($f : X \to Y$ is a ***connectivity function*** if for every connected $C \subseteq X$, the graph of $f|C$ is connected). Roberts gave a complete answer to this question by proving the following two results.

Theorem 1. **(Roberts;** 1965) *There is a compact zero-dimensional set $K \subseteq \mathbf{I}^2$ such that*

(1) *the graph of every continuous function* $f : \mathbf{I} \to \mathbf{I}$ *intersects* K;
(2) *there is a connectivity function* $g : \mathbf{I} \to \mathbf{I}$ *whose graph misses* K.

Theorem 2. **(Roberts;** 1965) *There is a compact zero-dimensional set* $K \subseteq \mathbf{I}^2$ *such that* K *intersects the graph of every connectivity function* $f : \mathbf{I} \to \mathbf{I}$.

As noted earlier, Roberts' early work was in continuum theory. For a discussion of his contributions to this area of research, see the paper ***History of Continuum Theory*** by J.J. Charatonik in volume 2 of the ***Handbook of the History of General Topology***, edited by C.E. Aull and R. Lowen (pages 726, 728, 730, 742).

3. Work in dimension theory

Starting around 1940, Roberts began working in dimension theory, and this would be his chief area of research for the remainder of his career. It is of interest that W. Hurewicz, who together with H. Wallman wrote the highly regarded book ***Dimension Theory*** held an academic position at the University of North Carolina at Chapel Hill from 1939 to 1945. This could well account for the somewhat abrupt change in Roberts' research interests.

Before discussing Roberts' work in dimension theory, let us review some basic ideas. There are three classical definitions of dimension, namely ind (***small inductive dimension***), Ind (***large inductive dimension***), and dim (***covering dimension***). The three are equivalent for separable metric spaces, and moreover Ind and dim coincide for metric spaces. On the other hand, a famous example of P. Roy shows that ind and Ind need not be the same in metric spaces. For metric spaces one works with dim or Ind, since ind is too weak to establish a satisfactory theory. The definitions are as follows.

Definition. ***dim*** $X \leq n$ if every finite open cover of X has an open refinement of order $\leq n+1$

Definition. ***Ind*** $X \leq n$ if for every pair of disjoint closed sets H and K in X, there is a closed set B in X such that B separates H and K and $\text{Ind}(B) \leq n-1$. (***Ind*** $\emptyset = -1$). To obtain ***ind*** $X \leq n$, replace H by a single point p.

Embedding theorems are among the most interesting results in dimension theory. An early and fundamental result in this direction is the following.

Embedding Theorem (Menger, Nöbeling, Hurewicz) *Let X be a separable metric space with dim $X = n$. Then X can be embedded in $\mathbf{I}^{2n+1}$. Moreover,*

(1) *the set of all homeomorphisms from X into $\mathbf{I}^{2n+1}$ contains a dense G_δ-subset of $C(X, \mathbf{I}^{2n+1})$ [space of continuous functions from X into $\mathbf{I}^{2n+1}$ with the sup norm topology];*
(2) *if X is compact, then the set of all homeomorphisms from X into $\mathbf{I}^{2n+1}$ is a dense G_δ-subspace of $C(X, I^{2n+1})$.*

The Menger-Nöbeling-Hurewicz Embedding Theorem is a sharpened version of Urysohn's 1925 metrization theorem, which asserts that every regular space with a countable base can be embedded in $\mathbf{I}^\omega$. For future reference we note that the dichotomy between (1) and (2) gives rise to a problem that was later solved by Roberts.

The following is a general problem about mappings and dimension. Given a continuous function f from X onto Y, what is the relationship between dim X, dim Y, and properties of f? The first result in this direction was obtained by Hurewicz in 1927.

Theorem (***Hurewicz; dimension-raising mappings***, 1927) Let X and Y be separable metric spaces and let f be a closed and continuous mapping from X onto Y that is at most $(k+1)$–to–1 $(k \geq 0)$. Then dim $Y \leq$ dim $X + k$.

In connection with this result, Hurewicz asked about the existence of such dimension-raising mappings: given a separable metric space Y with dim $Y = n$ and an integer k with $1 \leq k \leq n$, is there a separable metric space X of dimension $n - k$ and a closed continuous mapping from X onto Y that is at most $(k+1)$–to–1? Roberts' first publication in dimension theory gave an affirmative answer to Hurewicz's question. First, however, Roberts proved the following embedding theorem, which is of interest in its own right.

Theorem 1. *(**Roberts; embedding theorem,** 1941) Let X be a separable metric space with dim $X = n$. Then there is an embedding h of X into $\mathbf{R}^{2n+1}$ such that for any k-dimensional hyperplane $\mathbf{H}^k$ in $\mathbf{R}^{2n+1}$ $(n+1 \leq k \leq 2n+1)$,*

$$\dim \quad [h(X) \cap \mathbf{H}^k] \leq k - n - 1.$$

Note $\mathbf{H}^k$ is the translate of a k-dimensional vector subspace of $\mathbf{R}^{2n+1}$.

For example, suppose that dim $X = 1$ (so $n = 1$ and $2n+1 = 3$) and let $k = 2$. Then there is an embedding h of X into $\mathbf{R}^3$ such that for every plane Π in $\mathbf{R}^3$, $h(X)$ intersects Π in a set of dimension at most 0.

Roberts then used his embedding theorem to prove the following.

Theorem 2. *(**Roberts; existence of dimension-raising mappings,** 1941) Let Y be a separable metric space with dim $Y = n$ and let $1 \leq k \leq n$. Then there is a separable metric space X and a closed continuous function f from X onto Y such that dim $X = n - k$ and f is at most $(k+1)$-to-1.*

For example, let Y be the 2-cube $\mathbf{I}^2$ and let $k = 1$. Then there is a 1-dimensional separable metric space X and a closed continuous function f from X onto $\mathbf{I}^2$ that is at most 2-to-1.

The Menger-Nöbeling-Hurewicz Embedding Theorem asserts that for X compact, the set of all homeomorphisms from X into

$\mathbf{I}^{2n+1}$ is a dense G_δ-subset of $C(X, \mathbf{I}^{2n+1})$. In their book ***Dimension Theory***, Hurewicz and Wallman asked whether this result extended to the separable metric case. In 1948, Roberts gave a negative answer to this question.

Theorem 3. *(**Roberts;** 1948) Let $n \geq 0$ and let $k \geq n+1$. Then there exists an n-dimensional subset X of $\mathbf{I}^{n+1}$ such that the set of all homeomorphisms from X into $\mathbf{I}^k$ is **not** a G_δ-set in $C(X, \mathbf{I}^k)$.*

Let $\mathbf{H}$ denote Hilbert space (all infinite sequences $\{x_n\}$ of real numbers such that Σx_n^2 converges) and let $\mathbf{I}^\omega$ denote the Hilbert cube ($= \Pi_{1 \leq n < \infty}[-1/n, 1/n]$). The set $\mathbf{Q}$ of rational points in $\mathbf{I}^\omega$ (i.e., the points of $\mathbf{I}^\omega$ with all coordinates rational) is zero-dimensional, and in 1940 Erdös proved that the set $\Re$ of rational points in Hilbert space has dimension 1, hence $\Re$ can be embedded in $\mathbf{I}^3$. Erdös asked whether $\Re$ can be embedded in $\mathbf{I}^2$.

Theorem 4. *(**Roberts;** 1956) $\Re$ can be embedded in $\mathbf{I}^2$.*

Outline of Proof: The following notation is used:

C = Cantor set;

D = Cantor fan in $\mathbf{I}^2$ (the union of all line segments px, where $x \in C$ and $p = < 0, 1/2 > \in \mathbf{I}^2$).

Since dim $\mathbf{Q} = 0$, there is an embedding α of $\mathbf{Q}$ into C. The function $\phi(x) = 1/(1 + |x|)$ is a homeomorphism from $\mathbb{R}$ onto $(-1, 1)$, and from this it follows that there is a function $g : \mathbf{H} \to \mathbf{I}^\omega$ that is one-to-one, continuous, and preserves rationality coordinatewise. The required embedding $h : \Re \to D$ is defined as follows: for $x \in \Re$, $h(x)$ is the point in D that is on the line segment joining p and $\alpha(g(x))$ and has y-coordinate $\phi(\| x \|)$. $\square$

The year 1963 was an important milestone in Roberts' career. Beginning in 1963, and continuing to 1965, Keiô Nagami, from Ehime University in Japan, held a visiting position at Duke. Nagami and Roberts made a good team. Nagami, an expert in dimension theory (and the oriental board game go), had a knack for finding numerous interesting open problems in dimension theory. Their joint work, which might be characterized as the collaboration between a theorizer and a problem solver, greatly extended the area of dimension theory known as ***metric-dependent dimension functions***. Together they wrote three joint papers, which we now discuss in detail.

In their first joint paper, Nagami and Roberts gave a characterization of the strongly countable dimensional metric spaces (a metric space that is the union of a countable number of closed finite-dimensional subspaces; for example, $\oplus_{n\geq 1}\mathbf{I}^n$, the disjoint sum of the cubes $\mathbf{I}^1, \dots, \mathbf{I}^n, \dots$).

Theorem 5. *(**Nagami-Roberts**; 1965) A metric space X is strongly countable dimensional $\Leftrightarrow$ there is a sequence $\{\mathcal{G}_k : k \in \mathbf{N}\}$ of open covers of X such that*

(1) *$\mathcal{G}_{k+1}$ refines $\mathcal{G}_k$ for all $k \in \mathbf{N}$;*
(2) *for all $x \in X$, $\{\mathrm{st}^2(x, \mathcal{G}_k) : k \in \mathbf{N}\}$ is a local base for x;*
(3) *for all $x \in X$, $sup_{k\in N}[ord(x, \mathcal{G}_k)] < \infty$.*

Note that (2) is the Moore-Morita characterization of metrizability. To prove Theorem 5, Nagami and Roberts first gave the following characterization of dimension for metric spaces.

Theorem 6. *(**Nagami-Roberts;** 1965) A metric space X has dimension $\leq n \Leftrightarrow$ there is a sequence $\{\mathcal{G}_k : k \in \mathbf{N}\}$ of open covers of X such that*

(1) *$\mathcal{G}_{k+1}$ refines $\mathcal{G}_k$ for all $k \in \mathbf{N}$;*
(2) *for all $x \in X$, $\{\mathrm{st}^2(x, \mathcal{G}_k) : k \in \mathbf{N}\}$ is a local base for x;*
(3) *$ord\,\mathcal{G}_k \leq n + 1$ for all $k \in \mathbf{N}$.*

In addition to its application in Theorem 5, Theorem 6 also has as a corollary Vopěnka's characterization of dimension for metric spaces.

Corollary 1. *(**Vopěnka;** 1959) A metric space X has dimension $\leq n \Leftrightarrow$ there is a sequence $\{\mathcal{G}_k : k \in \mathbf{N}\}$ of open covers of X such that*

(1) $\mathcal{G}_{k+1}$ *refines* $\mathcal{G}_k$ *for all* $k \in \mathbf{N}$;
(2) *mesh* $\mathcal{G}_k \to 0$;
(3) *ord* $\mathcal{G}_k \leq n+1$ *for all* $k \in \mathbf{N}$

The result of omitting (1) in Vopěnka's theorem gives the following metric-dependent dimension function.

Definition. (***metric dimension***) Let $< X, \rho >$ be a metric space. Then μdim $(X, \rho) \leq n$ if there is a sequence $\{\mathcal{G}_k : k \in \mathbf{N}\}$ of open covers of X such that mesh $\mathcal{G}_k \to 0$ and ord $\mathcal{G}_k \leq n+1$ for all $k \in \mathbf{N}$ (alternatively, for all $\epsilon > 0$, there is an open cover $\mathcal{G}$ of X such that mesh $\mathcal{G} < \epsilon$ and ord $\mathcal{G} \leq n+1$).

In 1949 Sitnikov gave an example showing that μdim$(X, \rho) <$ dim X is possible, and in 1958 Katětov proved that dim $X \leq 2\ \mu$dim(X, ρ) always holds.

In their second paper, Nagami and Roberts began their study of metric-dependent dimension functions. They introduced two dimension functions d_1 and d_2; the first is based on Ind and the second on a characterization of dimension due to Eilenberg and Otto.

Definition. Let $< X, \rho >$ be a metric space.

(1) $d_1(X, \rho) \leq n$ if for every pair of closed sets H and K in X with $\rho(H, K) > 0$, there is a closed set B in X such that B separates H and K and $d_1(B) \leq n-1$. ($d_1(\emptyset) = -1$.)
(2) $d_2(X, \rho) \leq n$ if given any $n+1$ pairs $C_1, C_1', \ldots C_{n+1}, C_{n+1}'$ of closed sets with $\rho(C_k, C_k') > 0$ for $1 \leq k \leq n+1$, there exist closed sets $B_1, \ldots, B_{n+1}$ such that B_k separates C_k and C_k' for $1 \leq k \leq n+1$ and $\cap B_k = \emptyset$. ($d_2(\emptyset) = -1$.)

Clearly $d_1(X, \rho) \leq$ dim X, and $d_2(X, \rho) \leq$ dim X follows from the Eilenberg-Otto characterization of dim X, which is obtained from d_2 by replacing "$\rho(C_k, C_k') > 0$" with the weaker condition "C_k and C_k' disjoint". For their first result, Nagami and Roberts show that d_1 and dim are the same.

Theorem 7. *(**Nagami-Roberts**; 1965) For any metric space* $< X, \rho >$,

$$dim X = d_1(X, \rho).$$

Corollary 2. *If X is a metric space with dim $X = 1$, and ρ is any compatible metric for X, then there is a pair of closed sets H and K with $\rho(H,K) > 0$ that cannot be separated by the empty set.*

Let $< X, \rho >$ be a metric space. It follows immediately from the definitions of d_1 and d_2 that

$$d_1(X,\rho) = 0 \Leftrightarrow d_2(X,\rho) = 0.$$

This result, together with Theorem 7, then gives: if dim $X = 1$, then $d_2(X,\rho) = 1$. Nevertheless, d_2 and dim are not the same.

Theorem 8. *(**Nagami-Roberts;** 1965) There is a subset X of $\mathbf{I}^3$ such that $d_2(X,\rho) = 1$ and* dim $X = 2$ *(ρ = Euclidean metric).*

In their last joint paper, Nagami and Roberts introduced and studied two more metric-dependent dimension functions.

Definition. Let $< X, \rho >$ be a metric space.

(1) $d_3(X,\rho) \leq n$ if given any m pairs $C_1, C_1', \ldots C_m, C_m'$ of closed sets with $\rho(C_k, C_k') > 0$ for $1 \leq k \leq m$, there exist closed sets $B_1, \ldots, B_m$ such that B_k separates C_k and C_k' for $1 \leq k \leq m$ and ord $\{B_k\} \leq n$. $(d_3(\emptyset) = -1.)$

(2) $d_4(X,\rho) \leq n$ if given any countable number of pairs $C_1, C_1', C_2, C_2', \ldots$ of closed sets with $\rho(C_k, C_k') > 0$ for all $k \geq 1$, there exist closed sets $B_1, B_2, \ldots$ such that B_k separates C_k and C_k' for all $k \geq 1$ and ord $\{B_k\} \leq n$. $(d_4(\emptyset) = -1.)$

Note: If we replace "$\rho(C_k, C_k') > 0$" with "C_k, C_k' disjoint" in the definition of d_3 or d_4, we obtain characterizations of dim X for the class of normal spaces due to Morita.

Theorem 9. *(**Nagami and Roberts;** 1967) The following hold for every metric space $< X, \rho >$:*

(1) $d_4(X,\rho) =$ dim$X$;

(2) $d_2(X,\rho) \leq d_3(X,\rho) \leq \mu\text{dim}(X,\rho) \leq \text{dim}\ X$;
(3) $d_3(X,\rho) = \mu\text{dim}(X,\rho)$ *for* ρ *totally bounded.*

Nagami and Roberts gave a number of complicated examples showing that d_2, d_3 and dim are not the same. The following is typical.

Theorem 10. ***(Nagami and Roberts;** 1967) There is a separable metric space* $< X,\rho >$ *such that* $d_2(X,\rho) = 2$, $d_3(X,\rho) = 3$, *and* $\dim\ X = 4$.

Nagami and Roberts end their last joint paper with a list of difficult problems (the last is still unsolved).

(1) Is dim $X \leq 2\ d_2(X,\rho)$ for all (separable) metric spaces $< X,\rho >$?
(2) Let $d_2(X,\rho) < k \leq \text{dim}\ X$. Is there a topologically equivalent metric ρ_k on X such that $d_2(X,\rho_k) = k$?
(3) Is there a metric space $< X,\rho >$ such that $d_3(X,\rho) \neq \mu\text{dim}(X,\rho)$?

Roberts and his student Slaughter gave a partial solution to (2) by proving the following result.

Theorem 11. ***(Roberts and Slaughter;** 1967) Let* $\mu\text{dim}(X,\rho) < k \leq \text{dim}\ X$. *Then there is a topologically equivalent metric* ρ_k *on* X *such that* $\mu dim(X,\rho_k) = k$.

Roberts then used Theorem 11 and Theorem 12 below to give another partial solution to (2).

Theorem 12. ***(Roberts;** 1967) Let* $< X,\rho >$ *be a separable metric space. Then there is a totally bounded metric* σ *on* X *that is topologically equivalent to* ρ *and preserves* d_2 *and* d_3.

Corollary 3. *Let* $< X,\rho >$ *be a separable metric space and let* $d_3(X,\rho) < k \leq \text{dim}\ X$. *Then there is a topologically equivalent metric* σ *on* X *such that* $d_3(X,\sigma) = k$.

Outline of Proof: By Theorem 12, we may assume that ρ is totally bounded. By Theorem 9(3), $\mu\text{dim}(X,\rho) = d_3(X,\rho)$.

Now apply Theorem 11 (where the new metric ρ_k can be shown to be totally bounded). □

Problem (2) was eventually solved by T. Goto (**Proc. Amer. Math. Soc. 58** (1976), 265-271).

In one of his very last publications, Roberts obtained what is perhaps the deepest theorem in metric-dependent dimension functions

Theorem 13. *(**Roberts;** 1970)* $\dim X \leq 2\ d_2(X, \rho)$ *for every metric space* $< X, \rho >$.

4. Roberts in the classrooom

The Ph.D. students of Roberts that we talked to unanimously agreed that Roberts was a remarkable teacher, in many cases the most memorable of their career. For example, L.R. King described his personality in the classrooom as "magnetic". Since Roberts was himself a student of R.L. Moore, it is interesting to speculate on the influence that Moore had on Roberts as a teacher.

One trait that they shared in common was the almost total avoidance of formal lectures in the classroom. For Roberts, a mathematics class was a dialogue between teacher and students, and the goal was for everyone to understand the topic of the day. The following story that Roberts often told about himself illustrates the point. One day Roberts was asked to teach a geometry class in one of the local high schools. Roberts began the class by posing a problem; within a very short time the entire class was interested, with everyone eager to contribute to the understanding of the problem. Just as the class was about to end, a satisfactory solution was obtained. Roberts then asked if everyone understood the solution, and the entire class signaled ***yes*** by raising their hands. Just as Roberts was about to walk out the door, a student on the first row said that she did have one question: "Dr. Roberts, what am I suppose to write in my notebook?"

To what extent did Roberts use the Moore method in his teaching? The experiences of the two authors, Hodel and Vaughan, differ here. Hodel took topology from Roberts in the fall of 1959, and three members of that class wrote their thesis with Roberts: Hodel, King and Rosenstein. That year Roberts used the Moore method to a great extent. There was no textbook; instead, the class was given a handwritten list of axioms (the axioms of a Moore space) and a list of theorems to prove from the axioms. Members of the class were challenged to find proofs and present them at the blackboard.

L.R. King commented that this method of teaching made the class itself so much fun. Both King and Rosenstein pointed out that Roberts differed from Moore in that he allowed, even encouraged, cooperation in finding proofs. Rosenstein further observed that Roberts was probably "too nice" to encourage the type of competition that is characteristic of Moore's method of teaching. Hodel remembers that by the end of the first year he definitely wanted to work in topology, but he did wonder how much he had learned in the course. To find out, he picked up a copy of Kelley's ***General Topology*** and began reading. He did not recognize many theorems, but he quickly realized that the techniques used to prove them were familiar!

Vaughan took topology from Roberts in the fall of 1961, and two members of that class wrote their thesis with Roberts: Vaughan and Wenner. That year Roberts used a textbook, the newly published ***Topology*** by Hocking and Young. Students were responsible for presenting proofs from the text to the class. Occasionally Roberts would give a formal lecture (!!) on an especially difficult topic, such as Tychonoff's result that any product of compact spaces is compact.

5. The Ph.D. Students of John H. Roberts (by date of graduation)

1940: Paul Wilner Gilbert and Abram Venable Martin, Jr.

1942: Paul Civin

1948: Samual Wilfred Han

1949: Ivey Clenton Gentry and Milton Preston Jarnagin, Jr.

1950: Lewis McLeod Fulton, Jr.

1952: Henry Sharp, Jr.

1955: William R. Smythe

1958: Arthur L. Gropen and Auguste Forge

1959: Nosup Kwak

1960: M. Jarad Saadaldin

1962: Richard E. Hodel

1963: L. Richardson King and George M. Rosenstein

1964: Bruce Richard Wenner

1965: Jerry Vaughan

1966: Frank Gill Slaughter, Jr. and James Wilkerson

1967: James C. Smith

1968: Leonard E. Soniat

1970: Glenn A. Bookhout and Joseph C. Nichols

6. Publications of John H. Roberts

1. *On a problem of C. Kuratowski concerning upper semi-continuous collections*, Fundamenta Mathematicae, **14** (1929), 96-102.
2. (with J.L. Dorroh), *On a problem of G.T. Whyburn*, Fundamenta Mathematicae, **13** (1929), 58-61.
3. *On a problem of Menger concerning regular curves*, Fundamenta Mathematicae, **14** (1929), 327-333.
4. *Concerning atroidic continua*, Monatsheften für Mathematik and Physik, **37** (1930), 223-230.
5. *A note concerning cactoids*, Bulletin of the American Mathematical Society, **36** (1930), 894-896.

6. *Concerning collections of continua not all bounded*, American Journal of Mathematics, **52** (1930), 551-562.
7. *Concerning non-dense plane continua*, Transactions of the American Mathematical Society, **32** (1930), 6-30.
8. *A non-dense plane continuum*, Bulletin of the American Mathematical Society, **37** (1931), 720-722.
9. *A point set characterization of closed two-dimensional manifolds*, Fundamenta Mathematicae, **18** (1931), 39-46.
10. *Concerning metric collections of continua*, American Journal of Mathematics, **53** (1931), 422-426.
11. *Concerning topological transformations in E_n*, Transactions of the American Mathematical Society, **34** (1932), 252-262.
12. *Concerning uniordered spaces*, Proceedings of the National Academy of Sciences **18** (1932), 403-406.
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