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TOPOLOGICAL ENTROPY OF A CLASS OF TRANSITIVE MAPS OF A TREE

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Abstract

Let T be a tree, $x \in T$ and $\mathrm{Val}_T(x)$ be the valence of x. For a continuous map f of T, $h(f)$ is the entropy of f. We study the class of transitive maps f of T with $f^{-1}(x) = \{x\}$. We prove that $h(f) > \frac{1}{\mathrm{Val}_T(x)} \log\ 3$, and show that there exist a sequence of transitive maps f_n of T with $f_n^{-1}(x) = \{x\}$ and $h(f_n) \longrightarrow \frac{1}{\mathrm{Val}_T(x)} \log 3$, $n \longrightarrow \infty$.

1. Introduction

Topological entropy of a transitive map of a tree has been studied by many authors [ABLM], [AKLS], [BC], [KH], [W] and [Y]. Particularly, in [ABLM] the authors obtain a lower bound of the topology entropies of the transitive maps of a given tree with a given point such that the preimage of this point under the maps is itself. In this paper we aim at proving the lower bound mentioned before can NOT be reached, but it is the infimum.

Let f be a continuous map of a topological space X into itself. We say f is *transitive* if for each pair of non-empty open subsets (U, V) of X there exists $n \in \mathbb{N}$ such that $f^n(U) \cap V \neq \emptyset$. By a *tree* we mean a connected compact one-dimensional polyhedron, which does not contain any subset homeomorphic to a circle and which contains a subset homeomorphic to $[0, 1]$. A *subtree*

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of a tree T is a subset of T, which is a tree itself. If T is a tree and $x \in T$, then the number of connected components of $T \setminus \{x\}$ is called the *valence* of x in T and will be denoted by $Val_T(x)$. A point of T of valence 1 is called an *end* of T. The set of the ends of T and the number of ends of T, will be denoted by $\mathrm{E}\,(T)$ and $\mathrm{End}\,(T)$ respectively. The set of the *vertices* of T is consisted of all the points of valences different from 2 and finitely many points of valence 2, which depends on our need. The set of the *vertices* of T will be denoted by $V(T)$. The closure of each connected component of $T \setminus V(T)$ is called an *edge* of T. The set and the number of the edges of T, will be denoted by $\mathrm{Edge}\,(T)$ and $\mathrm{EdgeN}\,(T)$ respectively. Note that a point $x \in \mathrm{Int}\,(T) = T \setminus E(T)$ will be said in the interior of T or an interior point of T.

Let T be a tree and $A \subset T$. We will use $[A]$ and $cl(A)$ to denote the smallest connected and closed subset containing A and the closure of A respectively. If $A = \{a, b\}$, then we use $[a, b]$ to denote $[A]$. We define $(a, b) = [a, b] \setminus \{a, b\}$ and we similarly define $(a, b]$ and $[a, b)$.

We will use $F(T, x)$ to denote the set of transitive maps of a tree T such that the preimage of $x \in T$ under this map is x itself.

The following theorem is the Proposition 4.2 of [ABLM].

Theorem 1.1. *Let $f : T \longrightarrow T$ be a transitive map of a tree T and $x \in T$ with $\{x\} = f^{-1}(x)$. Then f has 3-horseshoe if x is an end of T, and f^k ($k = \mathrm{Val}_T(T)$) has 3-horseshoe if x is in the interior of T.*

We have the following immediate consequence.

Corollary 1.2. *Let T be a tree, and $x \in T$. Then $h(f) \geq \frac{1}{\mathrm{Val}_T(x)} \log\ 3$ for each $f \in F(T, x)$.*

2. Entropy Estimate

In this section we shall prove that the topology entropy $h(f)$ can't reach the lower bound $\frac{1}{\mathrm{Val}_T(x)}\log 3$, for each $f \in F(T,x)$, where T is a tree and $x \in T$. That is, we will prove

Theorem 2.1. *Let T be a tree, x be a point of T and f be a transitive map of T such that $f^{-1}(x) = \{x\}$. Then the topological entropy $h(f) > \frac{1}{\mathrm{Val}_T(x)}\log 3$.*

To prove this theorem, we need the following lemmas

Lemma 2.2. *Let T be a tree and $f : T \longrightarrow T$ be transitive. Then exactly one of the following alternatives holds.*

(1) *f^n is transitive for each $n \in \mathbb{N}$.*

(2) *There is $n_0 > 1$ such that there are an interior fixed point y of f and subtrees $T_1, \ldots, T_{n_0}$ of T with $\cup_{i=1}^{n_0} T_i = T$, $T_i \cap T_j = \{y\}$ for all $i \neq j$ and $f(T_i) = T_{i+1(mod\ \mathrm{n}_0)}$ for $1 \leq i \leq n_0$. Moreover, $f^{n_0}|_{T_i}$ is transitive for each $1 \leq i \leq n_0$.*

The proof of Lemma 2.2 can be found in [AKLS]. And the following corollary is obvious.

Corollary 2.3. *Let T be a tree, and $\mathrm{E}\,(T)$ contain a fixed point of f, then f^s is transitive for each $s \geq 1$.*

Lemma 2.4. *Let T be a tree and f be a map of T such that f^s is transitive for each $s \in \mathbb{N}$. Suppose H is a subtree of T with $H \cap \mathrm{End}\,(T) = \emptyset$, and J is an open subtree of T. Then there is $n \in \mathbb{N}$ such that $H \subset f^n(J)$.*

The proof of Lemma 2.4 is similar to that of Proposition 44 of [BC].

Lemma 2.5. *Let T be a tree , $x \in \mathrm{E}\,(T)$ and $f \in F(f,x)$, then $h(f) > log\ 3$.*

Proof. We identify each edge containing x with the unit interval $[0,1]$ (with $x=0$), and use the usual ordering $<$ to describe the relative position of points of this edge. Points not on this edge (if any) will be unimportant.

Set $y_0 = \min\{f(y) : y \in T\backslash[0,1], f(y) \in [0,1]\}$, $y_1 = \min\{y \in [0,1] : f(y) = 1\}$ and $y_2 = \min\{y_0, y_1\}$. We need from transitivity only that if $y \in [0,1]$ then none of the sets $[0,y]$ and $T \setminus [0,y)$ is f-invariant (plus we use the assumption $f^{-1}(0) = \{0\}$), and we will mean that when we say "by transitivity". Let A_l (resp. A_r) be the set of those points of $[0,y_2]$ that lead movement to the left (resp. right). That is, $y \in A_l$ if $y \in [0,y_2]$, $f(y) \le y$ and $f(z) \ge f(y)$ for all $z \in [y,1]$; similarly $y \in A_r$ if $y \in [0,y_2]$, $f(y) \ge y$ and $f(z) \le f(y)$ for all $z \in [0,y]$. Clearly, A_l and A_r are closed. By transitivity, $A_l \cap A_r = \{0\}$. Also by transitivity and by the definitions of A_l and A_r, one can easily show the following properties:

(1) $A_l \cap (0,y] \ne \emptyset$ for each $y \le y_2$,
(2) $A_r \cap (0,y] \ne \emptyset$ for each $y \le y_2$,
(3) $(f(y),y) \cap A_l \ne \emptyset$ for each $y \in A_l$, and
(4) $(y,f(y)) \cap A_r \ne \emptyset$ for each $y \in A_r$ such that $f(y) \le y_2$.

Since A_l and A_r are closed and by properties (1) and (2) above, there are points $w < t$ with $w \in A_r$, $f(w) \le y_2$, $t \in A_l$ and no points of $A_l \cup A_r$ in (w,t). By properties (3) and (4) above, there are points $v \in (f(t),t) \cap A_l$ and $u \in (w,f(w)) \cap A_r$. Hence , we get

$$f([v,w]) \supset [f(v),f(w)] \supset [v,u],$$

$$f([w,t]) \supset [f(t),f(w)] \supset [v,u],$$

and

$$f([t,u]) \supset [f(t),f(u)] \supset [v,u].$$

Notice that $f(w) > u, f(t) < v$. Therefore, there are three pairwise disjoint intervals I_1,I_2 and $I_3 \subset [v,u]$ such that $f(I_i) = [v,u], i = 1,2,3$. Thus, there exists an open interval $J \subset [v,u] \setminus (I_1 \cup I_2 \cup I_3)$. By Lemma 2.4 , there exists $k \in \mathbb{N}$ such that

$f^k(J) \supset [v,u]$. It's obvious that there are 3^k pairwise disjoint closed intervals $J_i \subset I_1 \cup I_2 \cup I_3, i = 1, \ldots, 3^k$ such that $f^k(J_i) = [v,u]$. Hence the intervals among $\{J\} \cup \{J_i\}_{i=1}^{3k}$ are pairwise disjoint, in other words, f^k has (3^k+1)-horseshoe. This implies

$$k\ h(f) = h(f^k) \geq \log(3^k + 1) > k\ \log 3.$$

That is, $h(f) > \log\ 3$. This ends the proof of Lemma 2.5. □

Proof of Theorem 2.1: Set $h = Val_T(x)$. By the assumption $f^{-1}(x) = \{x\}$ and the transitivity of f, we have $f(T_i) = T_{i+1(mod\ h)}$, where $\{T_i\}_{i=0}^{h-1}$ is the set of the closures of the components of $T \setminus \{x\}$. Moreover, T_i is f^h-invariant and $f^h|_{T_i}$ is transitive, $0 \leq i \leq h-1$. By Lemma 2.3, we have that $h\ h(f) = h(f^h) \geq h(f^h|_{T_i}) > \log\ 3$. Consequently, $h(f) > \frac{1}{h}\log 3 = \frac{1}{\mathrm{Val}_T(x)}\log 3$.

3. The Lower Bound is Infimum

In this section we shall prove that the lower bound obtained in Theorem 2.1 is infimum. That is,

Theorem 3.1. *Let T be a tree, $x \in T$, and $\epsilon > 0$. Then there exists $f \in F(T,x)$ such that $h(f) < \frac{1}{\mathrm{Val}_T(x)}\log 3 + \epsilon$. Consequently, $\frac{1}{\mathrm{Val}_T(x)}\log 3 = \inf\{h(f)|f \in F(T,x)\}$.*

To prove Theorem 3.1, we need the following lemmas.

Lemma 3.2. *[KH P.123] Let T be a tree, f be a continuous map of T and d be a metric on T such that there exists a positive real number L with $d(f(a), f(b)) \leq L\ d(a,b)$ for each $a, b \in T$. Then $h(f) \leq \max\{0, \log L\}$.*

For each edge e of a tree T, e is homeomorphic to some interval I_e under some homeomorphism $\varphi_e : e \to I_e$. D_T is called an *atlas* of T, if D_T is a map which maps each edge e of T to some homeomorphism φ_e from e to $I_e = [0, t_e](t_e > 0)$.

Let T be a tree and S be a subset of T. S is called *measurable*, if for each edge e, $\varphi_e(S \cap e)$ is Lebesgue measurable as a subset of I_e. For each edge e, we can define a measure m_e such that $m_e(A) = \mu_{I_e}(D_T(e)(A)) = \mu_{I_e}(\varphi_e(A))$ for each measurable subset A of e, where μ_{I_e} is the Lebesgue measure of I_e. For a measurable subset $S \subset T$, let

$$m_{D_T}(S) = \sum_{e \in \mathrm{Edge}\,(T)} m_e(S \cap e).$$

Clearly, m_{D_T} is a *measure* of T completely determined by D_T.

For any two points a and b of T, let

$$d_{D_T}(a, b) = m_{D_T}([a, b]).$$

Clearly, d_{D_T} is a *metric* of T which is completely determine by D_T. And it is compatible to the original topology on T (inherited from $\mathbb{R}^2$).

Definition 3.3. *Let T and T' be two trees and f be a map from T to T'. D_T and $D_{T'}$ are atlases on T and T' respectively. We call f a* PL-map*(or, piecewise linear map) if for each $e \in \mathrm{Edge}\,(T)$, $J = f(e)$ is an interval of T', and $f|_e$ is a non-degenerate linear function from e to J, i.e. there exists $c > 0$ such that $d_{D_{T'}}(f(a), f(b)) = c\ d_{D_T}(a, b)$,and we call c the* scale *of f on e.*

Here is a property of a PL-map:

Lemma 3.4. *Let T and T' be two trees, and f be a PL-map from T to T'. Then there is $L' > 0$, such that $m_{D_{T'}}(f(S)) \geq L'\ m_{D_T}(S)$ for each subtree S of T.*

Proof. Let S be a subtree of T. By the definition of m_{D_T}, $m_{D_T}(S) = \sum_{e \in \mathrm{Edge}\,(T)} m_{D_T}(S \cap e)$. Then we have

$$\begin{aligned} m_{D_{T'}}(f(S)) &= m_{D_{T'}}(f(\cup_{e \in \mathrm{Edge}\,(T)}(S \cap e))) \\ &= m_{D_{T'}}(\cup_{e \in \mathrm{Edge}\,(T)} f(S \cap e)) \\ &\geq c_e\ m_{D_T}(S \cap e), \end{aligned}$$

for each edge e of T, where c_e is the scale of f on e. It is easy to see that there exists $e_0 \in \mathrm{Edge}\,(T)$ such that $m_{D_T}(S \cap e_0) \geq \frac{1}{\mathrm{EdgeN}\,(T)} m_{D_T}(S)$. Therefore,

$$\begin{aligned} m_{D_{T'}}(f(S)) &\geq c_{e_0}\ m_{D_T}(S \cap e_0) \\ &\geq \frac{\min\{c_e | e \in \mathrm{Edge}\,(T)\}}{\mathrm{EdgeN}\,(T)} m_{D_T}(S) \\ &= L'\ m_{D_T}(S), \end{aligned}$$

by letting

$$L' = \frac{\min\{c_e | e \in \mathrm{Edge}\,(T)\}}{\mathrm{EdgeN}\,(T)}.$$

This ends the proof of Lemma 3.4. □

We call L' an L'-constant of f, and write $L'(f) = L'$. We have some properties showing relations between atlas, measure, metric and PL-map. Note that if D_T is an atlas of T, and $c, c' > 0$, then cD_T will denote the atlas with $(cD_T)(e) = cD_T(e)$ for each $e \in Edge(T)$.

Remark 3.5.

(i) $m_{cD_T} = cm_{D_T}$, $d_{cD_T} = cd_{D_T}$, *and*

(ii) if f *is a PL-map from* T *(related to* D_T*) to* T' *(related to* $D_{T'}$*), then* f *is also a PL-map from* T *(related to* cD_T*) to* T' *(related to* $c'D_{T'}$*).*

Moreover, (1) $L_{cD_T, c'D_{T'}}(f) = \frac{c'}{c} L_{D_T, D_T}(f)$, $L'_{cD_T, c'D_{T'}}(f) = \frac{c'}{c} L'_{D_T, D_{T'}}(f)$, *where* $L_{D_T, D_{T'}}(f)$, $L'_{D_T, D_{T'}}(f)$ *denote the Lipschitz constant and L'-constant (related to* D_T *and* $D_{T'}$*) respectively.*

(2) if $c = c'$, *then* $L_{cD_T, c'D_{T'}}(f) = L_{D_T, D_{T'}}(f)$ *and* $L'_{cD_T, c'D_{T'}}(f) = L'_{D_T, D_{T'}}(f)$.

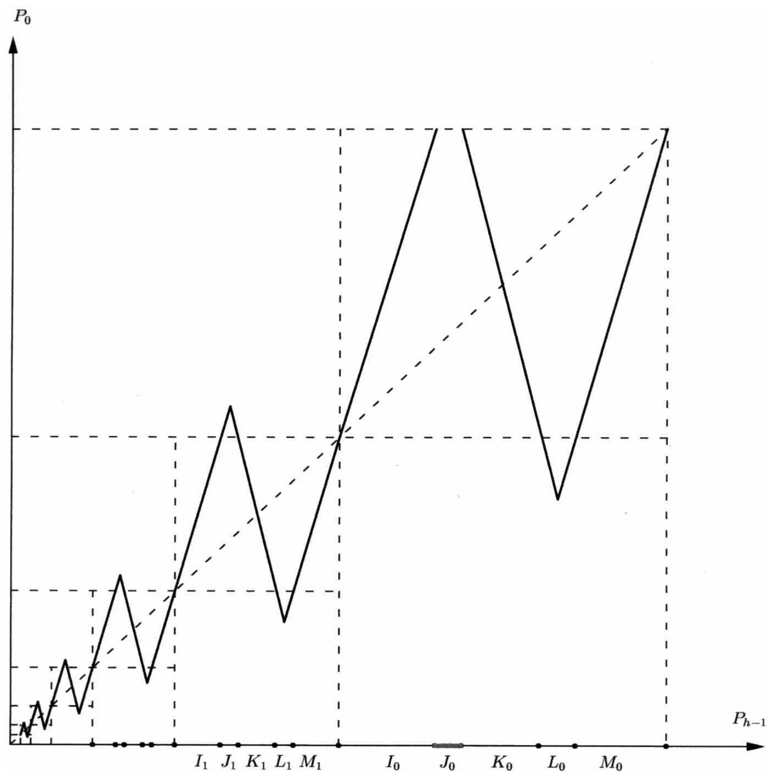

Fig. 1.

Proof of Theorem 3.1: Let $h = \mathrm{Val}_T(x)$. Then $T \setminus \{x\}$ has exactly h connected components, whose closures will be denoted by T_i, $i = 0, ..., h-1$. We choose an interior point y_i in the edge of T_i, which contains x. And we add y_i as a new vertice of T and T_i. Then T_i can be represented as $P_i \cup T_i'$, where $P_i = [x, y_i]$ which is an edge of T_i. For any given $\epsilon > 0$, which is small enough, we want to construct a continuous map $f \in F(T, x)$ which has the topological entropy less than $\frac{1}{\mathrm{Val}_T(x)} \log 3 + \epsilon$.

To do this, first we choose a homeomorphism $\varphi_{P_i} : P_i \longrightarrow [0,1]$ with $\varphi_{P_i}(x) = 0$ for each $0 \leq i \leq h-1$, and then we construct maps f_i from T_i to $T_{i+1(mod\ h)}$ (see Figure 1) with the following properties:

(a) For $0 \leq i \leq h-2$, let f_i map P_i onto P_{i+1} with $f_i = \varphi^{-1}_{P_{i+1}} \circ \varphi_{P_i}$ satisfying that

(1) $f_i|_{T'_i} : T'_i \longrightarrow T'_{i+1}$ is a surjective PL-map, and

(2) there is a constant $L_i \leq 3^{\frac{1}{h}}...(*)$, such that $d_{D_{T_{i+1}}}(f_i(x), f_i(y)) \leq L_i\ d_{D_{T_i}}(x,y)$ for any $x,\ y \in T_i$, $0 \leq i \leq h-2$.

By Lemma 3.4, there exists $L'_i > 0$ such that $m_{D_{T_{i+1}}}(f_i(S)) \leq L'_i\ m_{D_{T_i}}(S)$ for any subtree S of T'_i .

(b) For convenience, we identify $P_i (0 \leq i \leq h-1)$ with $[0,1]$, i.e. for each $a, b \in [0,1]$ denote the point $(D_{T_i}(P_i))^{-1}(a)$ simply by a, and denote $[(D_{T_i}(P_i))^{-1}(a), (D_{T_i}(P_i))^{-1}(b)] \subset P_i$ simply by $[a,b] \subset [0,1]$. Note that, this will be also used in the rest part of this paper.

Let f_{h-1} map T_{h-1} onto T_0 satisfying

(1) $f_{h-1}(u^l_0) = f_{h-1}(u^r_0) = 1$, $f_{h-1}(v_j) = v_j, j \geq -1$,

$f_{h-1}(u_j) = v_{j-1} + \frac{\epsilon}{2^j}, j \geq 1$, and $f_{h-1}(w_j) = v_j + \frac{\epsilon}{2^j}, j \geq 0$,

where

$v_j = \frac{1}{2^{j+1}}$, $u_j = v_j + \frac{1}{3}\frac{1}{2^{j+1}}$, $w_j = v_j + \frac{2}{3}\frac{1}{2^{j+1}}$, $j \geq 0$,

$u^l_0 = u_0 - \delta$, and $u^r_0 = u_0 + \delta$ (δ will be determined later).

(2) Moreover, f_{h-1} is linear on intervals
$\{[v_0, u^l_0], [u^r_0, w_0], [w_0, v_{-1}]\} \cup \{[v_j, u_j], [u_j, w_j], [w_j, v_{j-1}]\}^{\infty}_{j=1}$
respectively (see Figure 2).
For each $j \geq 1$, let $u^l_j < u^r_j$ be those of $[v_j, v_{j-1}]$ such that $f(u^l_i) = f(u^r_i) = v_i$. And
for each $j \geq 0$, let $w^l_j < w^r_j$ be those of $[v_j, v_{j-1}]$ such that $f(w^l_j) = f(w^r_j) = v_j$.

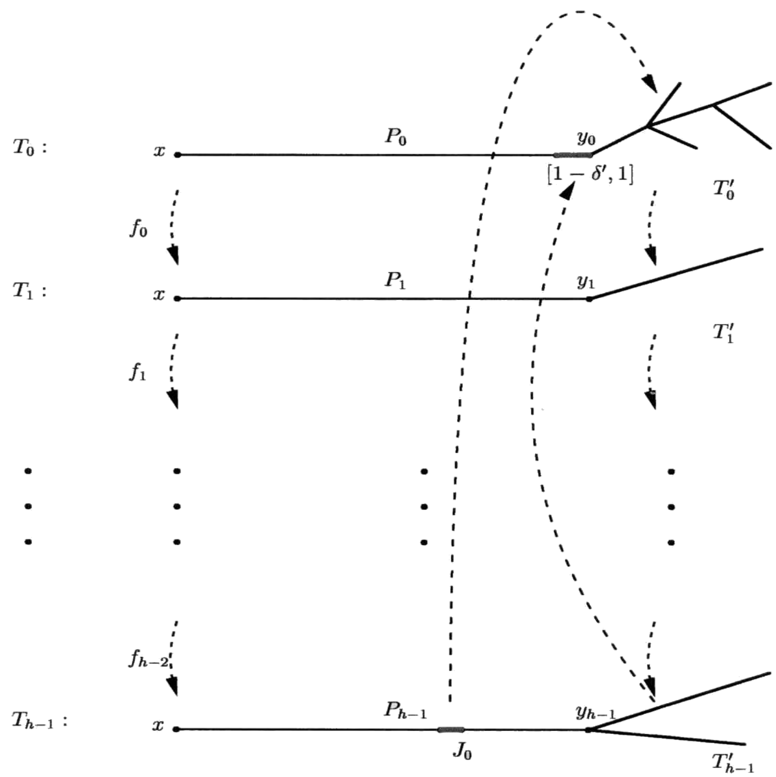

Fig. 2.

(3) For each $j \geq 0$, let

$I_j = [v_j, u_j^l]$, $J_j = [u_j^l, u_j^r]$, $K_j = [u_j^r, w_j^l]$, $L_j = [w_j^l, w_j^r]$, and $M_j = [w_j^r, v_{j-1}]$. Then we can choose a PL-map $f_{h-1}|_{J_0}$ from J_0 onto T'_0 such that $L_{D_{J_0}, D_{T'_0}}(f_{h-1}|_{J_0}) \leq 3^{\frac{1}{h}}$ for some atlases D_{J_0} (we take J_0 as a new edge of T) and $D_{T'_0} \ldots$(**). This can be done by using Definition 3.3 and Remark 3.5 (ii).

(4) It is easy to see that we can obtain a PL-map $f_{h-1}|_{T'_{h-1}}$ onto $\phi_{T_0}^{-1}[1-\delta', 1]$ (by choosing δ'), such that $L(f_{n-1}|_{T'_0}) \leq 3^{\frac{1}{h}} \ldots$(***).

Let $f = \cup_{i=0}^{h-1} f_i$. Then f is a continuous map of T with $f^{-1}(x) = \{x\}$.

Therefore, to prove Theorem 3.1 it's enough to show that we can choose ϵ, δ, and δ' such that f is a transitive map of T and its entropy is "very" close to $\frac{1}{\mathrm{Val}_T(x)} \log 3$.

Let $F = f^h|_{T_0} = f_{h-1} \circ f_{h-2} \circ ... \circ f_0$. By (*),(**) and (***) , we know that the Lipschitz constant $L(F) \leq \frac{3+6\epsilon}{1-6\delta}$, where

$$\lim_{\epsilon \longrightarrow 0^+, \delta \longrightarrow 0^+} \frac{3+6\epsilon}{1-6\delta} = 3.$$

By Lemma 3.1, $h(F) \leq \log(\frac{3+6\epsilon}{1-6\delta})$. Hence,

$$\begin{aligned} h(f) &= \frac{1}{h} h(f|_{T_0}^h) \\ &= \frac{1}{\mathrm{Val}_T(x)} h(F) \longrightarrow \frac{1}{\mathrm{Val}_T(x)} \log 3 (\epsilon \longrightarrow 0^+, \delta \longrightarrow 0^+). \end{aligned}$$

By Lemma 3.2 there exists $L_i' > 0$ such that $m_{D_T}(f_i(S)) \geq L_i' \; m_{D_T}(S)$, $0 \leq i \leq h-1$, for any subtree S of T_i', $0 \leq i \leq h-1$. Thus, there exists $L_{J_0}' > 0$ such that $m(f(S)) \geq L_{J_0}' \; m(S)$ for each subtree S of T_0'. Let $L' = \prod_{i=0}^{h-1} L_i'$. Now, we need to choose $\delta' > 0$ and $n' \in \mathbb{N}$ such that

(1) $\delta'(3+6\epsilon)^{n'} < \frac{1}{6}$,

(2) $L' \; 3^{n'} > 1, and$

(3) $L_{J_0}' L' \; 3^{n'} > 1$.

This can be done by letting

(i) $n' > \frac{\max(\frac{1}{L'}, \frac{1}{L_{J_0}' \; L'})}{\log(3+6\epsilon)}$, and

(ii) $\delta' < \frac{1}{12} \; \frac{1}{(3+6\epsilon)^{n'}}$

In order to preserve the constants L_i, L_i', L_{J_0} *and* L_{J_0}' , we need to change the atlas D_T of T on the subforest $(\cup_{0 \leq j \leq h-1} T_j') \cup f_{h-1}(J_0)$ and decrease δ (by Remark 3.5 (ii)).

Now, it is easy to see that

(I) For each closed subtree T' of $T_0 \setminus \{x\}$, and each subtree S of T_0 containing some point from $V = \{v_i, u_i^l, u_i^r, w_i^l, w_i^r\}_{i=0}^{\infty} \cup \{1\}$, there exists $n \in \mathbb{N}$ such that $F(S) \supset T'$.

(II) We claim that, for each subtree S of T_0, there exists $N \in \mathbb{N} \cup \{0\}$ such that $F^N(S) \cap V \neq \emptyset$. Assume the contrary, i.e., $F(S) \subset U \setminus V$, for each $n \in \mathbb{N} \cup \{0\}$, where $U \in \{I_i, J_i, K_i, M_i, L_i\}_{i=0}^{\infty} \cup \{T_0'\}$. Then
(1) If $U \in \{J_i, L_i\}_{i=1}^{\infty} \cup \{L_0\}$ then $m_{D_{T_0}}(F(S)) \geq \frac{3}{2}\, m_{D_{T_0}}(S)$,
(2) if $U = J_0$, then $F^{n'}(S) \subset M_0$ (by (****)) and $m_{D_{T_0}}(F^{n'}(S)) \geq L'_{J_0}\ 3^{n'}\ m_{D_{T_0}}(S)$.
(3) if $U \in \{I_i, K_i, M_i\}_{i=0}^{\infty} \cup \{I_0, K_0\}$, then $m_{D_{T_0}}(F(S)) \geq 3m_{D_{T_0}}(S)$,
(4) if $U = T_0'$, $m_{D_{T_0}}(F^{n'}(S)) \geq L'\ 3^{n'}\ m_{D_{T_0}}(S)$.
For $m_{D_{T_0}}(S) > 0$, $L' > 1$, we have

$$\limsup_{n \longrightarrow +\infty} m_{D_{T_0}}(F^n(S)) = +\infty,$$

a contradiction. This proves the claim.

By (I) and (II) we get that F is transitive. It follows that f is transitive, too.

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