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TOPOLOGY IN EASTERN EUROPE 1900 – 1950

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In this paper[1] we give a brief account of the development of topology in Eastern Europe between 1900 and 1950. The exposition is by countries where research as well as events relevant for the development of research took place and where the main protagonists were predominantly active or where they came from. Under Eastern Europe we understand areas which during the period between the two World Wars belonged to Poland, USSR, Czechoslovakia, Hungary, Romania, Bulgaria, Yugoslavia, Greece and Turkey. The countries are ordered by the approximative dates of the appearence of the first organized forms (seminars) of research in topology.

Topology emerged as a separate branch of mathematics at the end of the nineteenth and the beginning of the twentieth century as a result of efforts to understand such basic notions like continuity, convergence, connectedness and dimension. In this process real analysis, complex analysis and algebraic geometry played a major role. The main contributions were made in Western Europe, especially by Bernhard Riemann (1826-1866), Georg Cantor (1845-1918), Jules Henri Poincaré (1854-1912), Felix Hausdorff (1868-1942), Maurice Fréchet (1878-1973) and Luitzen Egbertus Jan Brouwer (1881-1966). Before 1900 in Eastern Europe there was no research which could be classified as topology. It appears that in Eastern Europe the first paper on topology was written

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in 1904 by the Hungarian mathematician Frigyes (Frédéric) Riesz [151].

Poland. At the beginning of the twentieth century in Poland three main centers of research in mathematics were emerging. One in Cracow, headed by Stanisław Zaremba (1863-1942) and oriented towards analysis. One in Lwów, headed by Hugo Steinhaus (1887-1972) and Stefan Banach (1892-1945), oriented towards functional analysis and one in Warsaw, oriented towards set theory and topology. The Warsaw group was formed by Wacław Sierpiński (1882-1969), Zygmunt Janiszewski (1888-1920) and Stefan Mazurkiewicz (1888-1945). Sierpiński studied in Warsaw and obtained his Ph.D. degree from the Jagellonian University in Cracow in 1906. Mazurkiewicz obtained his Ph.D. in 1913 in Lwów under Sierpiński, while Janiszewski obtained the Ph.D. in 1911 in Paris. Mazurkiewicz and Janiszewski started a topology seminar in Warsaw already in 1917 and were soon joined by Sierpiński, who was in 1915 interned in Russia by the Tsarist authorities and returned to Poland and Warsaw only in 1918. This was the first topology seminar in Eastern Europe and probably the first in the World.

In the beginning of his career Sierpiński was primarily interested in number theory. Since 1908 his main interest became set theory. During his stay in Russia he began a fruitful cooperation with N.N. Luzin in descriptive set theory and the theory of functions of a real variable and after his return to Poland, he continued to contribute to those areas. Sierpiński's contributions to topology refer primarily to disconnected spaces and continua. In [161] Sierpiński gave the first example of a punctiform space (this class of spaces was introduced by Janiszewski in [79]), which is not hereditarily disconnected. In [163] he introduced totally disconnected spaces and gave the first example of such a space which is not 0-dimensional. In the same paper he also exhibited a hereditarily disconnected space, which is not totally disconnected. In [159] Sierpiński constructed a planar 1-dimensional continuum, which contains homeomorhic copies of all planar 1-dimensional continua. This is the famous *Sierpiński carpet*. Another important topological result of Sierpiński asserts that no continuum can be decomposed into countably many disjoint non-empty closed subsets [160]. Moreover, in

[162] Sierpiński gave a criterion for the local connectedness of continua, known as propery S. In [164] he topologically characterized completely metrizable separable spaces. In 1928 Sierpiński published one of the first books on general topology. It was written in Polish and its English translation appeared in 1934 [165].

Janiszewski worked primarily in continua theory and in the topology of the plane. In 1912 he published his important thesis on continua irreducible between two points [79]. Next year appeared his habilitation paper [80] containing fundamental results on the topology of the plane. In 1913 (independently of H. Hahn) Mazurkiewicz characterized continuous images of the arc as locally connected metrizable continua [117], [118]. He was also interested in indecomposable and hereditarily indecomposable continua. In spite of the fact that the latter are difficult to imagine, Mazurkiewicz proved in [121] that most subcontinua in the square are hereditarily indecomposable, more precisely, the remaining subcontinua form a set of the first Baire category. This paper made him one of the pioneers of the Baire category method in topology. Mazurkiewicz also made important contributions to dimension theory. E.g., he exhibited the first examples of separable (completely) metrizable totally disconnected spaces having an arbitrarily high dimension [119]. Moreover, he proved that no open connected subset of the n-cell can be cut by an $(n-2)$-dimensional subset [120].

Among the first members of the Warsaw topology seminar were Bronisław Knaster (1893-1980) and Kazimierz Kuratowski (1896-1980). Knaster began his work with a major paper on connectedness of spaces, written jointly with Kuratowski in 1921 [92]. In 1922 Knaster discovered the first hereditarily indecomposable continuum, today known as the *pseudoarc* [91]. In the best tradition of Polish topology throughout his career he continued to construct unexpected examples, which showed all the wealth of the new mathematical world opened by set-theoretical methods. The majority of Knaster's papers deal with 1-dimensional continua. In 1929 B. Knaster, K. Kuratowski and S. Mazurkiewicz published a short and elegant proof of Brouwer's fixed point theorem which asserts that the n-cell has the fixed point property [93].

Kuratowski's first interest in topology referred to continua, especially to irreducible continua [100], [101]. To lay foundations for his research he defined topological spaces using the closure operator [99]. He devoted several papers to the topology of the 2-sphere, including its topological characterization [102]. Another result, which became widely known, was Kuratowski's characterization of planar graphs [103]. In 1933 Kuratowski (independently of W. Hurewicz) introduced the very useful notion of a canonical mapping subordinated to an open covering [104]. He also made important contributions to higher dimensional local connectedness [106] and to the topology of hyperspaces [107]. Certainly, an important event in the history of topology was the publication of Kuratowski's textbook on topology. The first volume appeared in 1933 [105]. In 1939 the second volume was ready for publication. However, due to the Second World War, it was published only in 1950 [108]. The revised English edition of the complete work appeared in 1966 and 1968, respectively.

An important event in the early history of topology was the appearance in 1920 of the first specialized journal *Fundamenta Mathematicae*, founded by Janiszewski. Unfortunately, Janiszewski died shortly before the publication of the first volume.

In the next generation of Polish topologists one encounters three outstanding names, Witold Hurewicz (1904-1956), Karol Borsuk (1905-1982) and Samuel Eilenberg (1913-1998). Hurewicz was born in Łodź. He studied in Vienna, where he obtained the Ph.D. in 1926. He then spent ten years in Amsterdam and in 1936 emigrated to U.S. There he held positions at the Institute for Advanced Study in Princeton, at the University of North Carolina, at Brown University in Providence, RI and at M.I.T. in Cambridge, MA. Of the many fundamental contributions he made, we mention the characterization of n-dimensional compacta as images of 0-dimensional spaces under mappings whose fibers have $\leq n+1$ points [70] and the generalization of dimension theory from metric compacta to separable metric spaces. In particular, introducing the technique of function spaces and the Baire category theorem, he gave an elegant proof of the fact that n-dimensional separable metric spaces embed in $\mathbb{R}^{2n+1}$ [71], [72]. His 1941 book on dimension theory, written jointly with the American and Swedish mathematician and engineer Henry Wallman (1915-1992), became a classic [78]. Hurewicz is generally considered to be one of the founders of homotopy theory. In particular, he introduced (independently of E. Čech) the n-dimensional homotopy groups of spaces [73] and proved the theorem, which now carries his name, and asserts that in an $(n-1)$-connected space X, $n \geq 2$, the n-th homotopy group $\pi_n(X)$ coincides with the n-th singular homology group $H_n(X)$ with integer coefficients [74]. Concerning his other contributions to algebraic topology, let us mention that he introduced exact sequences (for cohomology groups of a pair) [75]. The name *exact* was given only later, by Eilenberg and MacLane (after careful considerations). Furthermore, based on previous work of H. Seifert and H. Whitney, Hurewicz and N.E. Steenrod defined fiber spaces in terms of *slicing functions* [77]. The final development of the notion of *fibration* (generalizing Serre fibrations) was given in Hurewicz's paper [76].

Throughout his whole career Borsuk was associated with the University of Warsaw and the Institute of Mathematics of the Polish Academy of Science. His primary interest was geometric topology. Borsuk is best known for laying the foundations of two topological theories, the theory of retracts [24], [25] and the theory of shape. His book on retracts [32], published in 1967, surveys the state of the theory at that time. Shape theory is more recent and beyond the period considered in this article. In 1933 Borsuk proved the famous Borsuk-Ulam theorem [26], which was conjectured by Stanisław Ulam (1909-1984). The theorem asserts that every mapping of the n-sphere to $\mathbb{R}^n$ assumes the same values at some pair of antipodal points. In 1933 Borsuk published another important paper in which he proved the theorem on simultaneous extension of bounded real-valued mappings from a closed separable subset A of a metric space to all of X [27]. In 1936 he introduced the cohomotopy groups [30]. One of the most useful results of Borsuk is his homotopy extension theorem [31]. In the tradition of his predecessors, Borsuk constructed many important examples. For instance, he exhibited an acyclic continuum in $\mathbb{R}^3$, which does not have the fixed-point property [28]. Another famous example of Borsuk is the *dunce hat*, a $2-$dimensional CW-complex, which is contractible but not collapsible [29]. Borsuk stimulated much research by raising relevant and challenging problems. For example, at the International Congress of Mathematicians held in Amsterdam in 1954, he asked, does every compact ANR has the homotopy type of a compact polyhedron? The problem was solved in the affirmative only in 1975 by J.E. West (born in 1944).

Eilenberg wrote his Ph.D. thesis on mappings into the 1-sphere [51] in 1936 in Warsaw, under the influence of Borsuk. In 1939, while still in Poland, he developed the theory of obstructions [52]. The same year he emigrated to U.S. and became one of the founders of modern algebraic topology [56] and homological algebra [53], [36]. His book with Norman Earl Steenrod (1910-1971) on the foundations of algebraic topology, published in 1952, and his book with Henry Cartan (born in 1904) on homological algebra, published in 1956, belong to the classics. Together with Saunders MacLane (born in 1909), he laid the foundations of the theory of categories, which greatly influenced twentieth century mathematics [54]. Another important contribution was the introduction of the Eilenberg-MacLane complexes [55].

To the considered period also belong Edward Marczewski (earlier name Szpilrajn) (1907-1976) and Manachem Wojdysławski (born in 1918), who fell victim of the Nazi holocaust in 1942 or 1943. Between the two World Wars, Marczewski was active in Warsaw, and after the Second World War he moved to Wrocław. His main fields of interest were measure theory, descriptive set theory, general topology, probability theory and universal algebras. Among other things, Marczewski studied the relationship between the Hausdorff dimension $\dim_{\mathcal{H}} X$ (defined in terms of the Hausdorff measure) and the covering dimension $\dim X$ of a separable metric space X. In particular, he proved [175] that $\dim X \leq n$ if an only if the greatest lower bound of the Hausdorff dimensions $\dim_{\mathcal{H}} X'$ of homeomorphic copies X' of X in the $(2n+1)$-cube is $\leq n$ (the inequality $\dim X \leq \dim_{\mathcal{H}} X'$ was previously obtained by G. Nöbeling). Marczewski also proved the important fact that the direct product of an arbitrary family of spaces of countable weight has the Suslin property [158]. In 1938 M. Wojdysławski proved that, for locally connected metric continua X, the hyperspace 2^X of nonempty closed subsets is contractible and locally contractible [191]. Next year he improved this result by showing that 2^X is actually an absolute retract [192].

USSR. For the emerging of topology in Russia of great importance was the school of the theory of functions of a real variable, initiated in Moscow in 1911 by Dmitriĭ Fedorovich Egorov (1869-1931) and continued in 1914 by Nikolaĭ Nikolaevich Luzin (1883-1950). The first Russian topologists Mikhail Yakovlevich Suslin (1894-1919), Pavel Sergeevich Aleksandrov (1896-1982) and Pavel Samuilovich Uryson (1898-1924) began their careers in Luzin's seminar. In their first papers, published in C.R. Acad. Sci. Paris in 1916 and 1917, Aleksandrov and Suslin laid foundations of the descriptive set theory. In particular, in [4] Aleksandrov proved that every uncountable Borel set contains a perfect subset. To obtain this result, he introduced an important operation, which in his honor Suslin named the operation A. Sets generated by application of this operation to closed sets became known as analytic. In his only published paper [174] Suslin showed the existence of analytic sets which are not Borel sets. In 1920 in the problem section of the first volume of Fundamenta Mathematicae Suslin raised the following question. If an ordered continuum contains only countable families of disjoint open sets, is it necessarily separable? This is the famous Suslin problem, which kept set-theorists and topologists busy for many decades.

In the academic year 1921/22 Uryson gave at Moscow University a course entitled *Topology of continua*. This was the first topology course taught in Russia. In 1924 in Moscow Aleksandrov and Uryson founded the first topology seminar. Unfortunately, Uryson tragically lost his life the same year and Aleksandrov continued conducting the seminar alone for many decades. However, in the period 1923-1935, he was spending most of his time in Göttingen. The Aleksandrov seminar became the core of the famous Moscow school of general topology. In the early years of the seminar, among its members were such distinguished mathematicians as Mikhail Alekseevich Lavrent'ev (1900-1980), Andreĭ Nikolaevich Tikhonov (1902-1993), Andreĭ Nikolaevich Kolmogorov (1903-1987), Lev Semionovich Pontryagin (1908-1988) and Viktor Vladimirovich Nemyckiĭ (1900-1967).

We will now mention some of the main results in general topology achieved in Russia. Uryson's paper [184], published in 1925 and written only a few days before his untimely death, contains the famous Uryson lemma and the Tietze-Uryson extension theorem, which asserts that continuous bounded real-valued functions extend from a closed subset of a normal space to the entire space. The paper also contains the definition of completely regular spaces as well as an example of a connected countable Hausdorff space. At the same time Uryson proved his metrization theorem, which asserts that every normal space with a countable basis is metrizable [185]. The proof contains implicitly the result that such a space embeds in the Hilbert cube. This paper as well as some other papers of Uryson were edited after Uryson's death by his colleague and friend Aleksandrov, who used sketches written by Uryson as well as his own recollections from their many mathematical conversations.

Uryson's most important contribution to topology was the founding and development (independently of K. Menger) of dimension theory and of the theory of curves. The main results were obtained during 1921/22. They were announced in 1922 in two notes published in C.R. Acad. Sci. Paris [182], [183] and in a lecture to the Moscow Mathematical Society. A complete account was published in 1925, 1926 and 1928 in two unusually long papers, entitled *Memoir on Cantor manifolds* [186], [187]. The first of these papers is devoted primarily to dimension theory of metric compacta, including the result that the small inductive dimension ind coincides with the covering dimension dim and the result that $\dim \mathbb{R}^n = n$. The second paper is devoted to curves. In 1925 Lev Abramovich Tumarkin (1904-1974) (independently of W. Hurewicz) generalized the Menger-Uryson dimension theory to separable metric spaces [180], [181].

In 1923 in a joint paper Aleksandrov and Uryson gave the first solution of the metrization problem [11]. Of great importance for the development of topology was their joint paper [13], which represents the first systematic study of compact spaces (metrizable and non-metrizable, then called bicompact spaces). The main results of this work were presented to the Moscow mathematical society already during 1922 and were announced in 1923 [12]. In 1930 Tikhonov introduced the product topology and proved the important fact that the product of an arbitrary collection of compact spaces is again a compact space. He also proved that completely regular spaces embed in cubes I^τ [179].

In 1939 Izrail' Moiseevich Gel'fand (born in 1913) and Andreĭ Nikolaevich Kolmogorov (1903-1987) established a bijection between abstract normed rings and rings of continuous functions on compact spaces [65]. In 1940 Mark Grigor'evich Kreĭn (1907-1989) and David Pinkhusovich Mil'man (1913-1982) proved their theorem on extremal points of convex compact sets [98]. Andreĭ Andreevich Markov (1903-1979), best known for his work in constructive mathematics and the theory of algorithms, studied free topological groups [116] and so did Mark Iosifovich Graev (born in 1922) [69]. In 1949 A.S. Esenin-Vol'pin proved that every first countable dyadic bicompact is metrizable [58]. In 1951 Yuriĭ Mikhailovich Smirnov (born in 1921) proved his famous metrization theorem [168], obtained independently and at the same time by J. Nagata (born in 1925) and in a somewhat different form by R.H. Bing (1914-1986).

Of the contributions to dimension theory, we mention the work of Nikolaĭ Borisovich Vedenisov (1905-1941), who considered dimension of non-metric spaces. In particular, he showed that for normal spaces $\dim X \leq \operatorname{Ind} X$ and $\operatorname{Ind} \beta X = \operatorname{Ind} X$ [190]. Contributions of Aleksandr Lazarovich Lunc (born in 1924) and Oleg Vyacheslavovich Lokucievskiĭ (born in 1922) also refer to dimension theory of non-metric compact spaces. In particular they showed the existence of compact spaces X with $\dim \mathrm{X} < \operatorname{ind} X$ [114], [113]. Isaak Aronovich Vaĭnshteĭn (born in 1917) studied surjective mappings $f : X \to Y$ between separable metric spaces, which are closed and open and have countable fibers. He showed that $\operatorname{ind} X = \operatorname{ind} Y$ [188]. In 1952 he obtained important results estimating $\operatorname{Ind} X$, respectively $\operatorname{Ind} Y$, for finite-dimensional closed onto mappings $f : X \to Y$ between separable metric spaces [189]. Lyudmila Vsevolodovna Keldysh (1904-1976) began her work in set-theoretic topology [86]. She later obtained important results on dimension raising mappings of compacta. In particular, in 1954 she constructed an open mapping with 0-dimensional fibers from a 1-dimensional metric compactum onto the square I^2 [87].

In Moscow in the second half of the twenties and in the thirties the main problems in algebraic topology refer to the finding of an adequate definition of homology for spaces more general than polyhedra, to duality theorems and to homological dimension theory.

The notions of inverse system and nerve of a covering, fundamental to the development of homology of metric compacta, appear for the first time in Aleksandrov's papers [5] and [6]. A systematic account of his results concerning the algebraic topology of metric compacta appeared in 1929 in his important and comprehensive paper [7].

Aleksandrov's paper [6] already contained an Alexander duality theorem for compact subsets of S^n and $\mathbb{Z}/2$ coefficients. In 1928/29 Emmy Noether (1882-1935) came from Göttingen to Moscow as a visiting professor. Her visit considerably influenced the Moscow topology group. In particular, Betti numbers and torsion coefficients were replaced by homology groups. In his graduation paper published in 1931, written under the influence of Aleksandrov and Noether, Pontryagin proved the Poincaré and the Alexander duality laws for coefficients in the cyclic group $\mathbb{Z}/m$ [133]. Using Aleksandrov's methods he also handled the case of compact subsets $X \subseteq S^n$. However, the determination of the group $H_{n-r-1}(S^n \backslash X; \mathbb{Z})$ with integer coefficients, required a new idea. Pontryagin considered compact coefficient groups G and topologized the homology groups $H_r(X;G)$ with coefficients in G. Moreover, he developed the theory of characters, i.e., continuous homomorphisms of a locally compact group to the group of reals modulo the integers $\mathbb{R}/\mathbb{Z}$. The main result of this theory is the theorem that every compact abelian group (with a countable basis) is the group of characters of some discrete abelian group. Pontryagin proved that the group $H_{n-r-1}(S^n \backslash X; \mathbb{Z})$ is the character group of the compact group $H_r(X; \mathbb{R}/\mathbb{Z})$ [135]. A complete account of Pontryagin's theory of characters appeared in his paper [134] and in his book on continuous groups [139], which first appeared in 1938 in Moscow (in Russian) and was the first book on topological algebra. Izrail' Isaakovich Gordon (1910-1985), a student of Pontryagin, developed an intersection theory of homology, which under the Alexander duality law becomes the cup product of cohomology [68]. In 1935 Aleksandrov (independently of Čech) introduced and studied local homology [9]. The 1942 paper by Aleksandrov [2] was also devoted to duality theorems. In this paper he actually proved the exactness of the homology sequence of a pair of spaces.

The generalizations of the Alexander duality theorem in a different direction, required a different type of (discrete) homology groups $\overline{H}_r(X;\mathbb{Z})$, introduced in 1940 by Steenrod, as well as cohomology groups $H^{n-r-1}(S^n \backslash X;\mathbb{Z})$. Both of these notions were anticipated in two 1936 papers of Kolmogorov [94], [95]. This approach to the Alexander duality theorem was generalized to arbitrary subsets of the n-sphere by Kirill Aleksandrovich Sitnikov (born in 1926), who also gave an alternative description of the Steenrod homology groups, for metric compacta [166], [167].

The foundations of homological dimension theory were laid by Pontryagin and Aleksandrov in the years 1930-1932. Pontryagin exhibited examples of compacta whose homological dimension depended on the coefficient groups. This led him to his famous example of two 2-dimensional compacta, whose direct product has dimension 3 [132]. Later Vladimir Grigor'evich Boltyanskiĭ (born in 1925) exhibited a 2-dimensional compactum whose square is 3-dimensional. In his important paper [8], Aleksandrov proved that all homological dimensions $\dim_G X$ of a compactum X are bounded from above by the covering dimension $\dim X$. If $\dim X$ is finite, then $\dim X = \dim_G X$, for $G = \mathbb{R}/\mathbb{Z}$. In the years which followed, much work on homological dimension theory was done by Meer Feliksovich Bokshteĭn (1913-1990), who found a set of groups having the property that the homological dimensions with respect to these groups determine all other homological dimensions [23]. Aleksandrov's problem, does there exist a metric compactum X with $\dim_{\mathbb{Z}} X$ finite and $\dim X = \infty$, remained open for many years. It was solved only in 1988 by Aleksandr Nikolaevich Dranishnikov (born in 1958), who used sophisticated tools of algebraic topology (complex K-theory with mod p coefficients) [48].

A great service to the topological community was done in 1935, when P.S. Aleksandrov and Heinz Hopf (1894-1971) published the first volume of their topology book [10]. The Second World War prevented them from writing the second of the two planned volumes. Another reason for abandoning the project was the quick growth of topology, which soon made their first volume somewhat out of date. However, in 1946 Aleksandrov published alone a book in Russian under the title "Combinatorial topology", which went beyond the book with Hopf. E.g., it contained cohomology groups [3].

Lazar' Aronovich Lyusternik (1899-1981) and Lev Genrikovich Shnirel'man (1905-1938), in their work on closed geodesics on surfaces, used topological methods. In particular, with a space X they associated the minimal number of closed contractible subsets which cover X. Today this number is called the Lyusternik-Shnirel'man category $\mathrm{cat}(X)$ [115].

In 1935 Pontryagin computed the homology groups of the four series of classical compact Lie groups. This was the first case of a computation of homology groups which was not based on triangulations. Instead, Pontryagin used ideas from Morse's theory [136], [138]. The thirties and fourties mark in Moscow the beginning of research in homotopy theory and in differential topology. The main protagonist was again Pontryagin. After Hopf showed that the homotopy group $\pi_3(S^2) \approx \mathbb{Z}$, Pontryagin succeeded to show that $\pi_{n+1}(S^n) \approx \mathbb{Z}/2$, for $n \geq 3$ [137]. For the groups $\pi_{n+2}(S^n)$ Pontryagin first announced an erroneous result. He obtained the correct value $\mathbb{Z}/2$, for $n \geq 2$, only later [143]. Pontryagin also classified mappings of 3-dimensional polyhedra to S^2 [140]. The proofs of these results were very involved. However, their main importance lies in the new techniques which Pontryagin introduced, i.e., framed differentiable manifolds, fiber bundles and a cohomology operation, known as the Pontryagin square. While Pontryagin was using information on smooth manifolds to get information on the homotopy groups of spheres, in the development which followed, things were turned around. The new methods of spectral sequences, introduced by Jean Leray (1906-1998) and Jean-Pierre Serre (born in 1926), made possible the calculation of many homotopy groups and the Pontryagin techniques were used to study various problems of differential topology. This happened in the study of cobordism, performed by René Thom (born in 1923). Pontryagin's work on characteristic classes and singularities of vector fields was done in the years 1942-1949 [141], [142].

The work of Pontryagin in topology was continued by his students, especially by Vladimir Abramovich Rokhlin (1919-1984) and Mikhail Mikhailovich Postnikov (born in 1927). Rokhlin defended his candidate's thesis in 1948 and his Ph.D. thesis in 1951. At that time he was a junior researcher at the Steklov Institute in Moscow in the division directed by Pontryagin. One should keep in mind that in the Soviet Union and most countries of the socialist block the candidate's thesis was comparable to the Western Ph.D., while the doctor's thesis was comparable to the habilitation. During the period from 1952 to 1960 Rokhlin worked at Arhangel'sk, Ivanovo and Kolomna near Moscow. In 1960 he became professor at the University of Leningrad. In the beginning of his scientific career his main interest was in measure theory and ergodic theory. His topological papers were written between 1950 and 1958 and after 1965. He obtained important results in cobordism, characteristic classes, low-dimensional manifolds and topology of real algebraic manifolds [156], [157]. In 1949 Postnikov solved the homotopy classification problem for mappings of 3-dimensional polyhedra to simply connected polyhedra [145]. His most important paper, written a few years later, is devoted to the classification of polyhedra by homotopy type, using homotopy groups and cohomology classes, known as the Postnikov invariants [146].

Since the thirties, Moscow was one of World's centers of topology. This is witnessed by the fact that from September 4 to September 10, 1935, in Moscow took place the First International Topology Conference. It appears that this was the first specialized mathematical conference. Its true international character and importance can best be judged from the names of some of the participants: J.W. Alexander, P.S. Aleksandrov, K. Borsuk, E. Čech, P. Heegard, H. Hopf, H. Freudenthal, C. Kuratowski, S. Lefschetz, J. Nielsen, A. Tikhonov, H. Whitney.

Topology was gradually transplanted from Moscow to other centers of USSR. Rokhlin led the Leningrad topology school, Georgiĭ Sever'yanovich Chogoshvili (1914-1998) started in Tbilisi the Georgian topology school. A school emerged in Novosibirsk. It was led by Anatoliĭ Ivanovich Mal'cev (1909-1967) and it was oriented towards universal topological algebras. At Voronezh, a group of mathematicians, including Yuriĭ Grigor'evich Borisovich (born in 1930), was continuing the work of Kreĭn, applying topological methods to functional analysis, especially to fixed points of multivalued mappings. These cities became, beside Moscow, the strongest topology centers in the USSR.

Czechoslovakia. In the first half of the twentieth century the leading mathematician in Czechoslovakia was Eduard Čech (1893-1960). He studied mathematics and descriptive geometry at Charles University in Prague, where he obtaned his Ph.D. in 1920. Until 1930 Čech was working in projective differential geometry, alone or together with Guido Fubini (1879-1943) at the University of Turin, where he spent the academic year 1920/21. He devoted the period from 1930 to 1947 to topology and then turned again to differential geometry. Čech started the first topology seminar in Czechoslovakia in Brno in 1936. Unfortunately this very successful seminar lasted only until 1939, when the Czech universities were closed by the Nazis. Čech moved to Prague in 1945, where he became Professor at Charles University and Director of the Mathematical Institute of the Czech Academy.

Čech worked both in general and in algebraic topology. From 1931 to 1933 he published three papers devoted to dimension theory beyond the realm of separable metric spaces [37], [38], [41]. In the first two of these papers he introduced and studied the large inductive dimension Ind. The third paper contains a formal definition of the covering dimension dim. One of his most important papers is [43], published in 1937. In this paper Čech introduced (independently of M.H. Stone) the maximal compactification of a Tychonoff space, now known as the Čech-Stone compactification. In the same paper he also introduced Čech-complete spaces. In [39] Čech defined homology groups of an arbitrary topological space X as limits of inverse systems formed by the homology groups of the nerves of all finite open coverings of X. This was the first paper where inverse limits (as we know them today) were defined. Since Čech used finite coverings, his groups were useful only in the case of compact Hausdorff spaces. Čech directed his research in algebraic topology towards building a general theory of (homology) manifolds. His results are contained in a comprehensive paper published in 1933 [42]. Moreover, Čech was the first to introduce the higher homotopy groups $\pi_n(X)$ [40], later rediscovered and extensively studied by W. Hurewicz [73].

Of great importance for the development of topology in Czechoslovakia was Čech's book on topological spaces [44], written during the

Second World War, but published only in 1959. The idea of organizing the well-known Prague international topology conferences is also due to Čech. However, the first of these conferences took place in 1961, after Čech's death.

Čech's first students in his topology seminar were Josef Novák (born in 1905) and Bedřich Pospíšil (1912-1944). Unfortunately, Pospíšil died already in 1944 after having spent several years in Nazi's prisons. Both Pospíšil and Novák worked in general topology. Pospíšil is best known for his theorem on the Čech-Stone compactification βD of a discrete space D, which asserts that the cardinality $|\beta D| = 2^{2^{|D|}}$ and the weight $\mathrm{w}(\beta D) = 2^{|D|}$ [144]. Novák studied and worked at the University of Brno until 1948 when he moved to Prague and was professor at the Czech Technical University and later at Charles University. In his research he was primarily interested in Fréchet's convergence spaces and often engaged in the construction of counter-examples. E.g., he exhibited (independently of E. Hewitt) a regular space on which every continuous function is constant [122]. He also constructed two countably compact Tychonoff spaces whose direct product is not countably compact [123].

Another outstanding topologist from Czechoslovakia, whose activity began during the period considered in this paper, is Miroslav Katětov (1918-1995). He was born in Belinskiĭ in the USSR, went to high school in Prague and studied at Charles University. His Ph.D. thesis was completed in 1939. However, due to the closing of Czech universities, his graduation took place only after the Second World War ended. He first worked at Charles University and since 1961 at the Mathematical Institute of the Czechoslovak Academy. His main contributions refer to H-closed spaces, locally convex vector spaces, application of topological techniques to Boolean algebras, paracompactness and normality, uniform and proximity spaces and dimension theory. In particular, in his first paper [83], Katětov constructed the maximal H-closed extension of a Hausdorff space X, now known as the Katětov extension κX. In [84] he gave an elegant characterization of the covering dimnsion dim of a compact space X in terms of the Banach algebra $C(X)$ of all continuous real-valued functions on X. His best known theorem in dimension theory asserts that on metric spaces the dimension functions dim and Ind coincide [85], a result obtained independently and simultaneously also by K. Morita.

The above mentioned Czechoslovak topologists were followed by many younger topologists among which we only mention Zděnek Frolík (1933-1989), who worked in descriptive set theory, measure theory, general topology and especially in uniform spaces. The long list of his publications begins with the year 1959.

Hungary. Topology in Hungary begins with F. Riesz and his 10 papers on topology. Riesz was born in Györ, Hungary in 1880 (then part of Austro-Hungary). He studied in Budapest, Zürich and Göttingen and obtained his Ph.D. in Budapest in 1902. He worked as a high school teacher in Löcse and Budapest until 1912, when he became an Associate Professor at the University of Koloszvár (Cluj). In 1920, after Cluj was ceded to Romania, Riesz moved to the newly founded University of Szeged, where he taught until 1946. As a result of his and Alfréd Haar's (1885-1933) activities, Szeged became an internationally recognized center of mathematical research, especially in the field of real analysis. The journal *Acta Scientiarum Mathematicarum*, published in Szeged since 1922, witnesses this development. In 1946 Riesz moved to Budapest where he spent the rest of his life. He died in Budapest in 1956. Riesz is generally considered one of the founders of functional analysis.

The first of Riesz' papers on topology, published in 1904, is devoted to a proof of the Schoenflies theorem, i.e., the converse of the Jordan theorem on simple closed curves [151]. In 1905 he generalized a theorem of L. Zoretti by showing that every 0-dimensional compactum in the plane lies on an arc [152]. In 1906 and 1908 he published four papers (two in Hungarian), whose aim was to define topological spaces axiomatically. The basic concept used was a notion similar to the notion of an accumulation point [153], [154]. The importance of these papers was recognized only much later. Riesz returned once more to topology, when in 1938 he published (in Hungarian) a proof of Jordan's curve theorem [155] (a French version was published in 1939).

In 1915, encouraged by F. Riesz, Károly Kaluzsay showed that a compact subset of $\mathbb{R}^3$ is a 2-sphere provided $\mathbb{R}^3 \backslash X$ has two components, every point of X is accessible from both of these components and every closed polygon in $\mathbb{R}^3 \backslash X$ can be deformed to a point within $\mathbb{R}^3 \backslash X$ [81]. In spite of being written in Hungarian, the paper attracted attention and is mentioned in R.L. Wilder's survey article on point sets in three and higher dimensions (Bull. Amer. Math. Soc. 38 (1932), 649-692).

Dénes König (1883-1944) published in 1918 a small book (in Hungarian) on the elements of analysis situs [96]. The book contains a proof of the topological classification of closed surfaces, following the Encyclopedia article of Max Dehn and Poul Heegaard. König wrote the first monograph on graph theory [97]. In his work on graphs, beside combinatorial methods, he also used topological techniques. György (George) Pólya (1887-1985), was a well known analyst, especially interested in heuristic methods in the process of discovery. In 1913 he contributed to topology the construction of a Peano curve whose fibers have at most three points [131].

Tibor Radó (1895-1965) obtained his Ph.D. from F. Riesz in Szeged and there became docent. In 1929 he emigrated and held positions in Munich and at Harvard and Rice. Since 1931 he was Professor at Ohio State University. He made contributions to conformal mappings, Riemann surfaces, real analysis, calculus of variations, partial differential equations, integration theory and topology. In particular, he proved the triangulability of two-dimensional manifolds [147], he wrote a paper on Peano spaces [148] and a paper on singular homology [149].

Béla von Kerékjártó (1898-1946) was born in Budapest and there he obtained his Ph.D. in 1920. In 1922 he became private docent at the University of Szeged. He spent several years abroad lecturing in Göttingen (1922/23), Barcelona (1923), Princeton (1923/24, 1924/25) and Paris (1925/26). In 1926 he returned to Hungary and was Professor of Geometry in Szeged and since 1938 in Budapest. He was interested in geometry and topology, primarily in transformation groups of surfaces [90]. Expanding the course he gave in Göttingen, he wrote a book on topology, which was published in 1923 in the prestigious Springer series *Grundlagen der mathematischen Wissenschaften* [88]. In the book special attention is given to the topology of surfaces and it contains the author's own results on the topological classification of non-compact surfaces [89]. Though one of the first books on topology, it did not influence too much the development of topology, because of its lack of precision. Kerékjártó also published papers on fixed points, on the Jordan simple curve theorem, on Helly's theorem and on the Hahn-Mazurkiewicz theorem.

György (George) Alexits (1899-1978), after being a high school teacher, obtained a professorship at the Technical University in Budapest at the age of 50. He is best known for his outstanding work in orthogonal series. However, between 1932 and 1942 he worked primarily in topology and wrote about 15 papers belonging to topology. He was interested in locally connected continua and in Menger's theory of curves [14], [15], [16]. In a joint paper with Jenő Egerváry (1891-1958) he approached some concepts of differential geometry (curvature, torsion) using topological methods.

Paul Erdős (1913-1996) was one of the most prolific mathematicians. He worked in number theory and combinatorial analysis and is considered the founder of discrete mathematics. Among his 1475 papers, there are also papers belonging to topology. In particular, in his paper [57] he constructed a simple example of a separable metric space X, whose quasi-components are single points, but $\dim X = \dim X^2 = 1$. Ákos Császár (born in 1924), the present Nestor of Hungarian topologists, obtained his Ph.D. in 1952 from F. Riesz, for his work in analysis, especially in the theory of functions of a real variable. In his 1947 paper [45], Császár gave a new proof of the theorem (obtained in 1937 by H.A. Vaughan) that every locally compact separable metrizable space admits a metric in which all bounded sets are compact. In 1957 Császár published a note introducing syntopogeneous spaces [46], which contain as special cases topological spaces, uniform spaces (introduced by A. Weil (1906-1998) in 1935) and proximity spaces (introduced by Vadim Arsen'evich Efremovich (1903-1989) in 1952 [50]). In 1960 Császár wrote a book on this subject [47], which was further expanded in subsequent editions.

The first Ph.D. theses in topology in Hungary were written by Mátyás Bognár in 1957 and (somewhat earlier) by J. Molnár, both under the supervision of György Hajós (1912-1972). István Juhász and others followed. In the early fifties at the L. Eötvös University in Budapest a seminar devoted to analysis was directed by Császár. In this seminar topics belonging to topology were also considered. Császár started the first seminar devoted to topology before 1960 at the Research Institute for Mathematics of the Hungarian Academy of Sciences.

Romania. The twentieth century mathematics in Romania begins with three distinguished mathematicians: Gheorghe Ţiţeica (1873-1939), Dimitrie Pompeiu (1873-1954) and Traian Lalescu (1882-1929). After obtaining their Ph.D.'s in Paris, they returned to Romania and there initiated modern research in mathematics. Ţiţeica was a student of J.G. Darboux (1842-1917) and worked in Bucharest. His speciality was differential geometry. Pompeiu was a student of H. Poincaré and worked in Iaşi, Bucharest and Cluj. His speciality was real and complex analysis. Lalescu worked in various areas of mathematics and his main contributions belong to integral equations.

In the period 1900-1950 ground for future research in topology was laid by Simion G. Stoilow (1887-1961). He was born in Bucharest and studied in Paris (1907-1914). There he obtained his Ph.D. in 1916 under É. Picard. Stoilow was a member of the Romanian Academy of Sciences since 1945. He was the first director of the Institute of Mathematics of the Academy. In 1927 he opened a new area of mathematics, the topological theory of analytic functions, by giving a topological characterization of analytic functions, answering thus a question of Brouwer. Stoilow introduced interior mappings. In today's terminology these are mappings which are at the same time open and light. Non-constant analytic functions are interior mappings. Stoilow proved that conversely, every complex function, which is interior, is the composition of a homeomorphism and of an analytic function [169], [170]. His work considerably influenced later researchers in this area, especially G.T. Whyburn (1896-1982). Another important contribution of Stoilow is his topological characterization of Riemann surfaces as connected 2-manifolds which admit and interior mapping onto the 2-sphere [171], [172]. All these results were presented in his 1938 book [173]. In the book one also finds the construction of the ideal boundary of a non-compact Riemann surface, known in the literature as the

Kerékjartó-Stoïlow ideal boundary, as well as the general notion of covering space.

Between 1927 and 1936 Octav Onicescu (1892-1983) published several papers in which he studied complex functions by topological methods. Moreover, he introduced a class of interior mappings called holotopic mappings [126], [127]. Alexandru Froda (1894-1973) studied real and abstract functions. In particular, he studied the topology of p-metric spaces [59].

Around 1950 at the University of Bucharest there existed a chair of Geometry and Topology, headed by the differential geometer Gheorghe Vrânceanu (1900-1979). Courses in general topology were given by the functional analyst Alexandru Ghika (1902-1964) and in algebraic topology by the algebraic geometer Gheorghe Galbură (born in 1916). Galbură obtained his Ph.D. in Rome in 1942 with a thesis in algebraic geometry. In 1950 he published a paper on 3-dimensional manifolds which admit the structure of a topological group [60]. Since 1959/60 Gheorghe Călugăreanu (1902-1976) a specialist in the theory of functions was giving in Cluj a special course on general and algebraic topology. He made contributions to knot theory by studying the topological invariance of some line integrals [33], [34]. Grigore C. Moisil (1906-1973), a specialist in algebra and the theory of automata, was also encouraging students in Bucharest to study algebraic topology.

In the famous seminar of Stoilow on complex functions, topics from topology were also considered and many of the future Romanian topologist began their scientific careers as members of that seminar. Among the first topologists, who obtained their candidate's theses in Romania, were Stoilow's students Tudor Ganea (1922-1971), Israel Berstein (1926-1991), Ion M. Bucur (1930-1976), Aristide Deleanu (born in 1932), Valentin Poenaru (born in 1932) and Costake Teleman (born in 1933). Only Ganea's early papers belong to the considered period. They primarily refer to covering spaces, topological groups, multicoherence, symmetric products and the Lyusternik-Shnirel'man category [61], [62], [63], [64]. In 1962 Ganea obtained a Ph.D. in Paris from H. Cartan and then emigrated to U.S. He first spent a year at Purdue University and then settled in Seattle at the University of Washington. In 1950

Filip Obreanu published several papers on general topology [124], [125].

Bulgaria. Ghéorghi Ghéorghiev (1906-1972) was the first Bulgarian mathematician interested in topology. He spent the period 1935-1939 at the Technical University in Warsaw, where he wrote his Ph.D. thesis. As Associate Professor at the Polytechnical University in Varna (1946-1953), he organized the first topology seminar in Bulgaria. From 1948 to 1950 he published several papers (in Bulgarian) in which he considered mappings of the real line, fixed point problems and topological spaces [66], [67].

Yaroslav Aleksandrov Tagamlitzki (1917-1983) was the main promoter of topological concepts and techniques in Bulgaria. Since 1951 he directed a regular seminar on functional analysis and topology at Sofia University, where he was at that time Associate Professor. Moreover, in the early fifties he taught a course entitled "Combinatorial topology". His scientific interest involved real and functional analysis and topology. He was interested in both, the topological and the linear structure of spaces. Tagamlitzki obtained a generalization of the famous Kreĭn-Mil'man theorem (not being aware of the latter) [177], [178]. He is the author of the first paper on topology written by a Bulgarian mathematician [176].

One of Tagamlitzki's students, Doitchin Bogdanov Doitchinov (1926-1996) became the first Bulgarian mathematician, who went to Moscow (in 1959) to specialize in topology. There he obtained his Ph.D. in 1961 under the supervision of Yu.M. Smirnov. He was followed by a stream of Bulgarian topologists, who all specialized in Moscow. They returned to Bulgaria and today form the core of the Bulgarian topology group (I.R. Prodanov, N.G. Hadzhiivanov, Stanislava G. Petkova, P.S. Kenderov, G. S. Skordev, Y. Kintishev, G.D. Dimov, S.Y. Nedev).

Yugoslavia. The forerunners of topology in former Yugoslavia were Vladimir Varićak (1865-1942), Željko Marković (1889-1974), Danilo Blanuša (1903-1987) and Stanko Bilinski (1909-1998) in Zagreb and Bogdan Gavrilović (1864-1947) and Miloš Radojčić (1903-1975) in Belgrade. In 1946 Blanuša exhibited a (non-planar) graph whose edges could not be coloured using only three colours [21] (the first graph with this property was discovered by J. Petersen in

1898). Later Blanuša became interested in isometric embeddings of Riemann manifolds with constant curvature in Euclidean spaces. Only the first of a series of his papers on the subject was published before 1950 [22]. Bilinski's work on homogeneous nets in the plane and on closed orientable surfaces is also related to topology [19], [20]. The work of Radojčić refers to geometric aspects of the theory of analytic functions, in particular, to Riemann surfaces [150].

Đuro (George) Kurepa (1907-1993) was a set-theorist, especially interested in ordered sets. He specialized in Paris (1932-1936) and Warsaw (1937) and obtained his Ph.D. in 1935 in Paris under M. Fréchet. Kurepa was Professor at the University of Zagreb until 1965, when he moved to the University of Belgrade. Kurepa generalized metric spaces by allowing the distance function to assume non-numerical values, especially values in a linearly ordered set [110]. He also considered some cardinal invariants, especially the cellularity number [111]. He proved that the famous Suslin problem is equivalent to a problem in the theory of trees [109]. The appearence of Kurepa's book (written in Serbo-Croatian) on set theory and general topology in 1952 had a considerable impact on the development of topology in former Yugoslavia [112]. For many years Kurepa conducted an undergraduate seminar, where among other things topics from topology were also considered. The first Ph.D. theses in topology were written in 1953 by Pavle Papić (born in 1919) and in 1955 by Zlatko Mamuzić (1915-1996), both at the University of Zagreb and under Kurepa's supervision. The first graduate topology seminars started only in 1961. The one in Zagreb was conducted by Sibe Mardešić (born in 1927) and P. Papić and the one in Belgrade by Z. Mamuzić .

Greece. The first Greek mathematician who made contributions to topology was Konstantinos Karatheodoris (Constantin Carathéodory). He was born in Berlin in 1873 in a Greek family and died in Munich in 1950. In 1895 he graduated from the Belgian Military Academy and became a military engineer. He then spent some time in Egypt working on the regulation of the Nile. In 1900 he started his studies of Mathematics in Berlin, where his professors were I.L. Fuchs (1833-1902), H.A. Schwarz (1843-1921) and G. Frobenius (1849-1917). In 1904 at the University of Göttingen Carathéodory obtained his Ph.D. under H. Minkowski (1864-1909). After lecturing in Hannover and Breslau (Wrocław) in 1913 he succeeded F. Klein (1849-1925) in Göttingen and in 1918 he returned to Berlin. At the request of the Greek government in 1920 he went to Smyrna to head the newly opened University of Smyrna. When in 1922 Smyrna became part of Turkey, Carathéodory moved to Athens. In 1924 he became Professor in Munich, where he remained for the rest of his career.

Carathéodory made important contributions to many areas of pure and applied mathematics. His most significant results belong to the calculus of variations, conformal mappings and the theory of measure and integration. His contributions to topology refer to the topology of the plane and were inspired by the theory of conformal mappings. In particular, Carathéodory carried out a detailed study of the frontier of simply connected bounded domains in the plane showing that it is covered by subcontinua, which he called prime ends of the domain. He showed that a conformal mapping of the domain onto the interior of the unit disk (Riemann's theorem) induces a bijection between the prime ends of the domain and the points of the frontier of the disk [35].

Dēmētrios Andreou Kappos (1904-1984) studied at the University of Athens. He spent the period from 1934 to 1938 specializing at the University of Munich. Kappos obtained his Ph.D. from Carathéodory in 1940 at the University of Athens and became Professor of that university in 1952. Kappos' many contributions to mathematics refer to Boolean algebras and lattice theory, analysis, topology and especially to the foundations of probability theory [82]. Leonidas Alaoglu (1914-1981) was born at Red Deer, Alberta, Canada in a family of Greek immigrants. He began his studies of mathematics at the University of Alberta and completed them at the University of Chicago, where in 1938 he obtained his Ph.D. He held positions at Pennsylvania State University (1938-1939), Harvard University (1939-1941) and Purdue University (1942-1944). He became a U.S. citizen in 1946. Since 1953 he worked as a mathematician at the Operations Research Division of Lockheed Aircraft Corporation. Alaoglu is best known for his contributions to topology of vector spaces. In particular, his name is associated with the theorem which asserts that a Banach space is reflexive if and only if its unit ball is compact in the weak topology [1].

The best known Greek topologist is Christos Papakyriakopoulos (1914-1976). He was born in Athens and obtained his Ph.D. in Athens in 1944 with a thesis on n-dimensional complexes. In particular, the thesis contains a proof of the *Hauptvermutung* for $n = 2$ [128]. He emigrated to U.S. and spent the period 1955-1958 at the Institute for Advanced Study in Princeton. Subsequently he worked at Princeton University until his death. Papakyriakopoulos acquired fame by proving Dehn's lemma, the sphere theorem and the asphericity of knots [129], [130].

James Dugundji (1919-1985), another well-known topologist of Greek origin was born in New York in a family of Greek immigrants. He obtained his B.A. degree from New York University in 1940. The same year he began his graduate studies at the University of North Carolina at Chapel Hill as a student of Witold Hurewicz. After spending four years of the war in the U.S. Air Force, in 1946 he entered the Massachusetts Institute of Technology, where he obtained his Ph.D. in 1948 under Hurewicz' supervision. The same year he started teaching at the University of Southern California in Los Angeles, where he became Professor in 1958 and remained there until the end of his career. His best known result is his extension theorem, which asserts that every convex set in a locally convex vector spaces is an absolute extensor for metric spaces [49].

Turkey. It appears that in Turkey in the first half of the twentieth century there was no research in the area of topology. The ground for future work in topology was prepared by Cahit Arf (1910-1997), the founder and leader of modern mathematics in Turkey. He was born in Thessaloniki, which at the time of his birth belonged to the Ottoman empire. However, due to the Balkan wars, his family moved to Istanbul when he was only two years old. In 1926 he went to France, where he finished high school and graduated from Ecole Normale Supérieure in Paris. He was a high school teacher in Instanbul before joining the Mathematics Department of the University of Istanbul.

In 1937 Arf went to the University of Göttingen, where in 1938 he obtained his Ph.D. with a thesis in the theory of fields written under the supervision of Helmut Hasse (1898-1979). During his stay in Göttingen he wrote his best-known paper [17] in which he introduced an invariant of quadratic forms in fields of characteristic 2, which is known today as the Arf-invariant and plays an important role in some problems of topology, especially in homotopy theory and knot theory. After returning to Turkey Arf was professor at the University of Istanbul until 1962. He then spent several years at the Institute for Advanced Study in Princeton and at the University of California at Berkeley. In 1967 he joined the Middle East Technical University in Ankara, where he worked until his retirement in 1980. In 1966 in Istanbul appeared Arf's notes on topology [18], containing an introduction to general topology and an exposition of singular homology and cohomology. These notes constitute the first material on topology written in Turkey. Since the early seventies several young Turkish mathematicians wrote Ph.D. theses in topology, mainly at American universities.

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