

# Topology Proceedings



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### CONTRIBUTED PROBLEMS

The Problems Editor invites anyone who has published a paper in *Topology Proceedings* or has attended a Spring or Summer Topology Conference to submit problems to this section. They need not be related to any articles which have appeared in *Topology Proceedings* or elsewhere, but if they are, please provide full references. Please define any terms not in a general topology text nor in referenced articles.

Problems which are stated in, or relevant to, a paper in this volume are accompanied by the title of the paper where further information about the problem may be found. Comments of the proposer or submitter of the questions are so noted; comments of the Problems Editor are not specially noted. Information on the status of previously posed questions is always welcome. Submission of questions and comments by email in  $\text{\TeX}$  form is strongly encouraged, either to `topolog@auburn.edu` or directly to the Problems Editor at `mayer@math.uab.edu`.

#### **M. Manifolds and Cell Complexes**

**14–16.** For definitions and additional information concerning the following three problems, see *Incompressibility of torus transverse to vector fields* by C. A. Morales in this issue.

**14.** Let  $M$  be a closed  $P^2$ -irreducible 3-manifold and let  $A$  be an abelian subgroup of  $\pi_1(M)$  (the fundamental group of  $M$ ). Is there a knot  $K$  in  $M$  such that  $A \subset \pi_1(M \setminus K)$ ?

**Comments of the proposer.** A positive answer would imply the  $Q$ -Conjecture: *Every subgroup of  $\pi_1(M)$  which embeds in the rationals is cyclic.* The  $Q$ -Conjecture can be used to prove Thurston's Hyperbolization Conjecture (see p. 1511, Corollary 2: Robert Myers, *Compactifying sufficiently regular covering spaces of compact 3-manifolds*, Proc. Amer. Math. Soc. **128** (2000), no. 5, 1507–1513.)

**15.** Let  $S$  be a closed submanifold transverse to an Anosov flow  $X = X_t$  on a closed manifold  $M$ . Is  $S$  incompressible?

**16.** Let  $S$  be a closed submanifold transverse to a flow  $X$  (not necessarily Anosov) on a closed manifold  $M$ . Suppose that there is a unique orbit  $O$  of  $X$  which does not intersect  $S$  and suppose that  $O$  is not null homotopic in  $M$ . Is  $S$  incompressible?

**Comments of the proposer.** *Incompressible* means that the homomorphism  $\pi_1(S) \rightarrow \pi_1(M)$  induced by the inclusion is injective. *Anosov* means that the tangent bundle  $TM$  has a direct sum decomposition  $E^s \oplus E^X \oplus E^u$  such that  $E^s$  is contracted by  $X$ ,  $E^u$  is expanded by  $X$  and  $E^X$  is the subspace generated by  $X$ . The answers to **15–16** are positive if  $\dim(M) = 3$  (see S. Fenley, *Quasigeodesic Anosov flows and homotopic properties of flow lines*, J. Differential Geom. **41** (1995), no. 2, 479–514.).