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UNIQUENESS OF THE HYPERSPACE GRAPH OF CONNECTED SUBGRAPHS

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Abstract. An analogous concept for hyperspace of continua can be defined in abstract graph theory. Given a connected graph G, the hyperspace graph of connected subgraphs, denoted by $\mathcal{C}(G)$, is defined with the set of all connected non-empty subgraphs of G as the vertex set of $\mathcal{C}(G)$. In this paper, it is shown that given G and G' connected graphs such that $\mathcal{C}(G)$ is isomorphic to $\mathcal{C}(G')$, then G and G' are isomorphic.

1. Introduction

The work in this paper was presented at the 2005 Spring Topology and Dynamical Systems Conference. It is part of the doctoral dissertation of the author under the direction of Sam B. Nadler, Jr. at West Virginia University in 2005 [4].

In [3], given a graph G, Sam B. Nadler, Jr. defines a new graph whose vertices represent non-empty subsets of vertices of G. This is the first attempt to define an analogous concept to the hyperspaces of sets in the field of abstract graph theory. In [4], we get an analogue of the hyperspace of subcontinua. Given an abstract graph G, we define a graph $\mathcal{C}(G)$ with the property that every vertex of $\mathcal{C}(G)$ represents a connected subgraph of G. The graph $\mathcal{C}(G)$ is the hyperspace graph of connected subgraphs.

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R. Duda shows that if two finite topological graphs X and Y are such that neither of them is an arc nor a simple closed curve and they have homeomorphic hyperspaces of subcontinua, then X and Y are homeomorphic [1]. It is natural to ask if this result can be obtained for the hyperspace graphs of connected subgraphs. In this paper, we give the sketch of the proof that answers this question affirmatively.

First, we need some basic graph theory.

A *graph* G is a finite non-empty set denoted by $V(G)$ together with a set (possibly empty) of unordered pairs of distinct elements of $V(G)$, denoted by $E(G)$. The set $V(G)$ is called *the vertex set of* G, and the set $E(G)$ is called *the edge set of* G. The *size* of a graph G is the cardinality of $E(G)$. The elements of $V(G)$ are called *vertices of* G, and the pairs in $E(G)$ are called *edges of* G. If two vertices are joined by an edge, they are said to be *adjacent*.

A graph is a *degenerate graph* if it has no edges and only one vertex. If v is such vertex, then we denote the graph as $\{v\}$.

The *degree* of a vertex v in G is the number of vertices of G that are adjacent to v. An *endvertex* of G is a vertex with degree 1 in G.

A graph K is said to be a *subgraph* of a graph G (denoted by $K \subset G$) if $V(K)$ is a subset of $V(G)$ and $E(K)$ is a subset of $E(G)$.

A *path* in a graph G is a finite sequence of vertices $\mathcal{P}$, namely,

$$\mathcal{P} = v_0 v_1 v_2 ... v_n$$

such that v_i and v_{i+1} are adjacent in G and $v_i \neq v_j$ if $i \neq j$ for all $i, j = 0, 1, 2, ..., n$.

A graph is said to be *connected* if for every two vertices of the graph, there is a path joining those two vertices.

Let v and w be two vertices of the connected graph G. The *distance* between v and w in G is the size of the shortest path in G joining v and w. The distance between v and w is denoted by $d(v, w)$.

Finally, we are ready to define the analogous of the hyperspace of subcontinua.

Given a connected graph G, we will define *the hyperspace graph of connected subgraphs of* G as the graph $\mathcal{C}(G)$ such that

- the vertex set of $\mathcal{C}(G)$ is the set of all connected subgraphs of G; namely, $V(\mathcal{C}(G)) = \{K : K \text{ is connected subgraph of } G\}$;
- two connected graphs K and L are adjacent in $\mathcal{C}(G)$ if one of the following holds.
  (1) $K \subset L$ and L has exactly one more edge (as a subgraph of G) than K;
  (2) K and L are non-degenerate subgraphs and have exactly the same number of edges (as subgraphs of G), and there is a common connected subgraph H of both K and L such that H has exactly one edge less than K (or L). If this condition holds, K and L are said to be *level adjacent*;
  (3) $K = \{v\}$ and $L = \{w\}$ with v and w being adjacent vertices of G.

Let us consider the following two particular kinds of graphs.

A graph is said to be *complete* if every two vertices of the graph are adjacent. It is well known that if a graph G contains a subdivision of the complete graph with five vertices, then G is not planar.

A *cycle* in a graph G is a finite sequence of vertices $\mathcal{P}$, namely,

$$\mathcal{Y} = v_0 v_1 v_2 ... v_n \qquad \text{with } n \geq 3$$

such that v_i and v_{i+1} are adjacent in G, $v_0 = v_n$, and $v_i \neq v_j$ if $i \neq j$ for all $i, j = 0, 1, 2, ..., n$.

A graph G' is an *elementary subdivision* of G if G' can be obtained by removing an edge $\mathsf{e} = vv'$ from G and adding a new vertex w and two new edges vw and vw'.

A *subdivision* of G is the graph obtained by a succession of elementary subdivisions.

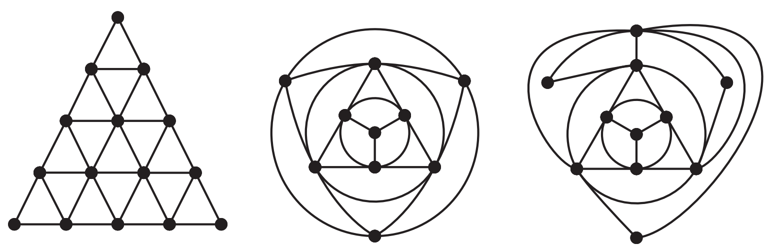
A graph B is a *bipartite graph* provided that there is a partition V and W of the vertex set of B such that $V \neq \varnothing$, $W \neq \varnothing$, and every edge of B joins a vertex in V with a vertex in W. A bipartite graph with a partition V and W is a *complete bipartite graph* if every vertex of V and every vertex of W are adjacent. We denote by $\mathcal{K}_{n,m}$ the complete bipartite graph such that V has n elements and W has m elements.

A graph G is *planar* if G contains no subgraph isomorphic to a subdivision of $\mathcal{K}_{3,3}$ or $\mathcal{K}_5$.

We get a nice characterization for graphs that have planar hyperspace graph of connected subgraphs.

Theorem 1.1. *$\mathcal{C}(G)$ is planar if and only if G is either a path, a cycle, or the 3-star.*

It is easy to prove that $\mathcal{C}(G)$ is planar if G is either a path, a cycle, or the 3-star. In Figure 1, we have the geometric representation of $\mathcal{C}(G)$ of the following graphs: (a) a path with size 4; (b) a cycle with size 3; and (c) the 3-star.



(a) **(b)** **(c)**

FIGURE 1

The proof follows from the following result.

Proposition 1.2. *If G is a connected graph with size greater than 3 containing a vertex with degree greater than 2, then $\mathcal{C}(G)$ contains a subdivision of the complete graph with five vertices (and therefore is not planar).*

Proof: The proof is divided in two cases: when G has a vertex with degree greater than 3 and when the maximum degree of the vertices of G is 3.

Suppose that G has a vertex v with degree greater than 3. Let a, b, c, and d be four vertices adjacent to v. Consider A, B, C, and D the four connected graphs with size 1 determined by the edges va, vb, vc, and vd, respectively. Since the graph $\{v\}$ is a common subgraph of A, B, C, and D, the graph $\mathcal{C}(G)$ contains a copy of

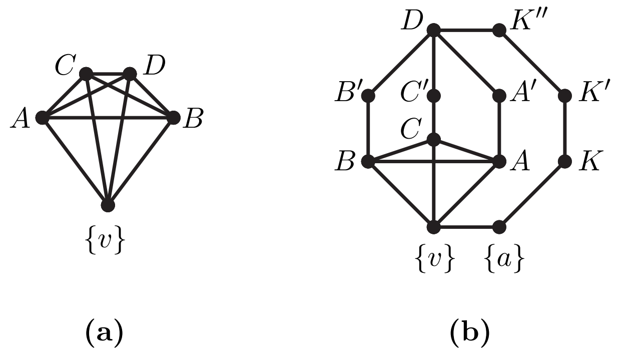


FIGURE 2

the complete graph with five vertices (Figure 2(a)). This finishes the first case.

Assume that the maximum degree of the vertices of G is 3. Let v be a vertex such that the degree of v is 3. Let a, b, and c be the three vertices adjacent to v.

Let A, B, and C be the three connected graphs with size 1 determined by the edges va, vb, and vc, respectively. Since the graph $\{v\}$ is a common subgraph of A, B, and C, the graph $\mathcal{C}(G)$ contains a copy of the complete graph with four vertices.

Let D be the connected graph defined by the edges va, vb, and vc. Next, we show that there are four paths in $\mathcal{C}(G)$ joining D with A, B, C, and $\{v\}$, having only the vertex D in common.

Consider the paths A', B', and C' in G defined by $A' = avb$, $B' = bvc$, and $C' = cva$. Then the paths $AA'D$, $BB'D$, and $CC'D$ are paths in $\mathcal{C}(G)$ (actually in $\mathcal{C}(D)$), joining D with A, B, and C, respectively, having only the vertex D in common.

Since G is a connected graph with size greater than 3, either a, or b, or c has degree at least 2. Without loss of generality, assume that there is a vertex w different from v adjacent to a and different from c.

Let K, K', and K'' be paths in G defined by $K = aw$, $K' = vaw$, and $K'' = cvaw$, respectively. Since K, K', and K'' are not subgraphs of D, $\{v\}\,\{a\}\,KK'K''D$ is a path in $\mathcal{C}(G)$ that has only the vertex D with the paths previously constructed.

The four paths constructed above, together with the complete graph with four vertices, create a subdivision of the complete graph with five vertices (Figure 2(b)). $\square$

Using Theorem 1.1, it is easy to show the following results.

Theorem 1.3. *Let P be a path with size N and let G be a connected graph. If $\mathcal{C}(P)$ and $\mathcal{C}(G)$ are isomorphic, then G is a path with size N.*

Theorem 1.4. *Let Y be a cycle with size N and let G be a connected graph. If $\mathcal{C}(Y)$ and $\mathcal{C}(G)$ are isomorphic, then G is a cycle with size N.*

Theorem 1.5. *Let T be the 3-star and G be a connected graph. If $\mathcal{C}(T)$ and $\mathcal{C}(G)$ are isomorphic, then G is the 3-star.*

These results reduce our problem to graphs that are not paths, cycles, or the 3-star.

2. Stitched graphs and size levels

There is a natural way to give a partition of the vertices of $\mathcal{C}(G)$. Since the vertices of $\mathcal{C}(G)$ are connected subgraphs of G, we can consider a partition by the cardinality of the edge set. Namely,

$$\mathcal{V}_n = \{K \in V(\mathcal{C}(G)) : K \text{ has size } n\}.$$

Note that if $|n - m| > 1$, then no vertex of $\mathcal{V}_n$ is adjacent to $\mathcal{V}_m$.

For each n, we define the graph $\mathcal{Q}_n(G)$ by taking $\mathcal{V}_n$ as the vertex set of $\mathcal{Q}_n(G)$ together with all the edges of $\mathcal{C}(G)$ that join two elements of $\mathcal{V}_n$ as the edge set of $\mathcal{Q}_n(G)$. The graph $\mathcal{Q}_n(G)$ is called *the n-th size level of $\mathcal{C}(G)$*.

The size level graphs can be seen as analogous to the concept of Whitney levels for hyperspaces of sets [2].

Given G, a connected graph with size N, we will refer to $(N-1)$-th size level as the *last non-degenerate size level of G*. Let us analyze some properties of such size level.

Let G be a graph and let K be a subgraph of G. The set $E(G) - E(K)$ is called the *complementary edge set of K in G* and it is denoted by $\mathcal{E}_{K,G}$. In particular, if $E(G) - E(K) = \{\mathsf{e}\}$, then e is the *complementary edge of K in G*.

If $\mathtt{e}$ is a complementary edge of K in G, then either $\mathtt{e}$ is a *cycle edge of* G (edge contained in a cycle of G) or $\mathtt{e}$ is a *terminal edge of* G (edge containing a vertex with degree 1). An edge $\mathtt{e}$ is a *removable edge of* G if $\mathtt{e}$ is either a terminal or a cycle edge of G.

Each connected subgraph of G in the last non-degenerate level can be determined by its complementary edge in G. In order words, if G is a connected graph with size N, then for every terminal edge $\mathtt{e}$ of G there is a connected subgraph K of G with size $N-1$ such that $\mathtt{e}$ is the complementary edge set of K in G.

Next, we analyze the structure of the last non-degenerate level.

Let K be a connected subgraph of G such that K is in the last non-degenerate level of G. The subgraph K is *C-type* in G if its complementary edge in G is a cycle edge. Similarly, the subgraph K is a T-type in G if its complementary edge in G is a terminal edge.

Let G be a connected graph with size N, let K be a T-type subgraph of G, let L be a connected subgraph of G with size $N-1$ different from K, and let $\mathtt{e}$ be the complementary edge of L in G. Since $K \neq L$, $\mathtt{e}$ is an edge of K. Note that since K is T-type, all the cycle edges of G are cycle edges of K and all terminal edges of G in K are terminal edges of K. Thus, there is a connected subgraph J of K with size $N-2$ with complementary edge $\mathtt{e}$ of J in K. This implies that J is a subgraph of L. We have shown the following proposition.

Proposition 2.1. *Let G be a connected graph with size N and let K be a T-type subgraph of G. Then for every connected connected subgraph L with size $N-1$ different from K, K is level adjacent to L.*

Let G be a connected graph with size N, let Y be a cycle of G, and let K be a connected subgraph of G with complementary edge $\mathtt{e}$ in G such that $\mathtt{e}$ is not in Y. Then all the edges of the cycle Y are cycle edges of K. Thus, for every edge $\mathtt{f}$ in Y, there is a connected subgraph J with size $N-2$ with complementary edge $\mathtt{f}$ in K. Let L be a connected subgraph of G with complementary edge $\mathtt{f}$ in G. We have that J is a connected subgraph of L. This proves the next proposition.

Proposition 2.2. *Let G be a connected graph with size N and let Y be a cycle in G and let L be a connected subgraph of G such that its complementary edge in G is an edge of Y. If K is a connected subgraph of G such that its complementary edge in G is not an edge of Y, then K and L are level adjacent.*

A subgraph S of a graph G is said to be *spanning* (or *spans* G) if $V(S) = V(G)$.

Recall the definition of bipartite given on page 285. A graph G is said to be *stitched* if G contains a spanning complete bipartite subgraph.

Theorem 2.3. *Let G be a connected graph with size N. Then G is not a cycle of size greater than 4 if and only if $\mathcal{Q}_{N-1}(G)$ is stitched.*

Proof: If G is a cycle with size $N \leq 4$, then $\mathcal{Q}_{N-1}(G)$ is a cycle with size N. Thus, $\mathcal{Q}_{N-1}(G)$ can be spanned by either $\mathcal{K}_{1,2}$ or $\mathcal{K}_{2,2}$. Therefore, $\mathcal{Q}_{N-1}(G)$ is stitched.

If G is a tree, by Proposition 2.1, $\mathcal{Q}_{N-1}(G)$ is a complete graph. Therefore, $\mathcal{Q}_{N-1}(G)$ is stitched.

Assume that G is not a cycle but contains a cycle Y as a subgraph. For every K subgraph of G with size $N-1$, let $\mathtt{e}_K$ be the complementary edge of K in G. Define the following partition of the vertices of $\mathcal{Q}_{N-1}(G)$:

- $W = \{K \in \mathcal{Q}_{N-1}(G) : \mathtt{e}_K \in E(Y)\}$;
- $V = \{K \in \mathcal{Q}_{N-1}(G) : \mathtt{e}_K \notin E(Y)\}$.

Note that $V \neq \varnothing$ and $W \neq \varnothing$.

Using Proposition 2.1 and Proposition 2.2, $\mathcal{Q}_{N-1}(G)$ is stitched.

On the other hand, if G is a cycle with size greater than 4, then $\mathcal{Q}_{N-1}(G)$ is a cycle of size greater than 4. Therefore, $\mathcal{Q}_{N-1}(G)$ is not stitched. □

3. Uniqueness of $\mathcal{C}(G)$

The *neighborhood* of a vertex v in a graph G is the subgraph of G determined by the vertices of G adjacent to v and the edges of G between those vertices.

In Theorem 2.3, we show that the neighborhood of the vertex G in the graph $\mathcal{C}(G)$ is stitched when G is not a cycle of size greater than 4. This characteristic is quite rare within the vertices of $\mathcal{C}(G)$.

In fact, only G and some very special small subgraphs of G will have the property of having stitched neighborhoods.

If K is a connected subgraph of G, we denote the neighborhood of K in $\mathcal{C}(G)$ by $\mathcal{N}_K$. Define the following set

$$\mathcal{L}(G) = \{K \in \mathcal{C}(G) : \mathcal{N}_K \text{ is stitched}\}.$$

We can restate Theorem 2.3 in the following remark.

Remark 3.1. Let G be a connected graph. Then G is not a cycle of size greater than 4 if and only if $G \in \mathcal{L}(G)$.

Lemma 3.2. *Let G be a connected graph and let K be a connected non-empty subgraph of G. Then $K \in \mathcal{L}(G)$ if and only if one of the following statements holds.*

(1) *$K = G$ and G is not a cycle of size greater than* 4.
(2) *$K = \{v\}$ for some endvertex v of G.*
(3) *$K = \{v\}$ for some vertex v with degree* 2 *and contained in a cycle of size* 3.

Proof: Let N be the size of G.

For (1), the result follows from Remark 3.1.

If K is like in (2), then $\mathcal{N}_K$ is a single edge and therefore stitched.

If K is like in (3), then $\mathcal{N}_K$ is a cycle with size 4 and therefore stitched.

Assume that K is a proper nondegenerate and non-empty subgraph of G with size n such that $\mathcal{N}_K$ is stitched.

Let $\mathcal{A}$ and $\mathcal{B}$ be a partition of $V(\mathcal{N}_K)$ guaranteed by the definition of a bipartite graph.

Claim 1. If $H \in V(\mathcal{Q}_{n-1}) \cap \mathcal{B}$, then $V(\mathcal{Q}_{n+1}) \cap V(\mathcal{N}_K) \subset \mathcal{B}$.

Let $H \in \mathcal{B}$ and H has size $n-1$. Since no subgraph of size $n+1$ is adjacent to H in $\mathcal{C}(G)$, for every $L \in V(\mathcal{N}_K)$ with size $n+1$, $L \in \mathcal{B}$.

Similarly, we can show the following claim.

Claim 2. If $L \in V(\mathcal{Q}_{n+1}) \cap \mathcal{B}$, then $V(\mathcal{Q}_{n-1}) \cap V(\mathcal{N}_K) \subset \mathcal{B}$.

Using the claims above, we can assume that $\mathcal{A} \subset V(\mathcal{Q}_n)$. Note that if $J \in \mathcal{A}$, then K and J are adjacent. Let H be the common subgraph of K and J with size $n-1$. Since H is a subgraph of K, $H \in \mathcal{B}$. Let e be the complementary edge of H in K. This implies that $E(K) = E(H) \cup \{\mathsf{e}\}$.

Consider $H' \in \mathcal{B} - \{H\}$. Since H and H' are two different connected subgraphs of K, e is an edge of H'.

Note that $J \in \mathcal{A}$ and $H' \in \mathcal{B}$. This implies that H' is adjacent to J. Thus, H' is a connected subgraph of J. Then $E(K) = E(H) \cup \{\mathsf{e}\} \subset E(J)$. Moreover, since K and J have the same size, K and J are the same subgraph of G. This is a contradiction. Therefore, if $\mathcal{N}_K$ is stitched, (1) holds, or K has to be a degenerate subgraph.

Assume that $K = \{v\}$ for some vertex v of G such that $\mathcal{N}_{\{v\}}$ is stitched.

Let $\mathcal{A}$ and $\mathcal{B}$ be a partition of $V\left(\mathcal{N}_{\{v\}}\right)$.

If the degree of v is greater than 2 in G, then there are three distinct vertices a, b, and c such that a, b, and c are adjacent to v in G. Let $A = va$, $B = vb$, and $C = vc$. Without loss of generality, assume $\{a\} \in \mathcal{A}$. Since B and C are not adjacent to $\{v\}$ in $\mathcal{C}(G)$, $B, C \in \mathcal{A}$. Also $\{b\}$ and $\{c\}$ are not adjacent in $\mathcal{C}(G)$ to C and B, respectively. This implies that $\{b\}, \{c\} \in \mathcal{A}$. Since $\{b\}$ is not adjacent to A in $\mathcal{C}(G)$, $A \in \mathcal{A}$. However, a, b, and c were any three vertices adjacent to v. This implies that $V(\mathcal{N}_K) = \mathcal{A}$. This contradicts the fact that $\mathcal{B}$ is not empty.

If the degree of v is 2, let a and b be the only two vertices of G that are adjacent to v and let $A = va$ and $B = vb$. Without loss of generality, assume $a \in \mathcal{A}$. Hence, $B \in \mathcal{A}$.

Note that if $b \in \mathcal{A}$, then $\mathcal{N}_{\{v\}} = \mathcal{A}$, contradicting the fact that $\mathcal{B}$ is not empty.

Hence, $b \in \mathcal{B}$. Since $\mathcal{N}_{\{v\}}$ is stitched with partition $\mathcal{A}$ and $\mathcal{B}$, we have that, in particular, a has to be adjacent to b. Then $vabv$ is a cycle of length 3. Therefore, (3) holds. This finishes the proof. $\square$

Lemma 3.3. *Let G and G' be two connected graphs that are not paths or cycles of size greater than 4. If there is an isomorphism $\phi : \mathcal{C}(G) \to \mathcal{C}(G')$, then $\phi(G) = \phi(G')$.*

Proof: Let $\phi : \mathcal{C}(G) \to \mathcal{C}(G')$ be an isomorphism. Then $\phi(\mathcal{L}(G)) = \mathcal{L}(G')$. By Remark 3.1, $G \in \mathcal{L}(G)$ and $G' \in \mathcal{L}(G')$.

Note that if $\mathcal{L}(G) = \{G\}$, then the result is trivial.

Claim 1. If $\mathcal{L}(G) = \{G, \{v\}\}$ for some vertex v of G, then the degree of $\{v\}$ is less than the degree of G in $\mathcal{C}(G)$.

If the degree of v in G is 1, then the degree of $\{v\}$ in $\mathcal{C}(G)$ is 2. Since G is not a path, the degree of G in $\mathcal{C}(G)$ is greater than 2.

If v is a vertex with degree 2 and contained in a cycle of size 3, then the degree of $\{v\}$ in $\mathcal{C}(G)$ is 4. Let a and b be the two vertices of G that are adjacent to v in G. Note that a and b must both have degrees greater than 2 (otherwise, they would be elements of $\mathcal{L}(G)$). Let K be the connected subgraphs of G with complementary edge set $\{va, vb\}$. Thus, the edge ab is an edge of K that is not a terminal edge of K. No matter if K has a cycle or K is a tree, there are two removable edges $\mathtt{e}$ and $\mathtt{f}$ of G in K. So G has at least five removable edges, namely va, vb, ab, $\mathtt{e}$, and $\mathtt{f}$. Thus, the degree of G in $\mathcal{C}(G)$ is at least 5.

Claim 2. Let $|\mathcal{L}(G)| \geq 3$ and let N be the size of G. Then $K \in \mathcal{L}(G)$ is such that for every $L \in \mathcal{L}(G) - \{K\}$, the distance between K and L in $\mathcal{C}(G)$ is N if and only if $K = G$.

First, note that for every vertex v of G, the distance between G and $\{v\}$ in $\mathcal{C}(G)$ is N.

By Lemma 3.2, $\mathcal{L}(G) - \{G\} \subset \mathcal{Q}_1(G)$. For any $\{v\}, \{w\} \in \mathcal{L}(G) - \{G\}$, the distance from $\{v\}$ and $\{w\}$ in $\mathcal{C}(G)$ is the same as the distance from v to w in G. Since G is not a path, the distance from v to w in G is less than N. This proves the claim.

Therefore, if $|\mathcal{L}(G)| = 2$ or $|\mathcal{L}(G)| \geq 3$, using the claims above, $\phi(G) = G'$. □

Theorem 3.4. *Let G and G' be connected graphs. Then G and G' are isomorphic if and only if $\mathcal{C}(G)$ and $\mathcal{C}(G')$ are isomorphic.*

Proof: One implication is obvious by the construction of the hyperspace graph.

The case when one of the graphs is a path, a cycle, or the 3-star, the result follows from Theorem 1.3, Theorem 1.4, and Theorem 1.5, respectively.

Therefore, assume that neither G nor G' are cycles or paths, and let $\phi : \mathcal{C}(G) \to \mathcal{C}(G')$ be an isomorphism.

By Lemma 3.3, we have that $\phi(G) = G'$. Consider

$$M = \max\{d(G, K) : K \in V(\mathcal{C}(G))\}$$

and

$$\mathcal{R}(G) = \langle K \in V(\mathcal{C}(G)) : d(G, K) = M\rangle.$$

Since ϕ is an isomorphism between $\mathcal{C}(G)$ and $\mathcal{C}(G')$, we have that $\phi(\mathcal{R}(G)) = \mathcal{R}(G')$. Note that $\mathcal{Q}_0(G) = \mathcal{R}(G)$ and $\mathcal{Q}_0(G') = \mathcal{R}(G')$.

Then $\phi|_{\mathcal{Q}_0(G)} : \mathcal{Q}_0(G) \to \mathcal{Q}_0(G')$ is an isomorphism. Therefore, G and G' are isomorphic. $\square$

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