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BEND INTERSECTION PROPERTY AND CONTINUA OF GENERALIZED TYPE N

FÉLIX CAPULÍN, FERNANDO OROZCO-ZITLI, AND ISABEL PUGA

Abstract. In this paper, we show an interrelation among the bend intersection property, continua of type N, and continua of generalized type N. Also, we give some partial answers to the following questions asked by Janusz J. Charatonik and Taejin Lee. Let X be a dendroid.

1) If there exists a retraction from either 2^X or $C(X)$ onto X, then does X have the bend intersection property?

2) If X admits a mean, then does X have the bend intersection property?

3) If X is contractible, then does X have the bend intersection property?

In particular, we show that every locally connected fan at its vertex, accepting retractions from either 2^X or $C(X)$ onto X or admitting means, has the bend intersection property

1. Introduction

All considered spaces are assumed to be metric and all mappings are continuous. A *continuum* means a nonempty compact and connected space. A continuum is said to be *hereditarily unicoherent* if the intersection of any two of its subcontinua is connected. An *arc* is understood as a homeomorphic image of a closed unit interval of the real line. For each x and y in a space Z we denote by xy any arc joining x and y. If any two points of a space Z can be joined by an arc lying in Z, then Z is said to be *arcwise connected*. A *dendroid* is defined as an arcwise connected and hereditarily unicoherent continuum. A point p of a dendroid X is called

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a *ramification point* of X if there exist three arcs emanating from p in X with the intersection of each two of them being just the singleton $\{p\}$; we denote by $R(X)$ the set of all ramification points of X. A *fan* means a dendroid having exactly one ramification point, and this point is called its *vertex*.

Let X be a continuum with a metric d. The hyperspace of all nonempty closed subsets of X is denoted by 2^X and the hyperspace of all subcontinua of X is denoted by $C(X)$. Let $F_1(X) = \{\{x\} : x \in X\}$ and $F_2(X) = \{\{x, y\} : x, y \in X\}$, all equipped with the Hausdorff metric. Since $F_1(X)$ is homeomorphic to X, we may assume that $X \subset C(X)$.

By a *selection* for $C(X)$, we mean a mapping $s : C(X) \to X$ such that $s(A) \in A$ for each $A \in C(X)$. X is said to be *selectible* provided that there is a selection for $C(X)$.

Let A be a closed subset of a topological space Z; a *retraction from Z onto A* is a mapping $r : Z \to A$ such that $r|_A = id|_A$. A *mean* on a space Z is a mapping $m : Z \times Z \to Z$ such that

(a) $m((x, x)) = x$ for each $x \in Z$,
(b) $m((x, y)) = m((y, x))$ for each $x, y \in Z$.

A topological space Z is said to be *contractible* provided that there are a mapping $H : Z \times [0, 1] \to Z$ and a point $p \in Z$ such that for each point $z \in Z$, $H(z, 0) = z$ and $H(z, 1) = p$.

The symbol $\mathbb{N}$ stands for the set of all positive integers. Let X be a continuum and $p, q \in X$. We say that X is of *type N between p and q* if there exist in X an arc $A = pq$, two sequences of arcs $\{A_i\}_{i=1}^{\infty} = \{p_i p_i'\}_{i=1}^{\infty}$ and $\{B_i\}_{i=1}^{\infty} = \{q_i q_i'\}_{i=1}^{\infty}$, and points $p_i'' \in B_i \setminus \{q_i q_i'\}$ and $q_i'' \in A_i \setminus \{p_i p_i'\}$ (where $i \in \mathbb{N}$) such that

(1) $A = \mathrm{Lim} A_i = \mathrm{Lim} B_i$;
(2) $p = \mathrm{lim} p_i = \mathrm{lim} p_i' = \mathrm{lim} p_i''$;
(3) $q = \mathrm{lim} q_i = \mathrm{lim} q_i' = \mathrm{lim} q_i''$;
(4) each arc in X joining p_i and p_i' contains q_i'';
(5) each arc in X joining q_i and q_i' contains p_i''.

We say that X is of *generalized type N between p and q* if there exist in X a subcontinuum K containing p and q, two sequences of arcs $\{p_i p_i'\}_{i=1}^{\infty}$ and $\{q_i q_i'\}_{i=1}^{\infty}$, and points $p_i'' \in q_i q_i' \setminus \{q_i, q_i'\}$ and $q_i'' \in p_i p_i' \setminus \{p_i, p_i'\}$ (where $i \in \mathbb{N}$) such that

(1) $K = \mathrm{Lim}\ p_i q_i'' = \mathrm{Lim}\ p_i' q_i'' = \mathrm{Lim}\ q_i p_i'' = \mathrm{Lim}\ q_i' p_i''$;
(2) $p = \lim\ p_i = \lim\ p_i' = \lim\ p_i''$;
(3) $q = \lim\ q_i = \lim\ q_i' = \lim\ q_i''$;
(4) each arc in X joining p_i and p_i' contains q_i'';
(5) each arc in X joining q_i and q_i' contains p_i''.

We say that a continuum X is of *(generalized) type* N if X is of (generalized) type N between two points in X. It is evident that each continuum of type N is a continuum of generalized type N, but not inversely, even for fans (see [1, Example 2, p. 270]).

The following definition was introduced by Tadeusz Maćkowiak in [13].

Let A be a subcontinuum of a continuum X and let $B \subset A$. We say that B is a *bend set* of A if there exist two sequences of subcontinua $\{A_n\}_{n=1}^{\infty}$ and $\{A'_n\}_{n=1}^{\infty}$ of X satisfying the following conditions:

(1) $A_n \cap A'_n \neq \emptyset$ for each $n \in \mathbb{N}$;
(2) $A = \mathrm{Lim} A_n = \mathrm{Lim} A'_n$;
(3) $B = \mathrm{Lim}(A_n \cap A'_n)$.

A continuum X is said to have the *bend intersection property* provided that, for each subcontinuum A of X, the intersection of all its bend sets is nonempty.

Recall that in [7, p. 78], the concept of generalized type N is introduced, but it is different from the one given here. In order to see the difference between these definitions, we also write here the one given in [7, pp. 78-79].

A continuum X is of *generalized type* N *between* p *and* q if there exist in X a subcontinuum K containing p and q, two sequences of arcs $\{p_i p'_i\}_{i=1}^{\infty}$ and $\{q_i q'_i\}_{i=1}^{\infty}$, and points $p''_i \in q_i q'_i \setminus \{q_i, q'_i\}$ and $q''_i \in p_i p'_i \setminus \{p_i, p'_i\}$ (where $i \in \mathbb{N}$) such that

(1) $K = \mathrm{Lim}\ p_i p'_i = \mathrm{Lim}\ q_i q'_i$;
(2) $p = \lim\ p_i = \lim\ p'_i = \lim\ p''_i$;
(3) $q = \lim\ q_i = \lim\ q'_i = \lim\ q''_i$;
(4) each arc in X joining p_i and p'_i contains q''_i;
(5) each arc in X joining q_i and q'_i contains p''_i.

We will show that the first condition in each of the two definitions is different.

In the first part of section 2, we are going to show that the fan given in [4, Example 3.10] satisfies the definition of Charatonik, et al.; however, it is selectible (by [4, Example 3.10]). On the other hand, it does not satisfy the first condition of our definition.

Adopting the definition of generalized type N given here, Proposition 14.11 in [7] is true.

On the other hand, it is immediate to see that if a continuum X has the bend intersection property, then X is not of type N and it is not of generalized type N. It is known that for the class of dendroids it is not true that not to be of type N implies having bend intersection property (see [12, Example 7, p. 126]). It is not true even for the class of fans (see [1, Example 2, p. 270]).

Since both examples are of generalized type N, it is natural to ask: If X is not of generalized type N, then does X have the bend intersection property? In section 2, we will present a fan which is not of generalized type N, but it does not have the bend intersection property. We will also provide some conditions on dendroids which imply on them the bend intersection property.

Additionally, we know that a dendroid of type N does not have the bend intersection property; therefore, it is nonselectible (see [13, Corollary]).

Regarding this result, the following question was asked: Does there exist a nonselectible fan which is not of type N? (See [4, Question 3.16].) In section 2, we answer this question in a positive way.

Further, it is well known that

(1) if X is a fan which is not of type N and it does not have Q-points, then X has the bend intersection property [11, Theorem 1];
(2) if X is a contractible fan, then it has the bend intersection property [11, Theorem 2];
(3) if X is a selectible dendroid, then it has the bend intersection property [13, Corollary].

Concerning these results, Janusz J. Charatonik [6] and Taejin Lee [12] asked the following questions. Let X be a dendroid.

Question 1. If there exists a retraction from $C(X)$ (2^X) onto X, then does X have the bend intersection property? [6, Question 5.9]

Question 2. If X admits a mean, then does X have the bend intersection property? [6, Question 3.27]

Question 3. If X is contractible, then does X have the bend intersection property? [12, Question 8]

In section 3, we provide some partial answers.

2. Continuum of (Generalized) Type N and Bend Intersection Property

Example 2.1. *There exists a selectible fan X satisfying the conditions of the definition of generalized type N given in* [7, p. 78] *which contradicts* [7, Proposition 14.11].

Proof. We consider the fan in [4, Example 3.10] (or fan X_1 in [3, Example 2.1]), which is repeated here for the reader's convenience. First of all, in order to define this fan, we denote by $\overline{xy}$ the convex arc (in the Euclidian space $\mathbb{R}^2$) joining the point x and y. Then we consider the following points in $\mathbb{R}^2$ in polar coordinates:

$$v = (0, 0),$$

$$a_n = (\tfrac{1}{2^{n-1}}, \tfrac{\pi}{2^n}) \text{ and } b_n = (\tfrac{1}{n}, \pi) \text{ for each } n \in \mathbb{N},$$
$$a_{n,m} = (\tfrac{1}{2^{n-1}}(1+\tfrac{1}{m}), \tfrac{\pi}{2^n}) \text{ and } p_{n,m} = (\tfrac{1}{m2^n}, \tfrac{3\pi}{2^{n+2}}) \text{ for every } m, n \in \mathbb{N}.$$

Consider $X = F_\omega \cup (\bigcup\{vb_m : m \in \mathbb{N}\})$, where $F_\omega = \bigcup\{\overline{va_n} : n \in \mathbb{N}\}$ and $vb_m = \overline{b_m a_{1,m}} \cup (\bigcup\{\overline{a_{n,m}p_{n,m}} \cup \overline{p_{n,m}a_{n+1,m}} : n \in \mathbb{N}\}) \cup \{v\}$ for each $m \in \mathbb{N}$ (see Figure 1).

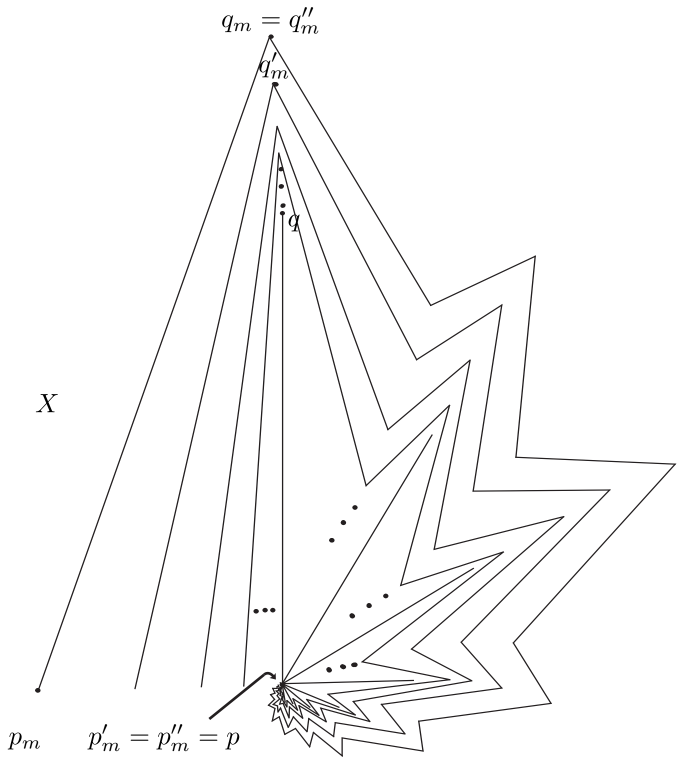

FIGURE 1

Let $q = a_1$; $q_m = a_{1,m} = q''_m$; $q'_m = a_{1,m+1}$; $p_m = b_m$; and $v = p = p_m = p''_m$ for each $m \in \mathbb{N}$. Then the sequences of arcs in X, which are $\{p_m p'_m : m \in \mathbb{N}\}$ and $\{q_m q'_m : m \in \mathbb{N}\}$, F_ω, and the points p and q satisfy the conditions of the definition in [7, p. 78], but do not satisfy the first condition of the definition of generalized type N given here.

On the other hand, by [4, Example 3.10], X is selectible, which contradicts [7, Proposition 14.11]. Hence, the condition Lim $p_m p'_m = K =$

Lim $q_m q'_m$, together with the other ones given in [7, p. 78], is not enough to contradict the selectibility. □

Example 2.2. *There exists a fan which is not of generalized type N and it does not have the bend intersection property.*

Proof. We apply the same notation as in Example 2.1. Consider the following points:

$d_{n,m} = (\frac{1}{2^{n-1}}(1+\frac{1}{m}), \frac{\pi}{2^n}, \frac{1}{2^{n+(2m+1)}})$ and

$e_{n,m} = (\frac{1}{m2^n}, \frac{3\pi}{2^{n+2}}, \frac{1}{2^{n+(2m+1)}})$ for each $m, n \in \mathbb{N}$.

For each $m \in \mathbb{N}$, let

$$b_m d_{1,m} = \overline{b_m d_{3,m}} \cup \overline{d_{3,m} e_{2,m}} \cup \overline{e_{2,m} d_{2,m}} \cup \overline{d_{2,m} e_{1,m}} \cup \overline{e_{1,m} d_{1,m}}.$$

Let $H = X \cup (\bigcup\{b_m d_{1,m} : m \in \mathbb{N}\})$ (see Figure 2).

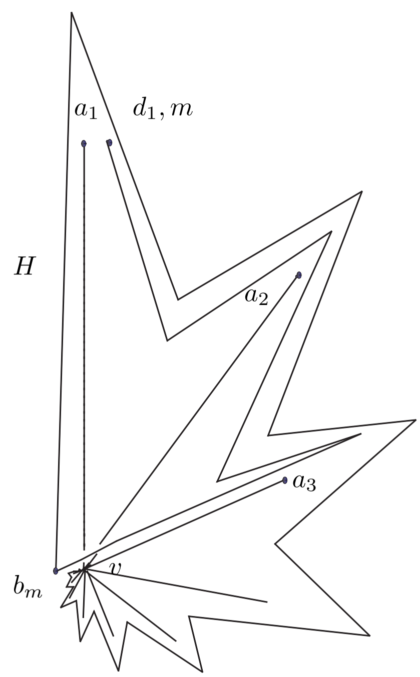

FIGURE 2

In this picture, we only show an arc of the sequence $\{v\boldsymbol{b_m} \cup \boldsymbol{b_m d_{1m}}\}$.

Notice that H is a fan. It is not difficult to see that the sets $\{a_1\}$ and $\{v\}$ are bend sets of the triod $T = \overline{va_1} \cup \overline{va_2} \cup \overline{va_3}$. Hence, the intersection of all its bend sets is empty. So, H does not have the bend intersection property. The proof that H is not of generalized type N is left to the reader. □

In [1, Example 2, p. 270], the authors show an example of a fan which is not of type N and it does not have the bend intersection property. So, by [13, Corollary], it is nonselectible. Using this fan, we respond positively to Question 3.16 asked by Charatonik [4]. Notice that this fan is of generalized type N. So it is natural to ask: If X is a nonselectible fan, then is X of generalized type N? If we consider the fan H above, it is easy to see that this question is answered in a negative way.

Now, we are going to give some conditions in dendroids which imply the bend intersection property.

First of all, we need the following two lemmas.

Lemma 2.3 ([12, Lemma 1]). *Let X be a continuum and let B be a bend set of a subcontinuum A of X. If the sequences $\{A_n\}_{n=1}^{\infty}$ and $\{A'_n\}_{n=1}^{\infty}$ satisfy the conditions of the bend set definition, then for each $p \in A\backslash B$ and each two sequences of points $\{p_n\}_{n=1}^{\infty}$ and $\{p_n\}_{n=1}^{\infty}$ such that $p_n \in A_n\backslash A'_n$ and $p'_n \in A'_n\backslash A_n$ and $Lim\, p_n = Lim\, p'_n = p$, there exist a sequence $\{a_{n_k}\}_{k=1}^{\infty}$, irreducible continua $I(p_{n_k}, p'_{n_k})$ between p_{n_k} and p'_{n_k}, and a point $a \in B$ such that $a_{n_k} \in A'_{n_k} \cap A_{n_k} \cap I(p_{n_k}, p'_{n_k})$ and $I(p_{n_k}, p'_{n_k}) \subset A'_{n_k} \cup A_{n_k}$ for each $k \in \mathbb{N}$ and $Lim\, a_{n_k} = a$.*

Let X be a continuum.

Let $L(X) = \{x \in X : X$ is locally connected in $x\}$ and $\mathbb{A} = \{A \in C(X) : A$ is an arc$\}$.

Lemma 2.4. *Let X be a hereditarily unicoherent continuum. If B is a bend set of a nondegenerate subcontinuum K of X, then $K \cap L(X) \subset B$.*

Proof. Suppose that B is a bend set of K such that $(K \cap L(X)) \setminus B \neq \emptyset$. Let $v \in (K \cap L(X)) \setminus B$. Since B is a bend set of K, there exist two sequences $\{A_n\}_{n=1}^{\infty}$ and $\{A'_n\}_{n=1}^{\infty}$ that satisfy the conditions of the bend set definition. Consider two sequences of points $\{v_n\}_{n=1}^{\infty}$ and $\{v'_n\}_{n=1}^{\infty}$ such that $v_n \in A_n \setminus A'_n$, $v'_n \in A'_n \setminus A_n$, and $\lim v_n = \lim v'_n = v$. By Lemma 2.3, there exist a sequence $\{b_{n_k}\}_{k=1}^{\infty}$ and a point $b \in B$ such that $\lim b_{n_k} = b$, and irreducible continua $I(v_{n_k}, v'_{n_k})$ between v_{n_k} and v'_{n_k} such that $b_{n_k} \in A'_{n_k} \cap A_{n_k} \cap I(v_{n_k}, v'_{n_k})$ and $I(v_{n_k}, v'_{n_k}) \subset A'_{n_k} \cup A_{n_k}$ for each $k \in \mathbb{N}$.

On the other hand, consider a connected open set W such that $v \in W$ and $\overline{W} \cap B = \emptyset$. Since $\lim v_n = \lim v'_n = v$ and $\lim b_{n_k} = b$, there exists $N \in \mathbb{N}$ such that $b_{n_k} \notin \overline{W}$ and $v_{n_k}, v'_{n_k} \in W$ for each $k \geq N$. Since X is hereditarily unicoherent, $I(v_{n_k}, v'_{n_k})$ is unique. So $I(v_{n_k}, v'_{n_k}) \subset \overline{W}$ with $k \geq N$; this is a contradiction. □

Theorem 2.5. *Let X be a dendroid such that for each $K \in C(X) \setminus (\mathbb{A} \cup F_1(X))$, $K \cap L(X) \neq \emptyset$. If X is not of (generalized) type N, X has the bend intersection property.*

Proof. Since each $K \in C(X) \setminus (\mathbb{A} \cup F_1(X))$ satisfies $K \cap L(X) \neq \emptyset$, by Lemma 2.4, the intersection of all bend sets of K is nonempty. Now, since X is not of (generalized) type N, by [12, Theorem 5], for each $A \in \mathbb{A}$, the intersection of all its bend sets is nonempty. Hence, X has the bend intersection property. □

Note that the fan of Example 2.1 has the bend intersection property and it is not of type N, but each $K \in C(F_\omega) \setminus (\mathbb{A} \cup F_1(X))$ satisfies $K \cap L(X) = \emptyset$. So this condition is not necessary in Theorem 2.5.

Notice that if X is a continuum such that for each $K \in C(X) \setminus F_1(X)$, $K \cap L(X) \neq \emptyset$, then, by [14, Exercise 5.22], X is locally connected. In particular, where X is hereditarily unicoherent, X is a dendrite (see [14, Theorem 10.35]). Then X is selectible (see [15, Corollary, p. 371]). Therefore, it has the bend intersection property (see [13, Corollary]).

Corollary 2.6. *Let X be a dendroid such that $R(X) \subset L(X)$. If X is not of (generalized) type N, then X has the bend intersection property.*

Proof. Let $K \in C(X) \setminus (\mathbb{A} \cup F_1(X))$. Then K contains a triod. Hence, $K \cap L(X) \neq \emptyset$. So, by Theorem 2.5, X has the bend intersection property. □

Notice that $R(Z)$ does not have to be contained in $L(Z)$ even if Z has the bend intersection property and $K \cap L(Z) \neq \phi$ for each $K \in C(Z) \setminus (\mathbb{A} \cup F_1(Z))$ (see Figure 3).

Corollary 2.7. *If X is a locally connected fan at the vertex, then the following conditions are equivalent:*

(1) *X is not of generalized of type N;*
(2) *X has the bend intersection property;*
(3) *X is not of type N;*
(4) *for each arc in X, the intersection of all its bend sets is nonempty.*

Proof. By Corollary 2.6, we have that (1) ⇒ (2) and (3) ⇒ (2). (2) ⇒ (3) and (2) ⇒ (1) are immediate. By [12, Theorem 5], (3) ⇔ (4). □

Corollary 2.8. *Let X be a fan. Then the following conditions are equivalent:*

(1) *Contractible;*
(2) *X is not type N, X is locally connected at the vertex, and X is pairwise smooth;*
(3) *X contains no Q-points, X has the bend intersection property, and X is pairwise smooth;*

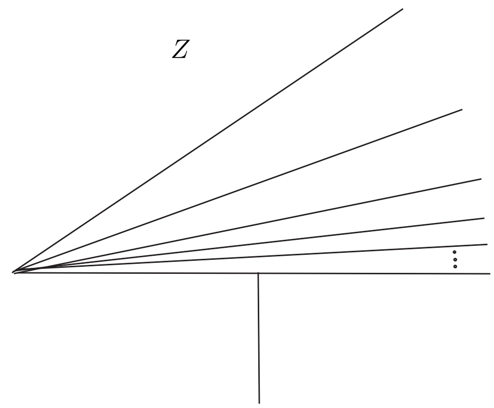

FIGURE 3

(4) *X contains no Q-points, X is not of generalized type N, and X is pairwise smooth;*
(5) *X is not type N, X contains no Q-points, and X is pairwise smooth;*
(6) *X contains no Q-points and no zig-zag, and X is pairwise smooth;*
(7) *X is monotone contractible;*
(8) *X is confluent contractible;*
(9) *X is weakly confluent contractible.*

Proof. (1) $\Rightarrow$ (2). It follows from [18, Theorem 6.1], [16, Corollary 2.2], and [9, Theorem 2.4]. Theorem 6.1 of [18], Corollary 2.2 of [16] and Theorem 2.4 of [9].
(2) $\Rightarrow$ (3). By [2, Theorem 2.2] and Corollary 2.7.
(3) $\Rightarrow$ (4). By Corollary 2.7.
(4) $\Rightarrow$ (5). It follows from Corollary 2.7.
(5) $\Rightarrow$ (6) $\Rightarrow$ (7) $\Rightarrow$ (8) $\Rightarrow$ (9) $\Rightarrow$ (1). By [17, Theorem 3.4]. □

3. The Bend Intersection Property, Retractions, Contractions, and Means

The following result is known.

Proposition 3.1. *Let X be a dendroid. Each of the following conditions implies that X is not of type N:*

(1) *X is contractible* [16, Corollary 2.2]*;*
(2) *there exists a retraction from 2^X onto X* [1, Corollary 2.4]*;*
(3) *there exists a retraction from $C(X)$ onto X* [1, Theorem 2.1]*;*
(4) *there exists a retraction from $F_2(X)$ onto X* [1, Theorem 2.2]*;*
(5) *X admits a mean* [1, Corollary 2.3]*,* [10, Theorem 2.2].

Finally, we are going to provide some partial answers to the questions given in the abstract.

Theorem 3.2. *Let X be a dendroid such that for each $K \in C(X) \setminus (\mathbb{A} \cup F_1(X))$, $K \cap L(X) \neq \emptyset$. Each of the following conditions implies that X has the bend intersection property:*

(1) *there exists a retraction from $C(X)$ onto X;*
(2) *there exists a retraction from 2^X onto X;*
(3) *there exists a retraction from $F_2(X)$ onto X;*
(4) *X admits a mean;*
(5) *X is contractible.*

Proof. It follows from Proposition 3.1 and Theorem 2.5. □

Theorem 3.3. *Let X be a dendroid such that $R(X) \subset L(X)$. Each of the following conditions implies that X has the bend intersection property:*

(1) *there exists a retraction from $C(X)$ onto X;*
(2) *there exists a retraction from 2^X onto X;*
(3) *there exists a retraction from $F_2(X)$ onto X;*
(4) *X admits a mean;*
(5) *X is contractible.*

Proof. It follows from Proposition 3.1 and Corollary 2.6. □

In particular, we have the following result on fans.

Corollary 3.4. *Let X be a fan locally connected at the vertex. Each of the following conditions implies that X has the bend intersection property:*

(1) *there exists a retraction from $C(X)$ onto X;*
(2) *there exists a retraction from 2^X onto X;*
(3) *there exists a retraction from $F_2(X)$ onto X;*
(4) *X admits a mean;*
(5) *X is contractible* [11, Theorem 2].

Proof. It follows from Proposition 3.1 and Corollary 2.7. □

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