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by

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**ON THE UNICOHERENCE OF
 $F_n(X)$ AND $SF_m^n(X)$ OF CONTINUA**

ENRIQUE CASTAÑEDA-ALVARADO AND JAVIER SÁNCHEZ-MARTÍNEZ

ABSTRACT. A *continuum* means a compact, connected, nondegenerate metric space. Given a continuum X and $n \in \mathbb{N}$, $F_n(X)$ denotes the hyperspace of all subsets of X with at most n points equipped with the Hausdorff metric. This hyperspace is called the n^{th} -symmetric product of X . If $m, n \in \mathbb{N}$ and $m < n$, we consider $SF_m^n(X)$ as the quotient space $F_n(X)/F_m(X)$ obtained by shrinking $F_m(X)$ to a point in $F_n(X)$, topologized with the quotient topology. In this paper, using inverse limits, we answer negatively a question of Janusz J. Charatonik (see E. Castañeda, *A unicoherent continuum whose second symmetric product is not unicoherent*, Topology Proc. **23** (1998), Spring, 61–67) about the unicoherence of the second symmetric products of λ -dendroids. We also analyze the unicoherence of $SF_m^n(X)$.

1. INTRODUCTION

A *continuum* means a compact, connected, and nondegenerate metric space. The symbols \mathbb{N} and \mathbb{C} will denote the set of positive integers and complex numbers, respectively. Also I will be the unit interval $[0, 1]$. Consider the following hyperspaces of a continuum X :

$$2^X = \{A \subset X : A \text{ is closed and nonempty}\}, \text{ and for } n \in \mathbb{N}$$

$$C_n(X) = \{A \in 2^X : A \text{ has at most } n \text{ components}\},$$

$$F_n(X) = \{A \in 2^X : A \text{ has at most } n \text{ points}\}.$$

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These hyperspaces are considered with the Hausdorff metric. The hyperspace $F_n(X)$ is also known as the n^{th} -symmetric product of X ; symmetric products were introduced by Karol Borsuk and Stanislaw Ulam in [3].

In 1979, Sam B. Nadler, Jr., [24] introduces the *hyperspace suspension of a continuum* X as the quotient space $C_1(X)/F_1(X)$. In 2004, Sergio Macías [19, p. 127] defines the n -fold *hyperspace suspension of a continuum* X as the quotient space $C_n(X)/F_n(X)$. In 2010, Franco Barragán [2] defines the n -fold *symmetric product suspension of a continuum* X as the quotient space $F_n(X)/F_1(X)$. Barragán studies the properties of unicoherence, local connectedness, and arcwise connectedness of this hyperspace and, in [2, section 4], he gives an example of a unicoherent continuum whose second symmetric product suspension is not unicoherent. We note in this paper that this example is incorrect (see Theorem 5.15). In general, for $m, n \in \mathbb{N}$ with $m < n$ and a continuum X , in section 5 we study mainly the property of unicoherence of the quotient space $F_n(X)/F_m(X)$ that we will denote by $SF_m^n(X)$ and consequently are characterized finite graphs for which $F_2(X)$ is homeomorphic to $SF_1^2(X)$.

In section 2, we give the basic concepts to understand this paper. In section 3, we analyze the unicoherence of $SF_1^2(X)$ for some continua. In section 4, we study the relationship between inverse limits and symmetric products of continua, useful for obtaining the results of section 5. Also in this section, in particular we answer negatively the problem of Janusz J. Charatonik: Does there exist an hereditarily unicoherent, hereditarily decomposable continuum X such that $F_2(X)$ is not unicoherent? (see [4, Problem 3, p. 66] and [16, Problem 34, p. 284]).

2. DEFINITIONS AND PRELIMINARIES

Let X be a continuum. A *subcontinuum* of X is a continuum contained in X . Given $A \subset X$ and $\varepsilon > 0$, the open ball around A of radius ε will be denoted by $\mathcal{V}_\varepsilon(A)$, and the closure of A in X will be denoted by $Cl_X(A)$.

A *map* is a continuous function. An onto map $f : X \rightarrow Y$ between continua is said to be *monotone* if $f^{-1}(y)$ is a connected subset of X for each $y \in Y$.

A continuum X is *unicoherent* provided that whenever A and B are closed, connected subsets of X such that $X = A \cup B$, then $A \cap B$ is connected. For each topological space Y , we define

$$b_0(Y) = (\text{number of components of } Y) - 1$$

if this number is finite, and $b_0(Y) = \infty$ otherwise. The *multicoherence degree*, $r(X)$, of a continuum X is defined by

$$r(X) = \sup \{b_0(H \cap K) : H, K \in C_1(X) \text{ and } X = H \cup K\}.$$

Notice that $r(X) = 0$ if and only if X is unicoherent. A continuum X is said to be *multicoherent* provided that $r(X) \neq 0$.

The following results about the unicoherence of symmetric products of continua are known.

Theorem 2.1 ([18, Theorem 8, p. 177]). *If X is a continuum, $F_n(X)$ is unicoherent for all $n \geq 3$.*

Theorem 2.2 ([18, p. 181]). *If X is a continuum, then $r(F_2(X)) \leq 1$.*

By the previous theorem, [14], and [15, Theorem 1.6, p. 16], we have the following theorem.

Theorem 2.3. *Let X be a continuum locally connected. $F_2(X)$ is unicoherent if and only if X is unicoherent.*

E. Castañeda [4, Example 2.1] gives an example of a unicoherent continuum X whose $F_2(X)$ is not unicoherent.

A *retraction* is a map r from a space X into itself such that r is the identity on its image (i.e., $r(r(x)) = r(x)$ for each $x \in X$). A closed subset A of X is said to be a *retract* of X provided that there is a retraction of X onto A . A space X is called an *absolute retract* provided that X is a retract of every space Y containing X as a closed subset.

If X is a continuum and $A \subset X$, then A is said to be a *deformation retract* of X provided that there is a map $h : X \times I \rightarrow X$ such that $h(x, 1) = x$ for all $x \in X$, $h(X \times \{0\}) \subset A$, and the function $r : X \rightarrow A$, defined by $r(x) = h(x, 0)$, is a retraction.

We said that a continuum X has *property (b)* if, for each map $f : X \rightarrow S^1$, there exists a map $g : X \rightarrow \mathbb{R}$ such that $f(x) = (\exp \circ g)(x)$ for all $x \in X$ where \exp is the exponential map and S^1 denotes the unit circumference.

The following lemmas will be used in this paper.

Lemma 2.4 ([1, Proposition 9, p. 2001]). *Let Z be a topological space and let Y be a deformation retract of Z . Then Z has property (b) if and only if Y has property (b).*

In general, if X is a connected, normal topological space and X has property (b), then X is unicoherent; see [11, Theorem 2, p. 69].

Lemma 2.5 ([11, Theorem 3, p. 70]). *If X is a continuum locally connected, then X has property (b) if and only if X is unicoherent.*

The following result is a particular case of [1, Proposition 8, p. 2001].

Lemma 2.6. *Let X be a continuum and $A, B \in C_1(X)$. If $X = A \cup B$, A and B have property (b), and $A \cap B$ is connected, then X has property (b).*

Given a continuum X and $n \in \mathbb{N}$, the product of X with itself n times will be denoted by X^n .

The Hausdorff metric for 2^X is defined by

$$H(A, B) = \inf \{ \varepsilon > 0 : A \subset \mathcal{V}_\varepsilon(B) \text{ and } B \subset \mathcal{V}_\varepsilon(A) \}.$$

Given a finite collection, U_1, \dots, U_m , of subsets of X , $\langle U_1, \dots, U_m \rangle_n$, denote the following subset of $F_n(X)$

$$\left\{ A \in F_n(X) : A \subset \bigcup_{i=1}^m U_i \text{ and } A \cap U_i \neq \emptyset \text{ for each } i = 1, \dots, m \right\}.$$

If each U_i is an open subset of X , it is known that the family of all subsets of $F_n(X)$ of the form $\langle U_1, \dots, U_m \rangle_n$ is a basis for the topology of $F_n(X)$ called the *Vietoris topology* (see [23, Theorem 0.11, p. 9]), and it is known that the Vietoris topology and the topology induced by the Hausdorff metric are the same (see [23, Theorem 0.13, p. 9]).

By a *graph* we mean a continuum which can be written as the union of finitely many arcs, any two of which are either disjoint or intersect only in one or both of their end points. By an *edge* of a graph X , we shall always mean one of those arcs. The end points of the edges of X are called *vertices* of X . Given a point $x \in X$ and a natural number n , the *order* of x in X (denoted by $\text{ord}(x, X)$) is n , provided that, for every $\varepsilon > 0$, there exists an open set U of X containing x with the diameter of U less than ε (denoted by $\text{diam}(U) < \varepsilon$) such that its boundary $\partial(U)$ consists of exactly n points. For each vertex $v \in X$, we have either $\text{ord}(v, X) = 1$ if v is an end point of X or $\text{ord}(v, X) \geq 2$ otherwise. If $\text{ord}(v, X) \geq 3$, then v is called a *ramification point* of X . By a *simple n -od* ($n \geq 3$), we mean a graph X with only one ramification point, exactly n end points, and without simple closed curves. A simple 3-od will be called simply *triad*. The *complete graph* K_m is the graph with exactly m vertices such that any two of them are joined by an edge of the graph.

If $m, n \in \mathbb{N}$, $m < n$, and X is a continuum, $SF_m^n(X)$ denotes the quotient space $F_n(X)/F_m(X)$, obtained by shrinking $F_m(X)$ to a point in $F_n(X)$, topologized with the quotient topology.

Given a continuum X , $\rho_{m,n}^X : F_n(X) \rightarrow SF_m^n(X)$ denotes the natural quotient map.

Remark 2.7. It is clear that, using the appropriate restriction of $\rho_{m,n}^X$, $SF_m^n(X) \setminus \rho_{m,n}^X(F_m(X))$ is homeomorphic to $F_n(X) \setminus F_m(X)$.

Given a map $f : X \rightarrow Y$ between continua and a positive integer n , the function $F_n(f) : F_n(X) \rightarrow F_n(Y)$ given by $F_n(f)(A) = f(A)$ is the *induced map by f between the n^{th} -symmetric products of X and Y ,*

respectively. If $m, n \in \mathbb{N}$ and $m < n$, we consider the natural induced function $SF_m^n(f) : SF_m^n(X) \rightarrow SF_m^n(Y)$, which is defined by

$$SF_m^n(f) (\rho_{m,n}^X(A)) = \rho_{m,n}^Y(f(A)).$$

By [10, Theorem 4.3, p. 126], $SF_m^n(f)$ is a map.

3. EXAMPLES

In this section we present examples of quotient of symmetric products for some continua. Examples 3.1 and 3.3 are geometric models of $SF_1^2(X)$ for the considered continua; these examples also appear in [2]. In examples 3.6, 3.9, 3.11, 3.13, 3.14, and 3.15, we analyze the unicoherence of $SF_1^2(X)$.

Example 3.1. If X is an arc, then $F_2(X)$ is homeomorphic to $SF_1^2(X)$ (see [2, Example 3.1, p. 598]). If X is a simple closed curve, then $SF_1^2(X)$ is homeomorphic to the real projective plane \mathbb{RP}^2 (see [2, Example 3.3, p. 599]). If X is the Hilbert Cube, then X , $F_n(X)$, and $SF_1^n(X)$ are homeomorphics (see [2, Example 3.5, p. 599]).

Remark 3.2. Observe that S^1 is an example of a nonunicoherent continuum such that $F_2(S^1)$ is not unicoherent, but $SF_1^2(S^1)$ is unicoherent (see [26, p. 227]).

The following example is a generalization of [2, Example 3.2, p. 598].

Example 3.3. Let $n \geq 3$ and let X be a simple n -od with e_1, \dots, e_n its end points. If

$$Z = \{A \in F_2(X) : e_i \in A \text{ for some } i \in \{1, \dots, n\}\},$$

then $F_2(X)$ is homeomorphic to the cone over Z (see [5, Lemma 1, p. 68]). By [5, Lemma 2, p. 70], Z is the union of K_n and n pairwise disjoint arcs B_1, \dots, B_n , each of them intersecting K_n in exactly one of its vertices $\{b_i\}$.

Moreover, $F_1(X)$ only intersects Z in $\bigcup_{i=1}^n B_i$ and $F_1(X) \cap B_i$ is the end point of B_i different from $\{b_i\}$ for each $i = 1, \dots, n$. So, using Example 3.1, $SF_1^2(X)$ is homeomorphic to cone over Z' where Z' is the union of K_n and n pairwise disjoint arcs B'_1, \dots, B'_n , each of them intersecting K_n in exactly one of its vertices. We conclude that $SF_1^2(X)$ is homeomorphic to $F_2(X)$.

Lemma 3.4. *Let X be a continuum and let A be a subcontinuum of X . If $\mathcal{X} = (X \times \{0\}) \cup (A \times I)$, then $X \times \{0\}$ is a deformation retract of \mathcal{X} , where \mathcal{X} is considered as a subspace of $X \times I$ with the product topology.*

Proof. We define $\phi : \mathcal{X} \times I \rightarrow \mathcal{X}$ by

$$\phi((x, t), s) = (x, ts), \text{ for all } (x, t) \in \mathcal{X} \text{ and } s \in I.$$

Since ϕ is a map, $\phi((x, t), 1) = (x, t)$ for all $(x, t) \in \mathcal{X}$, $\phi(\mathcal{X} \times \{0\}) \subset X \times \{0\}$, and the function $r : \mathcal{X} \rightarrow X \times \{0\}$ given by $r((x, t)) = \phi((x, t), 0) = (x, 0)$ is a retraction, then $X \times \{0\}$ is a deformation retract of \mathcal{X} . \square

The following notation will be used: If $x, y \in \mathbb{C}$, xy denotes the linear segment from x to y in \mathbb{C} for each $n \in \mathbb{N}$, $S_n = \{\frac{1}{n} + \frac{1}{n}e^{i\theta} \in \mathbb{C} : \theta \in \mathbb{R}\}$ will be the circumference in the complex plane with center $(\frac{1}{n}, 0)$ and radius $\frac{1}{n}$, and l_n will denote the segment joining $0 \in \mathbb{C}$ with $-1 + i\frac{1}{n} \in \mathbb{C}$.

Remark 3.5. If A is a subcontinuum of a continuum X , $SF_m^n(A)$ can be considered as the subspace of $SF_m^n(X)$ given by $\rho_{m,n}^X(F_n(A))$.

Example 3.6. If $Y = S_1 \cup l_1$, then $SF_1^2(Y)$ is unicoherent.

Proof. Notice that

$$SF_1^2(Y) = (SF_1^2(S_1)) \cup \mathcal{L} \cup (SF_1^2(l_1)),$$

where $\mathcal{L} = \{\rho_{1,2}^Y(\{x, y\}) : x \in S_1 \text{ and } y \in l_1\}$. Making $X = SF_1^2(S_1)$ and $A = \{\rho_{1,2}^Y(\{x, 0\}) : x \in S_1\}$, observe that A is a subcontinuum of X and $h : \mathcal{L} \rightarrow A \times l_1$, given by $h(\rho_{1,2}^Y(\{x, y\})) = (\rho_{1,2}^Y(\{x, 0\}), y)$, is a homeomorphism. Then, by Lemma 3.4, we have that $SF_1^2(S_1)$ is a deformation retract of $SF_1^2(S_1) \cup \mathcal{L}$, and, by Example 3.1, $SF_1^2(S_1)$ is unicoherent. By Lemma 2.4 and Lemma 2.5, $SF_1^2(S_1) \cup \mathcal{L}$ has property (b). By Example 3.1, $SF_1^2(l_1)$ is unicoherent and then, by Lemma 2.5, $SF_1^2(l_1)$ has property (b). Since $(SF_1^2(S_1) \cup \mathcal{L}) \cap SF_1^2(l_1)$ is an arc, by Lemma 2.6, $SF_1^2(Y)$ has property (b) and then it is unicoherent by Lemma 2.5. \square

Lemma 3.7. Let X be a continuum and $A, B \in C_1(X)$ such that $A \cap B$ has at most one point; then $A \times B$ is homeomorphic to $\langle A, B \rangle_2$.

Proof. It is not difficult to see that the function $h : A \times B \rightarrow \langle A, B \rangle_2$, given by $h(a, b) = \{a, b\}$, is a homeomorphism. In fact, by [20, Corollary 1.8.7, p. 61], h is a map. Clearly h is onto. Now, let $(a, b), (a', b') \in A \times B$ such that $(a, b) \neq (a', b')$ but $h(a, b) = h(a', b')$, then $a = b'$ and $b = a'$. Thus, since $A \cap B$ has at most one point, we obtain that $a = a' = b' = b$; this contradicts the choice of (a, b) and (a', b') . \square

Lemma 3.8. Let X be a continuum and $A, B \in C_1(X)$ such that $A \cap B$ has at most one point and A and B have property (b); then $\langle A, B \rangle_2$ has property (b).

Proof. By [26, 7.5, p. 228], $A \times B$ has property (b). By Lemma 3.7, $\langle A, B \rangle_2$ is homeomorphic to $A \times B$. Thus, $\langle A, B \rangle_2$ has property (b). \square

Example 3.9. Let $k \in \mathbb{N}$. If $X = S_1 \cup \left(\bigcup_{i=1}^k l_i \right)$, then $SF_1^2(X)$ is unicoherent.

Proof. By induction over $k \in \mathbb{N}$. The case $k = 1$ follows immediately from Example 3.6. Suppose that the result is true by k ($k \geq 2$). Let $X = S_1 \cup \left(\bigcup_{i=1}^{k+1} l_i \right)$, so it is clear that $X = X_1 \cup X_2$, where

$$X_1 = \left(S_1 \cup \left(\bigcup_{i=1}^k l_i \right) \right) \text{ and } X_2 = (S_1 \cup l_{k+1});$$

moreover, $SF_1^2(X)$ is equal to

$$SF_1^2(X_1) \cup SF_1^2(X_2) \cup \left\{ \rho_{1,2}^X(\{x, y\}) : x \in \bigcup_{i=1}^k l_i, y \in l_{k+1} \right\}.$$

By hypothesis of induction, $SF_1^2(X_1)$ is unicoherent and therefore has property (b) by Lemma 2.5. Now, by Example 3.6, $SF_1^2(X_2)$ has property

(b) and, by Lemma 3.8, $\left\{ \rho_{1,2}^X(\{x, y\}) : x \in \bigcup_{i=1}^k l_i, y \in l_{k+1} \right\}$ has property

(b) because this set is homeomorphic to $\left\langle \bigcup_{i=1}^k l_i, l_{k+1} \right\rangle_2$.

Since $SF_1^2(X_1) \cap SF_1^2(X_2) = SF_1^2(S_1)$, the set

$$\mathcal{F} = SF_1^2(X_1) \cup SF_1^2(X_2)$$

has property (b) by Lemma 2.6. On the other hand,

$$\mathcal{F} \cap \left\{ \rho_{1,2}^X(\{x, y\}) : x \in \bigcup_{i=1}^k l_i, y \in l_{k+1} \right\} = \left\{ \rho_{1,2}^X(\{0, x\}) : x \in \bigcup_{i=1}^{k+1} l_i \right\}$$

is a connected set. Thus, by Lemma 2.6, $SF_1^2(X)$ has property (b). Using Lemma 2.5, we can conclude that $SF_1^2(X)$ is unicoherent. \square

Lemma 3.10. If X_1 , X_2 , and $X_1 \cap X_2$ are subcontinua of a continuum X , then $SF_1^2(X_1) \cap SF_1^2(X_2)$ is homeomorphic to $SF_1^2(X_1 \cap X_2)$.

Proof. It follows from $F_2(X_1 \cap X_2) = F_2(X_1) \cap F_2(X_2)$ and Remark 3.5. \square

Example 3.11. If $X = S_1 \cup S_2$, then $SF_1^2(X)$ is unicoherent.

Proof. We consider

$$X_1 = S_1 \cup \left\{ \frac{1}{2} + \frac{1}{2}e^{i\theta} \in S_2 : \theta \in \left[\frac{\pi}{2}, \frac{3\pi}{2} \right] \right\}$$

and

$$X_2 = S_2 \cup \left\{ 1 + e^{i\theta} \in S_1 : \theta \in \left[\frac{\pi}{2}, \frac{3\pi}{2} \right] \right\}.$$

Notice that

- (1) $X = X_1 \cup X_2$;
- (2) $\left\{ \frac{1}{2} + \frac{1}{2}e^{i\theta} \in S_2 : \theta \in \left[\frac{\pi}{2}, \frac{3\pi}{2} \right] \right\}$ is an arc in S_2 which joins the points $\frac{1}{2} + \frac{1}{2}e^{i\frac{\pi}{2}}$ with $\frac{1}{2} + \frac{1}{2}e^{i\frac{3\pi}{2}}$ and, moreover, contains to 0 in its interior;
- (3) $\left\{ 1 + e^{i\theta} \in S_1 : \theta \in \left[\frac{\pi}{2}, \frac{3\pi}{2} \right] \right\}$ is an arc in S_1 which joins the points $1 + e^{i\frac{\pi}{2}}$ with $1 + e^{i\frac{3\pi}{2}}$. This arc also contains to 0 in its interior;
- (4) the intersection

$$\left\{ \frac{1}{2} + \frac{1}{2}e^{i\theta} \in S_2 : \theta \in \left[\frac{\pi}{2}, \frac{3\pi}{2} \right] \right\} \cap \left\{ 1 + e^{i\theta} \in S_1 : \theta \in \left[\frac{\pi}{2}, \frac{3\pi}{2} \right] \right\}$$

only contains the point 0.

By observations (2), (3), and (4),

$$X_1 \cap X_2 = \left\{ \frac{1}{2} + \frac{1}{2}e^{i\theta} \in S_2 : \theta \in \left[\frac{\pi}{2}, \frac{3\pi}{2} \right] \right\} \cup \left\{ 1 + e^{i\theta} \in S_1 : \theta \in \left[\frac{\pi}{2}, \frac{3\pi}{2} \right] \right\}$$

is homeomorphic to a simple 4-od.

Using (1), it is clear that

$$SF_1^2(X) = SF_1^2(X_1) \cup SF_1^2(X_2) \cup \mathcal{L},$$

where \mathcal{L} is the following set

$$\left\{ \rho_{1,2}^X \left(\left\{ 1 + e^{i\theta}, \frac{1}{2} + \frac{1}{2}e^{i\phi} \right\} \right) \in SF_1^2(X) : \theta, \phi \in \left[0, \frac{\pi}{2} \right] \cup \left[\frac{3\pi}{2}, 2\pi \right] \right\}.$$

By Example 3.9, both $SF_1^2(X_1)$ and $SF_1^2(X_2)$ have property (b). Moreover, by Lemma 3.10, $SF_1^2(X_1) \cap SF_1^2(X_2)$ is homeomorphic to $SF_1^2(X_1 \cap X_2)$. So, using Lemma 2.6, we conclude that $SF_1^2(X_1) \cup SF_1^2(X_2)$ has property (b).

On the other hand, \mathcal{L} is a 2-cell and $\mathcal{L} \cap (SF_1^2(X_1) \cup SF_1^2(X_2))$ is a simple closed curve. By Lemma 2.6, we conclude that $SF_1^2(X)$ has property (b) and, by Lemma 2.5, we obtain that $SF_1^2(X)$ is unicoherent. \square

Definition 3.12. Let $k \in \mathbb{N}$; $R_k = \bigcup_{i=1}^k S_i$ is a continuum which will be called the *Rose of k folds*.

Example 3.13. Let k be a positive integer, then $SF_1^2(R_k)$ is unicoherent.

Proof. It is easy to prove this using induction over $k \in \mathbb{N}$ with the same idea of Example 3.11. \square

Example 3.14. Let $k \in \mathbb{N}$. For each $r \in \mathbb{N}$, let $Y_r = R_k \cup \bigcup_{j=1}^r l_j$, then $SF_1^2(Y_r)$ is unicoherent.

Proof. By induction over $r \in \mathbb{N}$. First, suppose that $r = 1$. Observe that

$$SF_1^2(Y_1) = SF_1^2(R_k) \cup \mathcal{L}_1 \cup SF_1^2(l_1),$$

where $\mathcal{L}_1 = \left\{ \rho_{1,2}^{Y_1}(\{x, y\}) : x \in R_k, y \in l_1 \right\}$. Making $X = SF_1^2(R_k)$ and $A = \left\{ \rho_{1,2}^{Y_1}(\{x, 0\}) : x \in R_k \right\}$, notice that A is a subcontinuum of X and \mathcal{L}_1 is homeomorphic to $\langle A, l_1 \rangle_2$ and therefore is homeomorphic to $A \times l_1$ by Lemma 3.7. Then, by Lemma 3.4, $SF_1^2(R_k)$ is a deformation retract of $SF_1^2(R_k) \cup \mathcal{L}_1$; by Example 3.13, $SF_1^2(R_k)$ has property (b); by Lemma 2.4, $SF_1^2(R_k) \cup \mathcal{L}_1$ has property (b). By Example 3.1, $SF_1^2(l_1)$ has property (b). Since

$$(SF_1^2(R_k) \cup \mathcal{L}_1) \cap SF_1^2(l_1)$$

is a single point, by Lemma 2.6, $SF_1^2(Y_1)$ has property (b); thus, by Lemma 2.6, $SF_1^2(Y_1)$ is unicoherent.

Suppose that the result is true for some r .

Let $Y_{r+1} = R_k \cup \left(\bigcup_{j=1}^{r+1} l_j \right)$, then

$$Y_{r+1} = Y_r \cup W_2, \text{ where } Y_r = \left(R_k \cup \left(\bigcup_{j=1}^r l_j \right) \right) \text{ and } W_2 = (R_k \cup l_{r+1})$$

and $SF_1^2(Y_{r+1})$ is equal to

$$SF_1^2(Y_r) \cup SF_1^2(W_2) \cup \mathcal{L}_{r+1},$$

where \mathcal{L}_{r+1} is the following set

$$\left\{ \rho_{1,2}^{Y_{r+1}}(\{x, y\}) : x \in \bigcup_{j=1}^r l_j, y \in l_{r+1} \right\}.$$

Notice that $SF_1^2(Y_r)$ is unicoherent by hypothesis of induction; therefore, by Lemma 2.5, we have that $SF_1^2(Y_r)$ has property (b). On the other hand, base of induction implies that $SF_1^2(W_2)$ has property (b). Further, by Lemma 3.8, \mathcal{L}_{r+1} has property (b) because it is homeomorphic to

$$\left\langle \bigcup_{j=1}^r l_j, l_{r+1} \right\rangle_2.$$

Since $SF_1^2(Y_r) \cap SF_1^2(W_2) = SF_1^2(R_k)$, we obtain by Lemma 2.6 that $SF_1^2(Y_r) \cup SF_1^2(W_2)$ has property (b).

Observe that $(SF_1^2(Y_r) \cup SF_1^2(W_2)) \cap \mathcal{L}_{r+1}$ is the connected set

$$\left\{ \rho_{1,2}^{Y_r}(\{0, x\}) : x \in \bigcup_{j=1}^{r+1} l_j \right\}.$$

Thus, by Lemma 2.6, $SF_1^2(Y_{r+1})$ has property (b) and then, by Lemma 2.5, $SF_1^2(Y_{r+1})$ is unicoherent. The proof of this example is complete. \square

Example 3.15. *If X is a subcontinuum of R_k for some positive integer k , then $SF_1^2(X)$ is unicoherent.*

Proof. This result follows immediately from examples 3.1, 3.3, 3.9, 3.13, and 3.14. \square

To finish this section, we present a continuum X such that $SF_1^2(X)$ is not unicoherent.

Example 3.16. *The continuum homeomorphic to the Greek letter theta (θ) has the property that $SF_1^2(X)$ is not unicoherent.*

Proof. Suppose that $X = \{e^{i\theta} \in \mathbb{C} : \theta \in [0, 2\pi]\} \cup \{x \in \mathbb{C} : x \in [-1, 1]\}$. Let

$$\begin{aligned} \mathcal{A} = & \left\{ \{x, y\} \in F_2(X) : x, y \in [-1, 0] \text{ and } |x - y| \geq \frac{1}{2} \right\} \cup \\ & \left\{ \{x, y\} \in F_2(X) : x, y \in [0, 1] \text{ and } |x - y| \geq \frac{1}{2} \right\} \cup \\ & \left\{ \{x, e^{i\theta}\} \in F_2(X) : x \in \left[-\frac{1}{4}, \frac{1}{4}\right] \text{ and } \theta \in [0, 2\pi] \right\}. \end{aligned}$$

And let $\mathcal{B} = Cl_{F_2(X)}(F_2(X) \setminus \mathcal{A})$. It is not difficult to see that \mathcal{A} and \mathcal{B} are arcwise connected, then \mathcal{A} and \mathcal{B} are subcontinua of $F_2(X)$. Notice that $\mathcal{A} \cap \mathcal{B}$ has the following components:

$$\mathcal{K}_1 = \left\{ \left\{ \frac{-1}{4}, e^{i\theta} \right\} \in F_2(X) : \theta \in [0, 2\pi] \right\} \cup \left\{ \{x, y\} \in F_2(X) : x, y \in [-1, 0], |x - y| = \frac{1}{2} \right\}$$

and

$$\mathcal{K}_2 = \left\{ \left\{ \frac{1}{4}, e^{i\theta} \right\} \in F_2(X) : \theta \in [0, 2\pi] \right\} \cup \left\{ \{x, y\} \in F_2(X) : x, y \in [0, 1], |x - y| = \frac{1}{2} \right\}.$$

Since $F_1(X) \subset \mathcal{B}$, by [2, Lemma 4.3, p. 600], we have that $SF_1^2(X)$ is not unicoherent. □

4. INVERSE LIMITS AND UNICOHERENCE OF SYMMETRIC PRODUCTS

If $\{X_i\}_{i \in \mathbb{N}}$ is a sequence of topological spaces and $\{f_i\}_{i \in \mathbb{N}}$ is a sequence of maps such that $f_i : X_{i+1} \rightarrow X_i$ for each $i \in \mathbb{N}$, by the *inverse limit* of the inverse sequence $\{X_i, f_i\}_{i=1}^\infty$, denoted $\varprojlim \{X_i, f_i\}_{i=1}^\infty$,

we mean the subset of $\prod_{i=1}^\infty X_i$ to which the point $x = (x_i)_{i=1}^\infty$ belongs if and only if $f_i(x_{i+1}) = x_i$ for each $i \in \mathbb{N}$. It is well known that when the spaces X_i are continua, the inverse limit is a continuum. Denote by $\pi_\sigma : \varprojlim \{X_i, f_i\}_{i=1}^\infty \rightarrow X_\sigma$ the function given by $\pi_\sigma((x_i)_{i=1}^\infty) = x_\sigma$, called the *projection to X_σ* .

A map $f : X \rightarrow Y$ between continua is called an ε -map if the diameter of the fibers is less than ε .

If X is a continuum and \mathcal{P} is a collection of continua, then X is said to be \mathcal{P} -like provided that $X = \varprojlim \{X_i, f_i\}_{i=1}^\infty$, where $X_i \in \mathcal{P}$ and f_i is a surjective map for all $i \in \mathbb{N}$. The following theorem is a way to rewrite Theorem 1* of [21, p. 148].

Theorem 4.1. *If \mathcal{P} is a collection of continua, X is \mathcal{P} -like if and only if X satisfies either of the following statements:*

- (1) X is degenerate;
- (2) X is a continuum and, for every positive number ε , there is an ε -map from X onto a continuum $Y \in \mathcal{P}$.

When $\mathcal{P} = \{I\}$, we say that X is arc-like, and when $\mathcal{P} = \{S^1\}$, we say that X is circle-like.

The following theorem is a particular case of [18, Corollary 6, p. 177].

Theorem 4.2. *If $X = \varprojlim \{X_i, f_i\}_{i=1}^\infty$, where X_i is a continuum and f_i is onto for each $i \in \mathbb{N}$, then $\varprojlim \{F_2(X_i), F_2(f_i)\}_{i=1}^\infty$ is homeomorphic to $F_2(X)$.*

Remark 4.3. Regarding the previous theorem, it is possible to show that if $X = \varprojlim \{X_i, f_i\}_{i=1}^\infty$, then there is a homeomorphism

$$h : \varprojlim \{F_2(X_i), F_2(f_i)\}_{i=1}^\infty \rightarrow F_2(X)$$

such that $A \in F_1(X)$ if and only if $A = h(\{(x_i)_{i=1}^\infty\})$ for some $x_i \in X_i$ and for every $i \in \mathbb{N}$.

Applying Remark 4.3, the following theorem is a special case of [12, Exercise 3.12.13, p. 233].

Theorem 4.4. *If $X = \varprojlim \{X_i, f_i\}_{i=1}^\infty$, where X_i is a continuum and f_i is onto for each $i \in \mathbb{N}$, then $\varprojlim \{SF_1^2(X_i), SF_1^2(f_i)\}$ is homeomorphic to $SF_1^2(X)$.*

Definition 4.5. A λ -dendroid is an hereditarily unicoherent and hereditarily decomposable continuum.

Now, we answer in negative form the following problem of Charatonik: Does there exist an hereditarily unicoherent, hereditarily decomposable continuum X such that $F_2(X)$ is not unicoherent? (See [4, Problem 3, p. 66] and [16, Problem 34, p. 284].)

Corollary 4.6. *For every λ -dendroid X , $F_2(X)$ is unicoherent.*

Proof. Let X be a λ -dendroid. H. Cook [7, Corollary, p. 20] proves that every λ -dendroid is tree-like; thus, there is an inverse sequence $\{X_i, f_i\}_{i=1}^\infty$ such that X_i is a tree, f_i is onto for all $i \in \mathbb{N}$, and $X = \varprojlim \{X_i, f_i\}_{i=1}^\infty$. By [25, Definition 9.25, p. 153], each X_i is unicoherent. By Theorem 2.3, $F_2(X_i)$ is unicoherent. Then Theorem 4.2 implies that $F_2(X) = \varprojlim \{F_2(X_i), F_2(f_i)\}_{i=1}^\infty$ and, by [22, Corollary 1, p. 412], $F_2(X)$ is unicoherent. \square

Corollary 4.7. *If X is an arc-like continuum, then $F_2(X)$ is unicoherent.*

Concerning [4, Problem 1, p. 66] and [16, Problem 32, p. 284], we have the following corollary.

Corollary 4.8. *If X is the pseudoarc or the Buckethandle continuum, then $F_2(X)$ is unicoherent.*

Corollary 4.9. *If X is an AR-like continuum, then $F_2(X)$ is unicoherent.*

Proof. Let X be an AR-like continuum. By [9, Theorem 3, p. 311], there is an inverse sequence $\{X_i, f_i\}_{i=1}^\infty$ such that X_i is a Hilbert cube, f_i is onto for all $i \in \mathbb{N}$, and $X = \varprojlim \{X_i, f_i\}_{i=1}^\infty$. Hence, by [13, Corollary 5, p. 223] or [17, Theorem 2.4], $F_2(X_i)$ is a Hilbert cube. Therefore, $F_2(X_i)$ is unicoherent. Then Theorem 4.2 implies that $F_2(X) = \varprojlim \{F_2(X_i), F_2(f_i)\}_{i=1}^\infty$ and, by [22, Corollary 1, p. 412], $F_2(X)$ is unicoherent. \square

Definition 4.10. An *hereditarily equivalent continuum* is a nondegenerate continuum which is homeomorphic to each of its nondegenerate subcontinua.

By Cook [8, Theorem, p. 204], every hereditarily equivalent continuum is a tree-like continuum. We have the following corollary.

Corollary 4.11. *For every hereditarily equivalent continuum X , $F_2(X)$ is unicoherent.*

5. UNICOHERENCE OF $SF_m^n(X)$

Theorem 5.1. *Let X be a continuum and let $n \geq 3$ be a positive integer. Then $SF_m^n(X)$ is a unicoherent continuum for each $m < n$.*

Proof. By Theorem 2.1, we have that $F_n(X)$ is unicoherent. We note that $\rho_{m,n}^X$ is a monotone map. Then, by [25, Corollary 13.35, p. 294], $SF_m^n(X)$ is unicoherent. \square

For the case $n = 2$, the following result is known (see [2, Theorem 4.2, p. 599]).

Theorem 5.2. *Let X be a continuum. Then $r(SF_1^2(X)) \leq 1$.*

The proof of the following theorem is similar to Theorem 5.1.

Theorem 5.3. *If X is a continuum and $F_2(X)$ is unicoherent, then $SF_1^2(X)$ is unicoherent.*

Corollary 5.4. *If X is a λ -dendroid (in particular, a tree, a dendrite, or a dendroid), $SF_1^2(X)$ is unicoherent.*

Proof. By Corollary 4.6, $F_2(X)$ is unicoherent and, by Theorem 5.3, we have that $SF_1^2(X)$ is unicoherent. \square

Corollary 5.5. *If X is an arc-like continuum (in particular, if X is the pseudoarc or the Buckethandle continuum), then $SF_1^2(X)$ is unicoherent.*

By Corollaries 4.9 and 4.11, we have the following two results.

Corollary 5.6. *If X is an AR-like continuum, then $SF_1^2(X)$ is unicoherent.*

Corollary 5.7. *If X is an hereditarily equivalent continuum, $SF_1^2(X)$ is unicoherent.*

Theorem 5.8. *If X is a circle-like continuum, then $SF_1^2(X)$ is unicoherent.*

Proof. If Y is a unit circle, then by Example 3.1, $SF_1^2(Y)$ is unicoherent. Thus, if X is circle-like, by Theorem 4.4, $SF_1^2(X)$ is an inverse limit of unicoherent continua. By Theorem 5.3, we conclude that $SF_1^2(X)$ is unicoherent. \square

Lemma 5.9. *If $n \in \mathbb{N}$ and $n \geq 3$, then $F_2(T_n)$ cannot be embedded in $[0, 1]^2$.*

Proof. Since $n \geq 3$, $F_2(T_3)$ can be embedded in $F_2(T_n)$. By [6, Lemma 3.5, p. 61]), $F_2(T_3)$ cannot be embedded in $T_3 \times [0, 1]$; in particular, $F_2(T_3)$ cannot be embedded in $[0, 1]^2$. Thus, $F_2(T_n)$ cannot be embedded in $[0, 1]^2$. \square

Lemma 5.10. *Let X be a graph. If $F_2(X)$ is homeomorphic to $SF_1^2(X)$, then X has at most one ramification point.*

Proof. Let $h : F_2(X) \rightarrow SF_1^2(X)$ be a homeomorphism. Let p be a ramification point of X and suppose that $n = \text{ord}(p, X) \geq 3$; then there is a system of neighborhood β of $\{p\}$ in $F_2(X)$ such that every neighborhood U of β is homeomorphic to $F_2(T_n)$, where T_n is a simple n -od (see [6, Lemma 3.3, p. 60]). Suppose that $h(\{p\}) = \rho_{1,2}^X(\{x, y\})$ and $x \neq y$. We have the following three cases.

Case 1: If $\text{ord}(x, X) \leq 2$ and $\text{ord}(y, X) \leq 2$, there is a system of neighborhoods γ of $\{x, y\}$ in $F_2(X)$ such that every $V \in \gamma$ is homeomorphic to $[0, 1]^2$ (see [6, Lemma 3.3, p. 60]). By Lemma 5.9, this is impossible.

Case 2: If $\text{ord}(x, X) \leq 2$ and $\text{ord}(y, X) = k \geq 3$, there is a system of neighborhoods γ of $\{x, y\}$ in $F_2(X)$ such that every $V \in \gamma$ is homeomorphic to $[0, 1] \times T_k$ (see [6, Lemma 3.3, p. 60]). This is impossible since $F_2(T_n)$ cannot be embedded in $[0, 1] \times T_k$ (see [6, Lemma 3.5, p. 61]).

Case 3: If $\text{ord}(x, X) = r \geq 3$ and $\text{ord}(y, X) = k \geq 3$, there is a system of neighborhoods γ of $\{x, y\}$ in $F_2(X)$ such that every $V \in \gamma$ is homeomorphic to $\text{cone}(K_{r,k})$ (see [6, Lemma 3.3, p. 60]). This is impossible since $F_2(T_n)$ is not homeomorphic to $\text{cone}(K_{r,k})$ (see [6, Lemma 3.6, p. 62]).

Thus, $h(\{p\}) = \rho_{1,2}^X(F_1(X))$. We conclude that X has at most one ramification point. \square

Theorem 5.11. *Let X be a graph. $F_2(X)$ is homeomorphic to $SF_1^2(X)$ if and only if X is an arc or a simple n -od.*

Proof. If X is an arc or a simple n -od, then $F_2(X)$ is homeomorphic to $SF_1^2(X)$ by examples 3.1 and 3.3. Now, if $F_2(X)$ is homeomorphic to $SF_1^2(X)$, by Lemma 5.10, X has at most one ramification point. Since X is a graph, there is a Rose of k folds, R_k , for some k , such that X is a subcontinuum of R_k . By Example 3.15, $SF_1^2(X)$ is unicoherent and then by Theorem 2.3, X is unicoherent. Thus, X is an arc or a simple n -od. \square

Lemma 5.12. *Let X be a continuum and let Y be a subcontinuum of X such that Y is a deformation retract of X ; then $SF_1^2(Y)$ is a deformation retract of $SF_1^2(X)$.*

Proof. Let $h : X \times I \rightarrow X$ such that $h(x, 1) = x$ for all $x \in X$, $h(X \times \{0\}) \subset Y$, and therefore the function $r : X \rightarrow Y$, defined by $r(x) = h(x, 0)$, is a retraction. We consider the following function $\tilde{h} : F_2(X) \times I \rightarrow F_2(X)$ given by $\tilde{h}(A, t) = h(A \times \{t\})$. It is clear that \tilde{h} is a map. Now, we define $H : SF_1^2(X) \times I \rightarrow SF_1^2(X)$ by $H(\rho_{1,2}^X(A), t) = \rho_{1,2}^X(\tilde{h}(A, t))$. Since \tilde{h} and $\rho_{1,2}^X$ are maps, then H is a map. Moreover, $H(\rho_{1,2}^X(A), 1) = \rho_{1,2}^X(A)$, $H(SF_1^2(X) \times \{0\}) \subset SF_1^2(Y)$, and $R : SF_1^2(X) \rightarrow SF_1^2(Y)$, given by $R(\rho_{1,2}^X(A)) = H(\rho_{1,2}^X(A), 0)$, is a retraction because $H(\rho_{1,2}^X(A), 0) = \rho_{1,2}^X(r(A))$. \square

Lemma 5.13. *We consider the sets*

$$M = \left\{ \frac{1}{2} + \frac{1}{2}e^{i\theta} \in \mathbb{C} : \theta \in [0, 2\pi] \right\},$$

$$N = \{x \in \mathbb{C} : x \in [-1, 0]\} \text{ and}$$

$$O = \{-1 + iy \in \mathbb{C} : y \in [-1, 1]\}.$$

If $X = M \cup N \cup O$, then $SF_1^2(X)$ is unicoherent.

Proof. Let $Y = M \cup N$. Notice that Y is a deformation retract of X . By Lemma 5.12, $SF_1^2(Y)$ is a deformation retract of $SF_1^2(X)$. Example 3.6 shows that $SF_1^2(Y)$ is unicoherent and, by Lemma 2.4 and Lemma 2.5, we conclude that $SF_1^2(X)$ is unicoherent. \square

Lemma 5.14. *We consider the sets*

$$R = \left\{ 1 + \frac{1}{2}e^{i\theta} \in \mathbb{C} : \theta \in [0, 2\pi] \right\},$$

$$N = \left\{ x \in \mathbb{C} : x \in \left[-\frac{1}{2}, \frac{1}{2}\right] \right\} \text{ and}$$

$$O = \left\{ -1 + \frac{1}{2}e^{i\theta} \in \mathbb{C} : \theta \in [0, 2\pi] \right\}.$$

Define $X = R \cup N \cup O$, then $SF_1^2(X)$ is unicoherent.

Proof. It is easy to prove the result of this lemma using the sets

$$X_1 = R \cup N \cup \left\{ -1 + \frac{1}{2}e^{i\theta} \in \mathbb{C} : \theta \in \left[0, \frac{\pi}{2}\right] \cup \left[\frac{3\pi}{2}, 2\pi\right] \right\}$$

and

$$X_2 = N \cup O \cup \left\{ 1 + \frac{1}{2}e^{i\theta} \in \mathbb{C} : \theta \in \left[\frac{\pi}{2}, \frac{3\pi}{2}\right] \right\}$$

with the same idea of Example 3.11. \square

Let $X = S \cup Y \cup S^1$ where

$$S = \{3e^{it} \in \mathbb{C} : t \in \mathbb{R}\},$$

$$Y = \left\{ \left(\frac{t}{1+|t|} + 2 \right) e^{it} \in \mathbb{C} : t \in \mathbb{R} \right\} \text{ and}$$

$$S^1 = \{e^{it} \in \mathbb{C} : t \in \mathbb{R}\}.$$

Castañeda in [4, Example 2.1, p. 63] shows that this space is unicoherent, but $F_2(X)$ is not unicoherent. In the following theorem, we prove that $SF_1^2(X)$ is unicoherent. This fact contradicts the result given by Barragán in [2, Example 4.4, p. 600].

Theorem 5.15. *Let $X = S \cup Y \cup S^1$, where S_2 , Y , and S^1 are as above, then $SF_1^2(X)$ is unicoherent.*

Proof. Using the notation of Lemma 5.14, let $P = R \cup N \cup O$. Observe that for each $\varepsilon > 0$, there is $f_\varepsilon : X \rightarrow P$, a surjective ε -map. So, by Theorem 4.1, X is a $\{P\}$ -like continuum. Therefore, by Theorem 4.4, Lemma 5.14, and [22, Corollary 1, p. 412], we conclude that $SF_1^2(X)$ is unicoherent. \square

We have the following question.

Question 5.16. Does there exist a unicoherent continuum X such that $SF_1^2(X)$ is not unicoherent?

Concerning this problem see Example 3.16.

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