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PATH COMPONENTS IN THE UNIFORM SPACES OF CONTINUOUS FUNCTIONS INTO A NORMED LINEAR SPACE

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Abstract. In this paper, we study the path components in relation to two uniform topologies on $C(X, Y)$, where $C(X, Y)$ denotes the space of all continuous functions from a Tychonoff space X to a normed linear space Y. As a consequence of this study, we show that these two uniform topologies on $C(X, Y)$ are not homeomorphic. In particular, we show that in one case $C(X, Y)$ is pathwise connected, and in the other, $C(X, Y)$ has uncountably many path components. But we show that these two uniform topologies on the space $C^*(X, Y)$, the set of all bounded members of $C(X, Y)$, coincide.

1. Introduction

For two topological spaces X and Y, let $C(X, Y)$ denote the set of continuous functions from X into Y. When $Y = \mathbb{R}$, the space of real numbers, we write $C(X)$ instead of $C(X, \mathbb{R})$. The set $C(X, Y)$ has a number of natural topologies, such as point-open, compact-open and uniform topologies (see, for example, [4] and [7]). The space $C(X, Y)$ with the point-open topology and compact-open topology is denoted by $C_p(X, Y)$ and $C_k(X, Y)$, respectively. In order to define a uniform topology, it is necessary for Y to have some uniform structure. When Y is a metric space with compatible bounded metric d, then $C(X, Y)$ with the uniform topology generated by d is denoted by $C_d(X, Y)$ and is a metric space. Also $C_d(X, Y)$ is a complete metric space if and only if Y is a complete metric space under d.

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