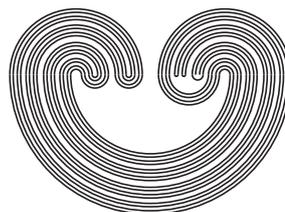

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by

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RETRACTS OF TOPOLOGICAL GROUPS AND COMPACT MONOIDS

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ABSTRACT. In this note a space which is homeomorphic to a retract of a topological group is called a GR-space and properties which a GR-space must possess are investigated. GR-spaces have earlier been called retral spaces by J. van Mill and G. J. Ridderbos (2006). Every compact space which admits a topological left-loop structure is a GR-space and every GR-space admits an H -space structure. For every positive dimension there are compact connected commutative monoids with zero which fail to be GR-spaces. A characterization is given for the compact GR-spaces which are homeomorphs of n -spheres, real projective n -spaces, compact surfaces, compact bordered surfaces and absolute retracts for the class of compact Hausdorff spaces. In the process, we observe and prove that the Möbius band is both a submonoid and a homotopy retract of the solid torus as compact topological commutative semigroup with identity.

1. INTRODUCTION

In this note all spaces are Hausdorff and the term map or mapping shall always mean continuous function.

The concepts of a topological left-loop and an H -space are two generalizations of the notion of a topological group and relationships between these two concepts are discussed in [10]. In this note another generalization of a topological group is introduced which we call a GR-space and

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Key words and phrases. Topological group, topological left-loop, H -space, retract, absolute (neighborhood) retract, compact (bordered) surface, compact topological semigroup.

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