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ON DOUBLE SPIRALS IN FIBONACCI-LIKE UNIMODAL INVERSE LIMIT SPACES

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Abstract. In this paper, we represent composants of unimodal inverse limit spaces as walks on a tree, related to the Hofbauer tower of the unimodal map. The goal is, for Fibonacci-like unimodal maps, to study the possibility of asymptotic composants. Whereas the question of their existence is still open, we show that Fibonacci-like inverse limit spaces possess so-called double spirals, which shows that points with different symbolic tails can still be on the same arc in such an inverse limit space. This shows that the converse of a result by Brucks & Diamond doesn't hold.

1. Introduction

The topological structure of inverse limit spaces is a central theme in continuum theory. The full classification of inverse limit spaces of a fixed unimodal bonding map revolves around the so-called Ingram conjecture, which was recently solved in [2]. Thus we know that the inverse limit spaces of tent maps $T : [0,1] \to [0,1]$, $x \mapsto \min\{sx, s(1-x)\}$ are non-homeomorphic for every two slopes $1 \leqslant s < s' \leqslant 2$. This result came after a long string of papers with partial answers and addressing various aspects of the fine-structure of such unimodal inverse limit spaces.

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Key words and phrases. Inverse limit space, continuum theory, unimodal map, symbolic dynamics.

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