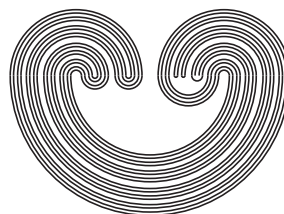

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ULTRAFILTERS AND PROPERTIES RELATED TO COMPACTNESS

by

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ABSTRACT. In this article we introduce and analyze the following concepts: Let $p \in \mathbb{N}^*$ and let X be a topological space. We say that

(a) X is *strongly p -compact* if X is p -pseudocompact and for each sequence $(x_n)_{n \in \mathbb{N}}$ of points in X , there exists a sequence of open subsets $(U_n)_{n \in \mathbb{N}}$ of X , with $x_n \in U_n$ for each $n \in \mathbb{N}$, such that the set of p -limit points of the sequence $(U_n)_{n \in \mathbb{N}}$ is a non-empty compact subspace of X ;

(b) X is *strongly p -pseudocompact* if for each sequence $(U_n)_{n \in \mathbb{N}}$ of open subsets of X , there exist a sequence $(x_n)_{n \in \mathbb{N}}$ of points in X and $x \in X$ such that $x_n \in U_n$ and $x = p\text{-}\lim x_n$;

(c) X is *pseudo- ω -bounded* if for each countable family \mathcal{U} of open subsets of X , there is a compact $K \subseteq X$ such that, for all $U \in \mathcal{U}$, $K \cap U \neq \emptyset$;

(d) X is *p -pseudo- ω -bounded* if for each family $\{U_n : n \in \mathbb{N}\}$ of open subsets of X , there is a compact subspace $K \subseteq X$ such that $\{n \in \mathbb{N} : K \cap U_n \neq \emptyset\} \in p$.

We prove:

- (1) Every strongly p -compact space is p -compact.
- (2) In the class of locally compact spaces, strong p -compactness and p -compactness are equivalent; and p -pseudo- ω -boundedness and p -pseudocompactness are equivalent too.
- (3) For two ultrafilters $p, q \in \mathbb{N}^*$, $p \leq_{RK} q$ if and only if every strongly q -pseudocompact space $\mathbb{N} \subseteq X \subseteq \beta\mathbb{N}$ is strongly p -pseudocompact.

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