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CLASSIFICATION OF TOPOLOGICAL SYMMETRY GROUPS OF K_n

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Abstract. In this paper we complete the classification of topological symmetry groups for complete graphs K_n by characterizing which K_n can have a cyclic group, a dihedral group, or a subgroup of $D_m \times D_m$ for odd m, as its topological symmetry group. Using this classification, one can algorithmically determine all possible topological symmetry groups of K_n for any n.

1. Introduction

The symmetries of a molecule affect many of its properties, including its reactions, its crystallography, its spectroscopy, and its quantum chemistry. Molecular symmetries are also an important tool in classifying molecules. Chemists have long used the group of rotations, reflections, and reflections composed with rotations as a means of representing the symmetries of a molecule. This group is known as the *point group* because its elements fix a point of $\mathbb{R}^3$. But using the point group to represent molecular symmetries only makes sense for rigid molecules. In fact, while most small molecules are rigid, larger molecules can be flexible, and some molecules are rigid except for pieces that can rotate around specific bonds. On the left of Figure 1 we illustrate a molecular Möbius ladder which is flexible because of its length, and on the right we illustrate a molecule that has pieces on either side that rotate around certain bonds.

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