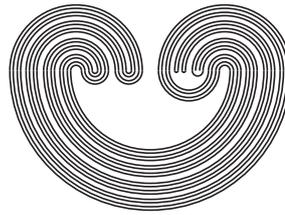


---

# TOPOLOGY PROCEEDINGS



Volume 43, 2014

Pages 331–339

---

<http://topology.auburn.edu/tp/>

## FREE PARATOPOLOGICAL GROUPS. II

by

ALI SAYED ELFARD

Electronically published on November 1, 2013

This file contains only the first page of the paper. The full version of the paper is available to Topology Proceedings subscribers. See <http://topology.auburn.edu/tp/subscriptioninfo.html> for information.

---

### Topology Proceedings

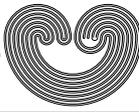
**Web:** <http://topology.auburn.edu/tp/>

**Mail:** Topology Proceedings  
Department of Mathematics & Statistics  
Auburn University, Alabama 36849, USA

**E-mail:** [topolog@auburn.edu](mailto:topolog@auburn.edu)

**ISSN:** 0146-4124

COPYRIGHT © by Topology Proceedings. All rights reserved.



## FREE PARATOPOLOGICAL GROUPS. II

ALI SAYED ELFARD

**ABSTRACT.** Let  $FP_G(X)$  and  $FP(X)$  be the free paratopological groups on a topological space  $X$  in the senses of Graev and Markov, respectively. In this paper, we prove that the groups  $FP_G(X)$  and  $FP(X)$  are discrete if  $X$  is discrete and the group  $FP_G(X)$  is indiscrete if  $X$  is indiscrete while the group  $FP(X)$  is the union of infinite indiscrete subspaces if  $X$  is indiscrete. Then we give a class of spaces  $X$  for which the groups  $FP_G(X)$  and  $FP(X)$  are locally invariant and another class of spaces  $X$  where they are not. Finally, we provide another proof for the existence of the free paratopological group  $FP(X)$  by embedding the space  $X$  in an infinite direct product of paratopological groups.

### 1. INTRODUCTION

The existence of the Graev free paratopological group  $FP_G(X)$  on a pointed topological space  $X$  were proved by Romaguera, Sanchis and Tkačenko [11] in 2003. An analogous proof of [11] was used by Elfard [3] to prove the existence of the Markov free paratopological group  $FP(X)$  on a topological space  $X$ .

In this paper, we prove that both groups  $FP_G(X)$  and  $FP(X)$  are discrete if  $X$  is discrete and the group  $FP_G(X)$  is indiscrete if  $X$  is indiscrete while the group  $FP(X)$  is the union of infinite indiscrete subspaces if  $X$  is indiscrete. Then we introduce the Graev paratopological group topology on the underlying set of the group  $FP(X)$  on a topological space  $X$  (see [1]) and then we give a class of spaces  $X$  for which the groups  $FP_G(X)$  and  $FP(X)$  are locally invariant and another class of spaces  $X$  where they are not.

---

2010 *Mathematics Subject Classification.* Primary 22A30; Secondary 54D10, 54E99, 54H99.

*Key words and phrases.* Paratopological group, Graev free paratopological group, Markov free paratopological group, discrete space, indiscrete space, quasipseudometric, Graev paratopological group topology, locally invariant property.

©2013 Topology Proceedings.

This file contains only the first page of the paper. The full version of the paper is available to Topology Proceedings subscribers. See <http://topology.auburn.edu/tp/subscriptioninfo.html> for information.