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A. A. Gryzlov

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Web:	http://topology.auburn.edu/tp/
Mail:	Topology Proceedings Department of Mathematics & Statistics Auburn University, Alabama 36849, USA
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ON THE CARDINALITY OF SUBSPACES OF TYCHONOFF PRODUCTS

A. A. GRYZLOV

Abstract. We prove theorems on the cardinality of subsets of Tychonoff products of topological spaces.

1. Introduction

To establish bounds of the cardinality of a topological space is one of the main problems of the theory of cardinal invariants.

A. V. Arhangel'skiĭ's [1] theorem on the cardinality of a Hausdorff compact space with the first axiom of countability, A. Hajnal and I. Juhász's [8] theorems on cardinality and Souslin number of a topological space, and V. V. Fedorčuk's [4] hereditarily separable compact space with cardinality 2^c, lay the foundation of an intensive development of this theory.

These results generated, in particular, two problems. The first one is a search of methods of proofs of cardinal inequalities. The other one is a search of weaker properties of spaces in theorems on cardinality of spaces.

V. I. Ponomarev [10] suggested another method of the proof of Arhangel'skiĭ's theorem. The author [6], and also B. Šapirovskiĭ [11] and R. Pol [9], got the modification of Arhangel'skiĭ's method. In [7], the author proved the theorem on the cardinality of T_1-compact space with countable pseudocharacter.

Here we suggest one more way of the proof of cardinal inequalities.

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2. Preliminaries

Recall some facts and notation used here (see [2], [5], [3]).

We regard Tychonoff products of topological spaces $\prod_{\alpha\in A} X_\alpha$. The product of closed intervals is called a *Tychonoff cube*.

For a subset $A' \subseteq A$, a space $\prod_{\alpha\in A'} X_\alpha$ is called an *A'-face of* $\prod_{\alpha\in A} X_\alpha$.

A mapping $\pi_{A'}\colon \prod_{\alpha\in A} X_\alpha \to \prod_{\alpha\in A'} X_\alpha$, defined by the rule

if $x = \{x_\alpha\colon \alpha \in A\} \in \prod_{\alpha\in A} X_\alpha$, then $\pi_{A'}(x) = \{x_\alpha\colon \alpha \in A'\}$ is called an *A'-projection of* $\prod_{\alpha\in A} X_\alpha$.

Definition 2.1. Let $X \subseteq \prod_{\alpha\in A} X_\alpha$ and $A' \subseteq A$. The restriction $p_{A'} = \pi_{A'}|_X$ is called an *A'-projection of X*,

$$p_{A'}\colon X \to p_{A'}(X) \subseteq \prod_{\alpha\in A'} X_\alpha.$$

Notation used in the paper is standard. By $|X|$, $\psi(X)$, $\chi(X)$, $w(X)$, and $t(X)$, we denote cardinality, pseudocharacter, character, weight, and tightness of a space X, respectively. By $[B]$, we denote the closure of a set B. We assume spaces in the paper are T_1-spaces.

We prove the following theorem.

Theorem 3.2. *Let a space $X \subseteq \prod_{\alpha\in A} X_\alpha$, where the weight $w(X_\alpha) \leqslant 2^\tau$ for all $\alpha \in A$, satisfy the following conditions:*

$$\psi(X) \leqslant 2^\tau;\ t(X) \leqslant \tau;\ |[B]| \leqslant 2^\tau \text{ for all } B \subseteq X \text{ such that } |B| \leqslant 2^\tau.$$

Then there is a closed subset $\tilde X \subseteq X$, $|\tilde X| \leqslant 2^\tau$, and a subset $\tilde A \subseteq A$, $|\tilde A| \leqslant 2^\tau$, such that $p_{\tilde A}(\tilde X)$ is a dense subset of $p_{\tilde A}(X)$ and $|p_{\tilde A}^{-1}(p_{\tilde A}(x))| = 1$ for all $x \in \tilde X$.

Recall that a mapping $f\colon X \to Y$ is called *quotient* if the following is true: A set $B \subseteq Y$ is closed (open) in Y if and only if $f^{-1}(B)$ is closed (open) in X.

Definition 2.2. A subspace $X \subseteq \prod_{\alpha\in A} X_\alpha$ is called *q-embedded* in $\prod_{\alpha\in A} X_\alpha$ if the following is true:

(q) the A'-projection $p_{A'}\colon X \to p_{A'}(X) \subseteq \prod_{\alpha\in A'} X_\alpha$ is a quotient mapping for all $A' \subseteq A$.

It follows from the properties of quotient mapping that a compact subspace of a product $\prod_{\alpha\in A} X_\alpha$ of Hausdorff spaces is a q-embedded subspace of $\prod_{\alpha\in A} X_\alpha$. Also, an open subspace and a Σ-product of $\prod_{\alpha\in A} X_\alpha$ are *q*-embedded subspaces of $\prod_{\alpha\in A} X_\alpha$. Finally, in a product $\prod_{\alpha\in A} X_\alpha$ for every X_{α_0}, the set $X_{\alpha_0} \times \prod_{\alpha\in A\setminus\{\alpha_0\}} \{z_\alpha\}$ where a point $z_\alpha \in X_\alpha$ is a q-embedded subspace of $\prod_{\alpha\in A} X_\alpha$.

For *q*-embedded subspaces of $\prod_{\alpha\in A} X_\alpha$, we get the following theorem.

Theorem 3.4. *Let a space $X \subseteq \prod_{\alpha\in A} X_\alpha$, where the weight $w(X_\alpha) \leqslant 2^\tau$ for all $\alpha \in A$, satisfy the following conditions:*

(1) *X is q-embedded in $\prod_{\alpha\in A} X_\alpha$;*
(2) *$\psi(X) \leqslant 2^\tau$; $t(X) \leqslant \tau$; $|[B]| \leqslant 2^\tau$ for all $B \subseteq X$ such that $|B| \leqslant 2^\tau$.*

Then there is $A' \subseteq A$, $|A'| \leqslant 2^\tau$, such that the A'-projection $p_{A'}\colon X \to p_{A'}(X) \subseteq \prod_{\alpha\in A'} X_\alpha$ is a homeomorphism, and $|X| \leqslant 2^\tau$.

The next theorem follows from Theorem 3.4.

Theorem 2.3 (Arhangel'skiĭ [1]). *If X is a Hausdorff compact space, then $|X| \leqslant 2^{\chi(X)}$.*

3. Main Results

Prove the following simple, but useful for us, lemma.

Lemma 3.1. *Let $X \subseteq \prod_{\alpha\in A} X_\alpha$ and $x \in X$. Then there is $A' \subseteq A$, $|A'| \leqslant \psi(x, X)$, such that $|p_{A'}^{-1}(p_{A'}(x))| = 1$.*

Proof. Let a point $x \in X$. There is a subset $A' \subseteq A$, $|A'| \leqslant \psi(x, X)$, and a pseudobase $Bx = \{V\}$ of x in the space X such that $|Bx| \leqslant \psi(x, X)$ and Bx consists of sets V of the following type

$$V = X \cap \pi_{\alpha_1}^{-1}(U_{\alpha_1}) \cap \ldots \cap \pi_{\alpha_n}^{-1}(U_{\alpha_n}),$$

where $\alpha_i \in A'$ and $U_{\alpha_i} \subseteq X_{\alpha_i}$ are open, $i = 1, \ldots, n$ where $n \in \omega$.

The set A' is as required. Really, if $y \in p_{A'}^{-1}(p_{A'}(x))$, then $p_\alpha(x) = p_\alpha(y)$ for all $\alpha \in A'$. Hence, $y \in V$ for all $V \in Bx$, and we get $y = \bigcap\{V : V \in Bx\} = x$. □

A subset $A' \subseteq A$ we call a *characteristic subset* for a point $x \in X$, $X \subseteq \prod_{\alpha\in A} X_\alpha$, if $|p_{A'}^{-1}(p_{A'}(x))| = 1$ for the A'-projection of X.

Theorem 3.2. *Let a space* $X \subseteq \prod\limits_{\alpha \in A} X_\alpha$*, where the weight* $w(X_\alpha) \leqslant 2^\tau$ *for all* $\alpha \in A$*, satisfy the following conditions:*

$$\psi(X) \leqslant 2^\tau;\ t(X) \leqslant \tau;\ |[B]| \leqslant 2^\tau \text{ for all } B \subseteq X \text{ such that } |B| \leqslant 2^\tau.$$

Then there is a closed subset $\tilde{X} \subseteq X$*,* $|\tilde{X}| \leqslant 2^\tau$*, and a subset* $\tilde{A} \subseteq A$*,* $|\tilde{A}| \leqslant 2^\tau$*, such that* $p_{\tilde{A}}(\tilde{X})$ *is a dense subset of* $p_{\tilde{A}}(X)$ *and* $|p_{\tilde{A}}^{-1}(p_{\tilde{A}}(x))| = 1$ *for all* $x \in \tilde{X}$*.*

Proof. The proof of this theorem has something in common with the version of Arhangelskiĭ's proof, mentioned above, and with V. I. Ponomarev's proof.

We construct, by induction, families $\{A_\delta \colon \delta < \omega_{\tau^+}\}$, $\{S_\delta \colon \delta < \omega_{\tau^+}\}$, and $\{Y_\delta \colon \delta < \omega_{\tau^+}\}$ such that, for all $\delta < \omega_{\tau^+}$, the following are true:

(a) $|A_\delta| \leqslant 2^\tau$, $|S_\delta| \leqslant 2^\tau$, $|Y_\delta| \leqslant 2^\tau$;
(b) $A_\delta \subseteq A_\beta$, $Y_\delta \subseteq Y_\beta$ if $\delta \leqslant \beta$;
(c) $A_\delta \subseteq A$, $S_\delta \subseteq \prod\limits_{\gamma \in A_\delta} X_\gamma$, $Y_\delta \subseteq X$;
(d) Y_δ closed in X;
(e) S_δ is a dense subset of $p_{A_\delta}(X)$;
(f) $p_{A_\delta}(\bigcup\{Y_\gamma \colon \gamma < \delta\}) \subseteq S_\delta$;
(g) $Y_\delta \cap p_{A_\delta}^{-1}(x) \neq \varnothing$ for all points $x \in S_\delta$;
(h) $|p_{A_\beta}^{-1}(p_{A_\beta}(x))| = 1$ for all $x \in Y_\delta$ and $\beta < \omega_{\tau^+}$ such that $\delta < \beta$.

Let $A_0 = \{\alpha_0\}$ for some $\alpha_0 \in A$, and let $S_0 \subseteq p_{A_0}(X)$ be some dense subset of $p_{A_0}(X)$ such that $|S_0| \leqslant 2^\tau$. For every point $x \in S_0$, fix a point $y(x) \in p_{A_0}^{-1}(x) \subseteq X$. Let $Y_0' = \{y(x) \colon x \in S_0\}$ and $Y_0 = [Y_0']$.

Assume we have constructed families $\{A_\delta \colon \delta < \beta\}$, $\{S_\delta \colon \delta < \beta\}$, and $\{Y_\delta \colon \delta < \beta\}$, satisfying conditions (a)–(h).

Now we define A_β, S_β, and Y_β. Let $A_\beta \supseteq \bigcup\{A_\delta \colon \delta < \beta\}$, $|A_\beta| \leqslant 2^\tau$, be a characteristic set for the set $\bigcup\{Y_\delta \colon \delta < \beta\}$; i.e., $|p_{A_\beta}^{-1}(p_{A_\beta}(x))| = 1$ for every point $x \in \bigcup\{Y_\delta \colon \delta < \beta\}$.

Such a set exists since, by Lemma 3.1, for every point $x \in \bigcup\{Y_\delta \colon \delta < \beta\}$, there is a characteristic set of indexes of a cardinality not more than 2^τ, $|\bigcup\{Y_\delta \colon \delta < \beta\}| \leqslant 2^\tau$ and also $|\bigcup\{A_\delta \colon \delta < \beta\}| \leqslant 2^\tau$.

Let $S_\beta \subseteq p_{A_\beta}(X)$ be some dense subset of $p_{A_\beta}(X)$ such that $|S_\beta| \leqslant 2^\tau$ and $S_\beta \supseteq p_{A_\beta}(\bigcup\{Y_\delta \colon \delta < \beta\})$. Such a set S_β exists since $w(\prod\limits_{\alpha \in A_\beta} X_\alpha) \leqslant 2^\tau$ and $|\bigcup\{Y_\delta \colon \delta < \beta\}| \leqslant 2^\tau$.

For every point $x \in S_\beta$, fix a point $y(x) \in p_{A_\beta}^{-1}(x)$. Let $Y_\beta' = \{y(x) : x \in \bigcup\{Y_\delta \colon \delta < \beta\}\}$. Note that for every point $x \in \bigcup\{Y_\delta \colon \delta < \beta\}$, we have $p_{A_\beta}^{-1}(p_{A_\beta}(x)) = x$. Let $Y_\beta = [Y_\beta']$.

It follows from the construction that families $\{A_\delta\colon \delta \leqslant \beta\}$, $\{S_\delta\colon \delta \leqslant \beta\}$, and $\{Y_\delta\colon \delta \leqslant \beta\}$ satisfy conditions (a)–(h).

After ω_{τ^+} steps of our construction, we get the families $\{A_\delta\colon \delta < \omega_{\tau^+}\}$, $\{S_\delta\colon \delta < \omega_{\tau^+}\}$, and $\{Y_\delta\colon \delta < \omega_{\tau^+}\}$.

Let $\tilde{A} = \bigcup\{A_\delta\colon \delta < \omega_{\tau^+}\}$, $\tilde{S} = \bigcup\{S_\delta\colon \delta < \omega_{\tau^+}\}$, and $\tilde{X} = \bigcup\{Y_\delta\colon \delta < \omega_{\tau^+}\}$.

From conditions (a)–(h) and the conditions of the theorem, we have the following.

(1) $\tilde{X}$ is closed in X and $|\tilde{X}| \leqslant 2^\tau$. This follows from the fact that $|Y_\delta| \leqslant 2^\tau$, Y_δ is closed in X for all $\delta \in 2^\tau$ and $t(X) \leqslant \tau$.

(2) $|p_{\tilde{A}}^{-1}(p_{\tilde{A}}(x))| = 1$ for all $x \in \tilde{X}$. Indeed, let $x \in \tilde{X}$. Then there is $\delta_0 < \omega_{\tau^+}$ such that $x \in Y_{\delta_0}$. From (h) it follows that

$$|p_{A_{\delta_0+1}}^{-1}(p_{A_{\delta_0+1}}(x))| = 1;$$

moreover, $|p_{\tilde{A}}^{-1}(p_{\tilde{A}}(x))| = 1$.

(3) $p_{\tilde{A}}(\tilde{X})$ is dense in $p_{\tilde{A}}(X)$. Let

$$V = p_{\tilde{A}}(X) \cap (U_{\alpha_1} \times \ldots \times U_{\alpha_n} \times \prod_{\substack{a \in \tilde{A} \\ \alpha \neq \alpha_1 \ldots \alpha_n}} X_\alpha)$$

be a set from a base of the space $p_{\tilde{A}}(X)$. There is $\beta < \omega_{\tau^+}$ such that $\alpha_i \in A_\beta$ for all $i = 1, \ldots, n$. Since S_β is dense in $p_{A_\beta}(X)$ (see (e)), there is $z \in S_\beta \cap (U_{\alpha_1} \times \ldots \times U_{\alpha_n} \times \prod_{\substack{a \in A_\beta \\ \alpha \neq \alpha_1 \ldots \alpha_n}} X_\alpha)$. We have $p_{A_\beta}^{-1}(z) \cap Y_\beta \neq \varnothing$ (see (g)). Let $x \in p_{A_\beta}^{-1}(z) \cap Y_\beta$, then $x \in \tilde{X}$. Since $p_{A_\beta}(x) = z$, we have $x_{\alpha_i} \in U_{\alpha_i}$ for all α_i-coordinates x_{α_i} $(i = 1 \ldots n)$ of x. Therefore, $p_{\tilde{A}}(x) \in U_{\alpha_1} \times \ldots \times U_{\alpha_n} \times \prod_{\substack{a \in \tilde{A} \\ \alpha \neq \alpha_1 \ldots \alpha_n}} X_\alpha$, and $p_{\tilde{A}}(x) \in V$. So $p_{\tilde{A}}(\tilde{X}) \cap V \neq \varnothing$.

We have proved that $\tilde{A}$, $\tilde{S}$, and $\tilde{X}$ are as required in the theorem. □

Since a Tychonoff space is homeomorphic to a subset of a Tychonoff cube, the next theorem follows from Theorem 3.2.

Theorem 3.3. *For a Tychonoff space X, there is a Tychonoff space Y where $w(Y) \leqslant 2^{\chi(X)}$, a dense subset $B \subseteq Y$ where $|B| \leqslant 2^{\chi(X)}$, and a continuous mapping $f\colon X \to Y$ such that $f^{-1}(B)$ is a closed subset of X and $|f^{-1}(y)| = 1$ for all $y \in B$.*

For q-embedded subsets of $\prod_{\alpha \in A} X_\alpha$, we get the following theorem.

Theorem 3.4. *Let a space $X \subseteq \prod_{\alpha \in A} X_\alpha$, where the weight $w(X_\alpha) \leqslant 2^\tau$ for all $\alpha \in A$, satisfy the following conditions:*

(1) *X is q-embedded in $\prod_{\alpha \in A} X_\alpha$;*

(2) *$\psi(X) \leqslant 2^\tau$; $t(X) \leqslant \tau$; $|[B]| \leqslant 2^\tau$ for all $B \subseteq X$ such that $|B| \leqslant 2^\tau$.*

Then there is $A' \subseteq A$, $|A'| \leqslant 2^\tau$, such that the A'-projection $p_{A'} \colon X \to p_{A'}(X) \subseteq \prod_{\alpha \in A'} X_\alpha$ is a homeomorphism and $|X| \leqslant 2^\tau$.

Proof. By Theorem 3.2, there is a closed subspace $\tilde{X} \subseteq X$, $|\tilde{X}| \leqslant 2^\tau$, and a subset of indexes $\tilde{A} \subseteq A$, $|\tilde{A}| \leqslant 2^\tau$, such that $|p_{\tilde{A}}^{-1}(p_{\tilde{A}}(x))| = 1$ for every point $x \in \tilde{X}$, and $p_{\tilde{A}}(\tilde{X})$ is dense in $p_{\tilde{A}}(X)$. From the facts that the mapping $p_{\tilde{A}} \colon X \to p_{\tilde{A}}(X)$ is quotient, $\tilde{X} = p_{\tilde{A}}^{-1}(p_{\tilde{A}}(\tilde{X}))$, and $\tilde{X}$ is closed in X, it follows that $p_{\tilde{A}}(\tilde{X})$ is closed in $p_{\tilde{A}}(X)$. But the set $p_{\tilde{A}}(\tilde{X})$ is dense in $p_{\tilde{A}}(X)$. Hence, $p_{\tilde{A}}(\tilde{X}) = p_{\tilde{A}}(X)$, and therefore $\tilde{X} = X$. Since the mapping $p_{\tilde{A}} \colon X \to p_{\tilde{A}}(X)$ is one-to-one, the mapping $p_{\tilde{A}}$ is a homeomorphism X onto $p_{\tilde{A}}(X)$. Also we get $|X| \leqslant 2^\tau$. □

From Theorem 3.4, Theorem 3.5 follows.

Theorem 3.5. *If a Tychonoff space X is homeomorphic to a q-embedded subspace of a Tychonoff cube $\prod_{\alpha \in A} I_\alpha$, then $|X| \leqslant 2^{\chi(X)}$.*

Lemma 3.6. *Let a subspace of a Tychonoff cube $X \subseteq \prod_{\alpha \in A} I_\alpha$ be such that the A'-projection $p_{A'} \colon X \to p_{A'}(X) \subseteq \prod_{\alpha \in A'} I_\alpha$ is a closed mapping for all $A' \subseteq A$. Then $\chi(x, X) \leqslant \psi(x, X)$ for every point $x \in X$.*

Proof. Let $\psi(x, X) = \tau$ for a point $x \in X$. Then for x there is a characteristic set $A' \subseteq A$ such that $|A'| \leqslant \tau$, i.e., $p_{A'}^{-1}(p_{A'}(x)) = x$. The character of the point $p_{A'}(x)$ in the space $p_{A'}(X) \subseteq \prod_{\alpha \in A'} I_\alpha$ is not bigger than τ. Let $B = \{V\}$ be a base of the point $p_{A'}(x)$ in the space $p_{A'}(X)$ such that $|B| \leqslant \tau$. By the closedness of the mapping $p_{A'} \colon X \to p_{A'}(X)$, the family $\{p_{A'}^{-1}(V) \colon V \in B\}$ is a base of x in X and is as required. □

From Theorem 3.3 and Lemma 3.6, the next theorem follows.

Theorem 3.7. *Let a space $X \subseteq \prod_{\alpha \in A} I_\alpha$ be such that the A'-projection $p_{A'} \colon X \to p_{A'}(X) \subseteq \prod_{\alpha \in A'} I_\alpha$ is a closed mapping for all $A' \subseteq A$. Then $|X| \leqslant 2^{\psi(X)}$.*

If we change the conditions of projections of a subspace $X \subseteq \prod_{\alpha\in A} X_\alpha$, we get a theorem from which the version of Arhangel'skiĭ's theorem [1] for Lindelöf spaces, in particular, follows.

We call a continuous mapping $f\colon X \to Y$ *δ-quotient* if the following condition holds:

If $f^{-1}(A)$ is a closed subset of X, then A is G_δ-closed in Y.

Here, a subset $B \subseteq X$ of a topological space X is called G_δ-closed if B is closed in the G_δ-modification of a space X.

Theorem 3.8. *Let $X \subseteq \prod_{\alpha\in A} X_\alpha$, where weight $w(X_\alpha) \leqslant 2^\tau$ for all $\alpha \in A$, satisfy the following conditions:*

(1) *for all $A' \subseteq A$, the A'-projection $p_{A'}\colon X \to p_{A'}(X) \subseteq \prod_{\alpha\in A'} X_\alpha$ is a δ-quotient mapping;*
(2) *$\psi(X) \leqslant 2^\tau$; $t(X) \leqslant \tau$; $|[B]| \leqslant 2^\tau$ for all $B \subseteq X$ such that $|B| \leqslant 2^\tau$.*

Then $|X| \leqslant 2^\tau$.

The proof of this theorem is almost the same as those of Theorem 3.2 and Theorem 3.3. By induction, we construct families $\{A_\delta\colon \delta < \omega_{\tau^+}\}$, $\{S_\delta\colon \delta < \omega_{\tau^+}\}$, and $\{Y_\delta\colon \delta < \omega_{\tau^+}\}$. But we have to replace condition (e) in the proof of Theorem 3.2 with the condition

(e$'$) $[S_\alpha]_\delta = p_{A_\alpha}(X)$ where $[S_\alpha]_\delta$ is a closure of S_α in the G_δ-modification of $p_{A_\alpha}(X)$.

Corollary 3.9 (Arhangel'skiĭ [1])**.** *If X is a Lindelöf space, then $|X| \leqslant 2^{\chi(X)}$.*

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Department of Mathematics; Udmurt State University; Russia, Izhevsk
E-mail address: gryzlov@udsu.ru