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by

Alejandro Illanes

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Web:	http://topology.auburn.edu/tp/
Mail:	Topology Proceedings Department of Mathematics & Statistics Auburn University, Alabama 36849, USA
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CLOSED SUBSETS OF EUCLIDEAN SPACES CONTAINED IN PSEUDO-ARCS

ALEJANDRO ILLANES

Abstract. In this paper we prove that if K is a compact subset of the Euclidean space $\mathbb{R}^k$ ($k \geq 3$) with the property that every nondegenerate component of K is a pseudo-arc, then there exists a pseudo-arc P with $K \subset P \subset \mathbb{R}^k$.

1. Introduction

J. R. Kline and R. L. Moore proved [7] that, in the plane, a compact set M is a subset of an arc if and only if every component of M is either a one-point set or an arc α such that no point of α, except its end points, is a limit point of $M - \alpha$. In his dissertation, published in [3], H. Cook studied the corresponding problem for the pseudo-arc and proved that if K is a compact plane set, then there exists a pseudo-arc P with $K \subset P \subset \mathbb{R}^2$ if and only if each of the nondegenerate components of K is a pseudo-arc. H. Cook has conjectured that this result is also true for $\mathbb{R}^k$ if $k \geq 3$. This conjecture was stated in the paper by David P. Bellamy in [1].

In this paper we prove Cook's conjecture by showing that, if $k \geq 3$ and K is a compact subset of the Euclidean space $\mathbb{R}^k$, then there exists a pseudo-arc P such that $K \subset P \subset \mathbb{R}^k$ if and only if each nondegenerate component of K is a pseudo-arc.

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Key words and phrases. Chainability, Crooked Chain, Euclidean Space, Indecomposability, Pseudo-arc.

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