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FUNCTION SPACES AND L-PREORDERED SETS

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AND STEPHEN E. RODABAUGH

Abstract. In classical domain theory, the function-space constructor is the most interesting domain constructor. In this paper, we review some results involving the classical function-space constructor and the Scott topology, and then we begin to consider how these results could be extended if we replace preordered and partially ordered sets with L-preordered sets and L-partially ordered sets for a frame L. In this paper, we focus on L-preordered sets.

1. Introduction

One of the interesting issues in domain theory is how to define function-space domains so that they behave nicely with respect to the function-space constructor. Since the function-space constructor involves spaces of functions, cardinality issues may be problematic. A specific goal of this current study is to begin to examine conditions which could be applied in lattice-valued settings so that the function-space constructor would behave nicely.

There is a relatively rich literature on L-fuzzy preorders and L-fuzzy partial orders, where the L may be a lattice structure different from a frame; see, for example, [1, 6, 11, 12]. In this paper, when compared to Lai and Zhang [6], we focus on L-preorders instead of L-partial orders, though we do compare our L-antisymmetry condition to theirs. When compared to Yao and Shi [11], we work with traditional topologies instead of many-valued topologies even when beginning with many-valued orders. This may be considered an intermediate step as one transitions

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